Baryon Masses in a Relativistic Quark-Diquark Model

D. B. Lichtenberg, W. Namgung, E. Predazzi,[a] and J. G. Wills
Physics Department, Indiana University, Bloomington, Indiana 47405
(Received 8 March 1982)

Masses of ground-state spin-$\frac{3}{2}$ baryons are calculated in a relativistic quark-diquark model with a potential motivated by QCD. The parameters of the model are determined by fitting vector-meson masses, and so the calculation for baryons contains no free parameters. Results are in rather good agreement with experiment. Moreover, starting from dynamical quark masses close to the so-called current masses, effective quark masses are obtained in the right range to be identified with constituent masses.

PACS numbers: 12.70.+q, 12.35.Ht, 14.20.-c

In his original paper on the quark model, Gell-Mann[1] raised the possibility of the existence of free diquarks. Later, diquarks were considered as bound constituents of baryons,[2,3] exotic mesons,[3-7] or other hadronic matter.[6-9] During recent years arguments have been given[6-14] for the existence of diquark substructure in baryons. Despite this fact, except for a nonrelativistic calculation of P- and D-state baryons,[15] little has been done in the way of actual quantitative calculations of baryon masses in a dynamical quark-diquark model. This is surprising, since the use of diquarks considerably reduces the mathematical difficulties by converting a three-body problem into a two-body one.

Here we consider a baryon to be composed of a quark and a diquark and calculate baryon masses by solving a relativistic wave equation for the motion of the quark and diquark in a phenomenological potential which depends only on their separation. The form and color dependence of the potential are motivated by considerations from quantum chromodynamics (QCD). The parameters of the potential and the input quark current masses are first determined from the meson sector. With no free parameters left, we calculate the properties of diquark states, and then, using the diquark masses found in the preceding step, we find the spectrum of quark-diquark (baryon) states. In this way a relativistic three-body problem is reduced to solving a two-body problem twice. In this introductory note we restrict ourselves to spin-independent interactions and illustrate the method by calculating the masses of the ground-state spin-$\frac{3}{2}$ baryons. We plan to include spin effects and calculate the masses of other baryons in a more complete investigation in the future.

Our results can be summarized as follows: (i) Using a potential containing two adjustable parameters, we obtain a good spectrum of vector mesons (including mesons containing only light quarks). (ii) Starting from current quark masses, we calculate effective quark masses which turn out to be quite close to the usual constituent masses. (iii) Without any adjustable parameters we calculate the absolute spectrum of the ground-state spin-$\frac{3}{2}$ baryons with surprising success.

First of all we discuss the framework in which these results are obtained. Although a nonrelativistic Schrödinger equation is adequate to treat mesons of the J/ψ and Υ families, which contain only the heavy c and b quarks, this equation, with a potential motivated by QCD, fails to give even an approximately correct spectrum of ordinary mesons containing only light quarks. A relativistic generalization is thus needed. One such possible generalization, which as far as we know is new, leads to a relativistic equation almost as simple to solve numerically as the Schrödinger equation. We obtain our wave equation as follows.

In the center-of-mass system, the relativistic expression for the total energy W of two free particles of masses m_1 and m_2 and three-momentum $\vec{p}$ is

$$W=(\vec{p}^{\,2}+m_1^{\,2})^{1/2}+(\vec{p}^{\,2}+m_2^{\,2})^{1/2}. \tag{1}$$

We denote by V an interaction which transforms like the fourth component of a four-vector and by S an interaction which transforms like a Lorentz scalar. Because the total energy transforms like the fourth component of a four-vector and the masses transform as scalars, it is most natural to incorporate the interactions V and S into (1) by making the replacements

$$W\to W-V,\quad m_1\to m_1+\tfrac{1}{2}S,\quad m_2\to m_2+\tfrac{1}{2}S, \tag{2}$$

where our reason for inserting the factors $\frac{1}{2}$ will be made clear shortly.

We can obtain a second-order differential equation from (1) by squaring twice and making the usual replacement $\vec{p} \to -i\nabla$. Then, by making the substitutions (2), we obtain the following wave equation:

$$\left\{-\nabla^2 - \frac{(W-V)^2}{4} + \frac{(m_1+\frac{1}{2}S)^2+(m_2+\frac{1}{2}S)^2}{2} - \frac{[(m_1+\frac{1}{2}S)^2-(m_2+\frac{1}{2}S)^2]^2}{4(W-V)^2}\right\}\psi = 0. \tag{3}$$

Because (3) is obtained from (1) by squaring twice, it admits spurious solutions, which are not solutions of (1). Except for the care we must take to exclude such solutions, (3) is as easy to solve numerically as the nonrelativistic Schrödinger equation. In the limit of large masses and small potentials V and S, (3) reduces to the nonrelativistic Schrödinger equation with a potential U given by $U=V+S$. This is the reason for the factors $\frac{1}{2}$ in (2).

The potential $U(r)$ we adopt in this paper[16,17] has a rather simple form in configuration space:

$$U(r) = -\vec{F}_1\cdot\vec{F}_2\left(\frac{6\pi}{27}\frac{(1-\lambda r)^2}{r\ln\lambda r} + S_0\right), \tag{4}$$

where $\vec{F}_1\cdot\vec{F}_2$ is a QCD color factor and λ and S_0 are the two parameters of the potential. The parameter λ is related to the QCD parameter $\Lambda_{\rm QCD}$ by[18] $\lambda = \Lambda_{\rm QCD}e^{\gamma}$, γ being Euler's constant. The potential is Coulomb-like at small r except that it is weakened by a logarithm in accordance with asymptotic freedom.[19] At large r the potential is linear except for a logarithm. Many other QCD-motivated potentials have been used in the literature, of which we can cite only a few.[18,20,21]

In using the potential (4) in (3), we must pay particular attention to its transformation properties. According to QCD, at small r the potential arises from one-gluon exchange, and so transforms like V. However, because of the form of (3), a potential transforming like V cannot lead to confinement. The reason is that if V goes to ∞ as $r\to\infty$, then the dominant term in (3) is $-\frac{1}{4}V^2$, which goes to $-\infty$ and so is not confining. We can get confinement, however, if we assume that at large r the potential transforms like S. It is therefore natural to split the U of (4) into $V+S$ as follows:

$$V = -\vec{F}_1\cdot\vec{F}_2(6\pi/27)(1-\lambda r)/(r\ln\lambda r), \tag{5}$$

$$S = -\vec{F}_1\cdot\vec{F}_2[(6\pi/27)\lambda(\lambda r-1)/\ln\lambda r + S_0]. \tag{6}$$

Analogous splittings were previously used[22,23] with Todorov's wave equation.[24]

We can use (5) and (6) in (3) to calculate a mass spectrum of meson, diquark, or baryon states. The QCD color factor $\vec{F}_1\cdot\vec{F}_2$ is $-\frac{4}{3}$ for a quark and antiquark bound in a meson, $-\frac{2}{3}$ for two quarks bound in a (color-antitriplet) diquark, and $-\frac{4}{3}$ for a quark and a diquark bound in a baryon.

Our detailed procedure is as follows. We consider first the J/ψ and ψ' mesons, adjusting λ and the linear combination $m_c+\frac{2}{3}S_0$, where m_c is the mass of the c quark, until we obtain the experimental values of the J/ψ and ψ'. This leads to $\lambda = 731.3$ MeV or $\Lambda_{\rm QCD} = 411$ MeV. We then consider the ρ meson with the same value of λ, neglecting the mass difference between the u and d quarks. We find that in order to obtain the experimental mass of the ρ, we must set $m_c+\frac{2}{3}S_0 = -65$ MeV. We next set the u and d quark masses at 5 MeV, the average of the current quark masses[25] of u and d. This gives $S_0 = -105$ MeV and in addition the value of m_c. Then, from the experimental masses of the φ and Υ mesons, respectively, we obtain the masses of the s and b quarks. The quark input masses obtained in this way are very close to the usual quark current masses, as shown in Table I. Also shown in Table I are effective quark masses $m_{\rm eff}$ obtained from the formula

$$m_{\rm eff} = m + \tfrac{1}{2}\langle S\rangle, \tag{7}$$

where $\langle S\rangle$ is the expectation value of the scalar potential in the state considered. With this definition, the effective mass of a given quark, which we identify with the constituent quark mass of the literature,[26] varies from state to state, being somewhat larger in excited states than in the ground state. Also, the effective mass of a given quark decreases if the mass of the antiquark with

TABLE I. Calculated current and effective (constituent) quark masses from meson data, with neglect of the difference between the masses of the u and d quarks.

	$m_{\rm current}$ (MeV)	$m_{\rm eff}$ (MeV)
u	5 (input)	360–435
d	5 (input)	360–435
s	177	530–590
c	1407	1700–1785
b	4783	4995–5140

which it is bound increases. In Table I we give a range of calculated effective quark masses obtained from ground-state wave functions containing different antiquarks. Effective quark masses determined from baryons turn out 30 to 40 MeV lower and are therefore closer to the usual values of the constituent quark masses determined from the properties of baryons.[26]

Once the quark masses and potential parameters λ and S_0 are fixed, then without any new parameters, we can calculate additional vector-meson masses. We find that the agreement with experiment[27-29] is rather good. For example, except for the excited ρ states (for which a long-standing experimental problem exists), the error never exceeds 2% for S states.

Using the same parameters found in the meson case and the appropriate color factor, we next calculate the masses of spin-one diquarks. The ground-state diquark masses turn out to be similar to the meson masses, but in general the spacing between diquark levels is less. With the calculated diquark masses we can use our procedure once more and predict the masses of baryons, again without any new parameters.

Before presenting our results, we make some cautionary statements. (i) Because the parameters of the potential were fixed from spin-one mesons, the potential is appropriate for baryons in which quarks have their spins aligned. This means that we can calculate the masses of spin-$\frac{3}{2}$ baryons, but not of the spin-$\frac{1}{2}$ baryons until we make the potential explicitly spin dependent. (ii) In our model we can calculate the masses of most baryons in more than one way. For example, to calculate the mass of the Σ^{*+}, which is composed of (uus), we can first form a (uu) diquark which then interacts with an s quark, or first form a (us) diquark which then interacts with a u quark. In principle, these methods give different masses, but somewhat surprisingly, we get remarkably similar results using these two methods. For example, for baryons containing only u, d, and s quarks, the different configurations give mass values which agree within 1 MeV. For baryons containing two light quarks (u or d) and one c quark, different configurations vary in mass by 18 MeV, a difference of less than 1%.

But there is also a question of principle: In a baryon containing quarks with more than one flavor, we can ask which two quarks form the diquark. In our model the diquark is not elementary but a dynamical cluster of two quarks.

TABLE II. Comparison of experimental baryon masses with theoretical masses calculated without any adjustable parameters.

Baryon	Theory (MeV)	Exp.[a] (MeV)
Δ	1282	1232
Σ^*	1397	1385
Ξ^*	1511	1530
Ω	1623	1672
Σ_c^*	2450	2430[b]

[a]Reference 27.

[b]This may be the mass of the Σ_c of spin $\frac{1}{2}$ rather than the mass of the Σ_c^* of spin $\frac{3}{2}$.

Under these circumstances it is plausible that, on the average, any pair of quarks is equally likely to comprise the diquark. We take this to be the case and calculate baryon properties by averaging over all possible diquark configurations.[30]

With these caveats, we compare in Table II the predictions of the model with the observed masses[27] of the ground-state spin-$\frac{3}{2}$ baryons. The fact that the predicted mass of the Δ comes out a little high is understandable because coupling to the decay channel, which we have not considered, tends to lower the mass.[31] The poorest agreement is in the case of the Ω^-, where the predicted value is 3% lighter than the observed value. Overall, we regard the agreement as quite good, considering the fact that we have no adjustable parameters and have not included any explicit spin dependence in the potential.

We believe that the generally satisfactory agreement with experiment indicates that our relativistic treatment and our phenomenological potential are a sound tool for an overall description of hadron spectroscopy. The more promising aspect of our analysis, however, lies in the exciting future possibilities opened up by the success of the quark-diquark picture of baryons. We have, in fact, a simple prescription which reduces the complexity of a relativistic three-body problem by approximating it with a double-step two-body problem. Admittedly, however, we are still a long way from obtaining the detailed results for baryons achieved by the cumulative work of many authors within the framework of the three-body problem.[32]

This work was supported in part by the U. S.

Department of Energy.

[a] On leave of absence from the Istituto di Fisica Teorica, University of Torino, 10125 Torino, Italy.

[1] M. Gell-Mann, Phys. Lett. 8, 214 (1964).

[2] M. Ida and R. Kobayashi, Prog. Theor. Phys. 36, 846 (1966).

[3] D. B. Lichtenberg and L. J. Tassie, Phys. Rev. 155, 1601 (1967); D. B. Lichtenberg, L. J. Tassie, and P. J. Kelemen, Phys. Rev. 167, 1535 (1968); D. B. Lichtenberg, Phys. Rev. 178, 2197 (1969).

[4] S. K. Bose, Nuovo Cimento 46, 419 (1966); S. K. Bose and E. C. G. Sudarshan, Phys. Rev. 162, 1396 (1967).

[5] C. Rosenzweig, Phys. Rev. Lett. 36, 697 (1976); A. W. Hendry and I. Hinchliffe, Phys. Rev. D 18, 3453 (1978).

[6] R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977), and Phys. Rev. D 17, 1444 (1978).

[7] H.-M. Chan and H. Høgaasen, Phys. Lett. 72B, 121, 400 (1977).

[8] D. B. Lichtenberg, E. Predazzi, D. H. Weingarten, and J. G. Wills, Phys. Rev. D 18, 2569 (1978); D. B. Lichtenberg and R. J. Johnson, Hadronic 2, 1 (1979).

[9] P. J. G. Mulders, A. T. M. Aerts, and J. J. De Swart, Phys. Rev. Lett. 40, 1543 (1978); M. Fukugita, K. Konishi, and T. H. Hansson, Phys. Lett. 74B, 261 (1978).

[10] R. E. Cutkosky and R. E. Hendrick, Phys. Rev. D 16, 786–793 (1977).

[11] L. F. Abbott, E. L. Berger, R. Blankenbecler, and G. L. Kane, Phys. Lett. 88B, 157 (1979).

[12] A. Donnachie, Z. Phys. C 4, 161 (1980); A. Donnachie and P. V. Landshoff, Phys. Lett. 95B, 437 (1980).

[13] P. J. Litchfield, in *Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, 1974,* edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975), and in *Proceedings of the Topical Conference on Baryon Resonances, Oxford, England, 1976,* edited by R. T. Ross and D. H. Saxon (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1976).

[14] M. Zralek *et al.*, Phys. Rev. D 19, 820 (1979); Z. Dziembowski, W. J. Metzger, and R. T. Van de Walle, to be published.

[15] G. R. Goldstein and J. Maharana, Nuovo Cimento 59A, 393 (1980).

[16] G. Fogleman, D. B. Lichtenberg, and J. G. Wills, Lett. Nuovo Cimento 26, 369 (1979).

[17] B. Margolis, R. Roskies, and N. DeTakacsy, in *Proceedings of the Fourth European Antiproton Symposium,* edited by A. Friedman (Editions du CNRS, Strasbourg, 1978), Vol. 2, p. 454.

[18] W. Celmaster, H. Georgi, and M. Machacek, Phys. Rev. D 17, 879 (1978).

[19] H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973).

[20] E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, Phys. Rev. Lett. 34, 369 (1975).

[21] J. Richardson, Phys. Lett. 82B, 272 (1979).

[22] H. Crater and P. Van Alstine, Phys. Lett. 100B, 166 (1981).

[23] D. B. Lichtenberg, W. Namgung, and J. G. Wills, Indiana University Report No. IUHET 71, 1982 (unpublished).

[24] I. T. Todorov, Phys. Rev. D 3, 2351 (1971).

[25] S. Weinberg, in *A Festschrift for I. I. Rabi,* edited by L. Motz (New York Academy of Science, New York, 1977), p. 185.

[26] J. Franklin, Phys. Rev. 172, 1807 (1968); A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[27] R. L. Kelly *et al.* (Particle Data Group), Rev. Mod. Phys. 52, S1 (1980).

[28] D. Schamberger, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn, 1981,* edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn, 1981).

[29] B. Delcourt, in Ref. 28.

[30] See, however, I. Bediaga, E. Predazzi, and A. F. S. Santoro, Phys. Lett. 105B, 71 (1981).

[31] D. B. Lichtenberg and J. G. Wills, Phys. Lett. 100B, 87 (1981).

[32] The three-body calculations of course contain adjustable parameters. A good discussion and many references are contained in *Proceedings of the Fourth International Conference on Baryon Resonances, Toronto, 1980,* edited by N. Isgur (University of Toronto, Toronto, 1980).