Sine-Gordon Soliton Dynamics

In recent papers[1,2] Fernandez and co-workers have shown that sine-Gordon solitons do not behave like Newtonian particles under the influence of a constant force. The results are obtained numerically and show that the soliton acceleration starts up very slowly ($\sim t^2$) and not with a finite value as a Newtonian behavior would require. The results are supported by a perturbation treatment based on a theory by Fogel *et al.*[3,4] On the other hand previous perturbation treatments[3-6] and experiments *in numero*[7-9] show that the sine-Gordon soliton under the influence of a force behaves like a Newtonian particle. In fact the perturbation results very precisely describe the soliton motion.

Both results are correct. The apparent contradiction is due to differences in the vacuum "motion" in the two cases, i.e., the behavior of solutions at $x = \pm\infty$. In Refs. 1 and 2 the vacuum is in an excited state while the Newtonian behavior is observed with the vacuum in its ground state.

The influence of the vacuum on the soliton motion can easily be seen from Eq. (4) of Ref. 1:

$$\gamma(t)V(t) = \tfrac{1}{4}\pi(\chi t - du_\infty/dt), \tag{1}$$

where $V(t)$ is the velocity of the soliton, $\gamma(t)^{-1}$ is the width of the soliton $\{$which for small perturbations is $[1 - V(t)^2]^{1/2}\}$, χ is the "force" term in the perturbed sine-Gordon equation

$$u_{xx} - u_{tt} = \sin u - \chi, \tag{2}$$

and $u_\infty(t)$ is the vacuum part of the sine-Gordon field $u(x, t)$, u_∞ *only* depending on time, i.e., u_∞ alone is a solution to Eq. (2).

Equation (1) clearly shows that the vacuum "motion" through du_∞/dt influences the motion of the soliton. If the vacuum is in its ground state $u_\infty = \sin^{-1}\chi$, then $du_\infty/dt = 0$ for all times, yielding the results of Refs. 3-9. On the other hand, if the vacuum is started with $u_\infty = 0$ and $du_\infty/dt = 0$, du_∞/dt starts up as χt and thus eliminates the direct action of the χ term in Eq. (2), yielding the results of Refs. 1 and 2. Other initial conditions of the vacuum would give other initial motion of the soliton.

We have checked the results obtained above numerically[10] and found good agreement between Eq. (1) and the numerically determined velocities for the different initial conditions of the vacuum. Various numerical definitions of the velocity have been used and they all show the same qualitative agreement with the theory.[10]

O. H. Olsen
M. R. Samuelsen
Physics Laboratory I
The Technical University of Denmark
DK-2800 Lyngby, Denmark

Received 13 October 1981
PACS numbers: 03.20.+i

[1] J. C. Fernandez, J. M. Gambaudo, S. Gauthier, and G. Reinisch, Phys. Rev. Lett. 46, 753 (1981).

[2] G. Reinisch and J. C. Fernandez, Phys. Rev. B 24, 835 (1981).

[3] M. B. Fogel, S. E. Trullinger, A. R. Bishop, and J. A. Krumhansl, Phys. Rev. Lett. 36, 1411 (1976), and 37, 314 (1976).

[4] M. B. Fogel, S. E. Trullinger, A. R. Bishop, and J. A. Krumhansl, Phys. Rev. B 15, 1578 (1977).

[5] D. J. Kaup and A. C. Newell, Proc. Roy. Soc. London, Ser. A 361, 413 (1978).

[6] D. W. McLaughlin and A. C. Scott, Phys. Rev. A 18, 1652 (1978).

[7] P. L. Christiansen and O. H. Olsen, Wave Motion 2, 185 (1980).

[8] O. H. Olsen and M. R. Samuelsen, Phys. Scripta 23, 1033 (1981).

[9] O. H. Olsen and M. R. Samuelsen, Phys. Rev. B 25, 3181 (1982).

[10] O. H. Olsen and M. R. Samuelsen, to be published.