Tests of Parity and Time-Reversal Noninvariance Using Neutron Interference

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Experiments are proposed, using neutron interference, to test possible parity and time-reversal noninvariance effects in the interaction of the neutron with electromagnetic and gravitational fields, including detection of its electric and gravitational dipole moments. These experiments would also test special relativity and general relativity at the quantum mechanical level.

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Since the discovery of parity nonconservation in the weak interaction,[1] many physicists have questioned whether parity (P), time-reversal (T), and charge-conjugation (C) symmetries are violated in the gravitational interaction,[2,3] which is even weaker than the weak interaction. Even prior to the experimental discovery of P noninvariance, Purcell and Ramsey[4] pointed out the possible existence of an electric dipole moment (EDM) for the neutron, which would violate P and T invariances.[5] Since then, there have been a number of experimental searches for the EDM of the neutron.[6]

The purpose of this Letter is to point out that the phase shift in the interference of two coherent beams due to an external field provides a method of testing the violation of discrete symmetries. More specifically, I shall propose a test of the EDM of the neutron and tests of P and T noninvariance in the gravitational interaction by means of neutron interference. The technique suggested is general enough to apply to other particles, such as the electron and the proton. But as will be readily seen, the neutron is preferable because it is electrically neutral. Also, the neutron undergoes electromagnetic, strong, weak, and gravitational interactions; so by using it, tests of symmetry violations in all four interactions are possible. These proposed experiments would also test special relativity and general relativity at the quantum mechanical level.

The proposed test of the EDM of the neutron consists of measuring the phase shift due to the interaction of one of the two interfering neutron beams with a homogeneous electric field $\vec{E}$ over a distance L of the beam. This is similar to the experiment of Werner *et al.* and Rauch *et al.*[7] in which the phase shift due to the coupling of the magnetic moment of the neutron to a magnetic field $\vec{B}$ was measured. But a crucial difference between the two experiments is that in the present one, the relativistic interaction energy of the magnetic moment with the magnetic field $c^{-1}\vec{E}\times\vec{v}$ experienced by the neutron in its rest frame is at least as large as the interaction energy of the EDM with the $\vec{E}$ field in the laboratory frame, where $\vec{v}$ is the velocity of the neutron relative to the laboratory frame. In order to compute the phase shift due to all of the above effects, I shall assume that at low energies the neutron may be described by a two-component wave function ψ which satisfies

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g_M\mu_N\vec{\sigma}\cdot\vec{B}\psi - \frac{i\hbar g_M\mu_N}{mc}\vec{\sigma}\cdot\vec{E}\times\nabla\psi + g_E\mu_N\vec{\sigma}\cdot\vec{E}\psi, \qquad (1)$$

where $\mu_N = e\hbar/4mc$, g_M, g_E are dimensionless constants, and σ_x, σ_y, and σ_z are Pauli spin matrices.

The second and third terms of the right-hand side of (1) represent the interaction of the magnetic moment with the magnetic field that the neutron experiences in its rest frame, while the fourth term represents the interaction of the EDM with the electric field. The latter term violates P and T invariance but respects rotational invariance. It should be noted that in the corresponding equation for the electron[8] the magnetic moment which couples to $\vec{B}$ is twice the value of the magnetic moment which couples to $\vec{E}\times\vec{v}/c$. This can be understood semiclassically as being due to the acceleration of the electron in the $\vec{E}$ field which results in the Thomas precession. Since the neutron is neutral, there is no such relative factor of 2 between the second and third terms.[9] The experiment being proposed here to detect the phase shift, which is based on (1), would also be a test of (1) and in particular of the relativistic term in it. Hence it may also be regarded as a test of special relativity at the quantum mechanical level. Equation (1) is consistent with all low-energy experiments with $g_M = -1.913$ and $|g_E| < 5.7\times10^{-10}$, the last inequality coming from the recent experiment of Dress *et al.*[6] The ratio of the last term to the preceding term is of the order of $|g_E c/g_M v| < 1.5\times10^{-2}$ even for ultracold neutrons ($v \sim 6$ m/sec). Hence the former term, though relativistic, is important in the present experiment. Equation (1) should also contain the term $(ig_E\mu_N\hbar/mc)\vec{\sigma}\cdot\vec{B}\times\nabla\psi$. But this is negligible for the present experiment.

I shall now compute the phase shift when homogeneous $\vec{E}$ and $\vec{B}$ fields are applied on one of the beams, using the WKB approximation. Consider a stationary solution $\psi_0(\vec{r})e^{-i\omega t}$ of (1). Let $\vec{k}_0$ be the wave vector in the absence of the fields so that $\omega = \hbar\vec{k}_0^2/2m$. Choose a coordinate system with the z axis in the direction of

$$\vec{F} \equiv g_M\vec{B} + (g_M\hbar/mc)\vec{E}\times\vec{k}_0 + g_E\vec{E} \tag{2}$$

and a representation in which

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Clearly

$$\psi_0 = \begin{pmatrix} A\exp(i\vec{k}_+\cdot\vec{r}) \\ B\exp(i\vec{k}_-\cdot\vec{r}) \end{pmatrix},$$

where $\vec{k}_+$ and $\vec{k}_-$ are parallel to $\vec{k}_0$ and A, B are slowly varying functions of position, is a solution provided that, to a good approximation, $\hbar\omega = \hbar^2 k_\pm^2/2m \pm \mu_N F$, where F is the component of $\vec{F}$ along the z axis so that $|F| = |\vec{F}|$. Spin-up and spin-down components undergo different phase shifts $\Delta\theta_+$ and $\Delta\theta_-$ given by

$$\Delta\theta_\pm = (k_\pm - k_0)L = \pm\mu_N F m L/\hbar^2 k_0 \tag{3}$$

to first order in F. As in the experiment of Rauch *et al.* and Werner *et al.*, the present experiment can be done with *unpolarized* neutrons, in which case the intensity in the interference region depends only on the magnitude of $\vec{F}$ and not on its direction.

A glance at (2) shows that by varying $\vec{E}$ and $\vec{B}$, g_M and g_E can be determined. Even though g_M can be determined to very high accuracy in this experiment, g_M has already been determined to an accuracy which far exceeds any theoretical prediction.[10] On the other hand, there are theoretical predictions of g_E which are beyond the sensitivity of the previous experiments.[6] In order to measure g_E it would be useful to eliminate the effect of the term $(g_M\hbar/mc)\vec{E}\times\vec{k}_0$ in (2). This can be eliminated, as in the original Ramsey experiments, by having a constant field $\vec{B}$ parallel to $\vec{E}$, so that the effect due to this term on $|\vec{F}|$ and hence on $\Delta\theta$ is second order in $\vec{E}$.[11] So not only is this contribution small, but also it does not change as $\vec{E}$ is reversed. On the other hand, the relativistic term can be measured by having $\vec{B}=0$ and varying $\vec{E}$ since, in this case, the contribution from the EDM would be at least 100 times smaller.

With the present techniques in neutron interference it may be possible to measure $\Delta\theta \sim 10^{-3}$ with some difficulty.[12] If the experiment proposed here is performed with the thermal neutrons used in the experiment of Werner *et al.*[7] ($k_0 = 4.348\times10^{10}$ m^{-1}), is done on a larger scale so that $L = 1$ m, and the applied electric field is the same as the experiment of Dress *et al.*[6] which was 10^7 V/m, then the upper limit $|\mu_E/e| < 0.9\times10^{-22}$ m, where $\mu_E = g_E\mu_N$, can be obtained. It is very likely, however, that neutron interference can be done with ultracold neutrons ($v_0 \sim 6$ m/sec) in the near future.[13] This would improve the upper limit to $|\mu_E/e| \lesssim 2\times10^{-25}$ m.

This projected upper limit is still about 70 times bigger than the upper limit already achieved by the experiment of Dress *et al.*[6] However, the sensitivity of the present experiment can be improved further by repeating the experiment for different values of the magnetic field $\vec{B}$. By varying $\vec{B}$, while $\vec{E}$ is kept constant, it is possible to have as many as 600 oscillations in the intensity of the recombined beam. If $\vec{E}$ is now reversed, this entire intensity pattern would shift. This shift, which is due to the phase shift associated with the reversal of $\vec{E}$, can then be detected with very high sensitivity. But such an experiment would still not be as sensitive as the corresponding experiment which uses the Ramsey technique. This is because in the latter experiment the frequency of an oscillating magnetic field is varied, and frequency measurements can be made to a much greater accuracy than the measurement of the spatial integral $\int\vec{B}\cdot d\vec{s}$, the variation of which produces the oscillations mentioned above.

So, let us consider the effect of subjecting one of the neutron beams to a constant magnetic field $\vec{B}_0$ and a rotating magnetic field $\vec{B}_1$, in the plane normal to $\vec{B}_0$, over a length L. Since the Hamiltonian now is time dependent, in general there would be beats in the recombined beam. However, in the special case where the beam entering the magnetic field region is polarized in the direction of $\vec{B}_0$, it can be shown that the intensity of the recombined beam is proportional to $1 + a\cos(\Delta k \times L) + b\sin(\Delta k L)$, where a and b are constants with $|a|,|b| \leq 1$ and

$$\Delta k = \frac{g_M \mu_N m}{\hbar^2 k_0} \left\{ \frac{B_1{}^2 + (B_0 - \hbar\omega/2g_M\mu_N)^2}{1 - m\omega/\hbar k_0{}^2} \right\}^{1/2},$$

ω being the frequency of B_1. Now the intensity as a function of ω can be experimentally determined to a high accuracy. The shift in this curve when $\vec{E}$ is turned on or reversed can therefore be determined with very high sensitivity. Because of the difficulty in achieving uniform magnetic fields over great lengths, it would be preferable to confine the oscillatory field to small regions at the beginning and end of the length over which $\vec{B}_0$ is nonzero, as in the Ramsey method.[14] Such an experiment would, in principle, be as sensitive as the Ramsey experiment.

I shall now consider a possible P- and T-nonconserving interaction with the gravitational field of a body of large mass M, such as the Earth, described by the wave equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \frac{GM\hbar}{2c}\vec{\sigma}\cdot\left(\alpha_1\frac{\vec{r}}{r^3} - i\frac{\alpha_2\hbar}{mcr^2}\nabla - i\frac{\alpha_3\hbar}{mcr^2}\vec{r}\times\nabla\right)\psi - i\alpha_4\frac{GM}{cr^2}\hbar\vec{r}\cdot\nabla\psi - \frac{GMm}{r}\psi, \tag{4}$$

where $\vec{r}$ is the position vector from the center of mass of the massive body and α_1, α_2, α_3, and α_4 are dimensionless constants which must be determined by experiment. Clearly the α_1 term violates P and T conservation while the α_2 and α_4 terms violate P conservation and T conservation, respectively. (If CPT symmetry is assumed, then the latter two terms violate C conservation as well.) In the classical limit (4) gives the interaction potential energy

$$U(r) = \alpha_1\frac{GM\vec{s}\cdot\vec{r}}{cr^3} + \alpha_2\frac{GM}{c^2}\frac{\vec{s}\cdot\vec{v}}{r^2} + \alpha_3\frac{GM}{c^2r^3}\vec{s}\times\vec{r}\cdot\vec{v} + \alpha_4\frac{GMm}{c}\frac{\vec{r}\cdot\vec{v}}{r^2},$$

which was phenomenologically proposed by Hari Dass,[3] in addition the usual Newtonian potential energy. Comparing (4) with (2), we note that the α_1 term is like a "gravitational dipole moment" interaction while the α_3 term is like the interaction of a "gravitational magnetic moment." This gravitational dipole moment may be regarded as being due to the separation of inertial and passive gravitational masses so that its detection would constitute a violation of the equivalence principle for particles with intrinsic spin.[3,15]

The phase shift due to the α_1, α_2, and α_3 terms can be determined in the WKB approximation to be

$$\Delta\varphi_\pm = \pm\frac{GMmL|\vec{H}|}{2c\hbar k_0}, \tag{5}$$

where L is the length of the interaction and

$$\vec{H} = \alpha_1\frac{\vec{r}}{r^3} + \alpha_2\frac{\hbar}{mcr^2}\vec{k}_0 + \alpha_3\frac{\hbar}{mcr^3}\vec{r}\times\vec{k}_0,$$

$\vec{k}_0$ being the approximate wave vector; the ± signs in (5) correspond to the spin-up and spin-down states relative to the direction of $\vec{H}$. If the interferometer is horizontal, then $\vec{H}$ is a sum of three mutually perpendicular vectors. Therefore, α_1, α_2, and α_3 may be determined by doing the experiment in turn with the neutrons polarized in each of three mutually perpendicular (or at least independent) directions and flipping the spin for one of the neutron beams at the beginning and back again at the end of a portion of its path of length L. This can be achieved by having a magnetic field parallel to the direction of polarization and having a suitable rotating magnetic field in the plane normal to this direction[14] at these two positions. The phase difference due to the α_4 term is $\alpha_4(GMm/c\hbar)\int\vec{r}\cdot d\vec{r}/r^2$, where the integral is along the beam. If the interferometer is in the shape of a parallelogram $ABCD$ with $\measuredangle BAD = \beta$ and it has been rotated an angle γ about a horizontal axis from an initial position in which AB and CD were vertical, then the phase shift is

$$\Delta\varphi_4 = \alpha_4(mAg/\hbar)\cos\gamma\cos\beta(1-\cos\beta)$$

to a very high approximation, where g is the acceleration due to gravity and A is the area enclosed by the interfering beams. Hence by varying γ, α_4 can be measured.

The experiments to observe the phase shifts due to the α_2, α_3, and α_4 terms, which are independent of k_0, may be performed with thermal neutrons. But it is advantageous to use ultracold neutrons for the α_1 term. Under the optimistic assumption that $\Delta\varphi \sim 10^{-3}$ rad is measurable and $A \sim 1$ m^2, these experiments would give the upper limits $\alpha_1 < 10^5$, $\alpha_2 < 10^{13}$, $\alpha_3 < 10^{13}$, and $\alpha_4 < 1.9 \times 10^{-3}$. The upper limit on α_1 would be much more stringent than the previous upper limit of $\alpha_1 < 10^{10}$ due to Leitner and Okubo.[2,3] However, Hari Dass has already concluded from the observed perihelion motion of mercury that $\alpha_4 < 10^{-4}$ for the classical limit of (4).[3] Nevertheless, the present experiment has the advantage that it would be a laboratory experiment and would test this effect at the quantum mechanical level. The tests proposed are also tests of general relativity[16] which predicts $\alpha_1 = \alpha_2 = \alpha_4 = 0$ and $\alpha_3 = \frac{1}{2}$. The phase shift due to the latter term has been considered previously[17] but the experiment which was proposed therein is less sensitive than the present one by a factor of 10^6.

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[9]It has been pointed out by J. Anandan [Phys. Rev. D 24, 338 (1981)] that there would be a phase shift in neutron interference due to the Thomas precession. But this is due to reflections of the beams and does not invalidate the present argument.

[10]See, for instance, articles by G. L. Greene and N. F. Ramsey, in *Fundamental Physics with Reactor Neutrons and Neutrinos*, edited by T. von Egidy, IOP Conference No. 42 (Institute of Physics, Bristol, 1978).

[11]I wish to thank Eugene D. Commins for pointing this out to me.

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