photoionization of the H^- ion and the data are consistent with a Wannier threshold law over the 0.3-eV region above threshold. We can discriminate against a simple linear law but we cannot tell the difference between the Wannier and the modulated linear law because of the field-ionization background. The present experiment has provided us with limits on the parameters which appear in these hypotheses. These limits can serve as a guide to theory and to the design of second-generation experiments.

We have had stimulating discussions with U. Fano, A. R. P. Rau, and A. Temkin. We are indebted to T. H. Shields, R. Marchini, and R. Dunn for their assistance with the data acquisition and analysis. We thank the LAMPF staff for their help during the assembly and running of the experiment. This work was supported by the U. S. Department of Energy, in part by Contract No. DE-AS04-77ER0-3998.

[(a)]Present address: Cyclotron Corporation, 950 Gilman St., Berkeley, Cal. 94710.

[(b)]Present address: Sandia National Laboratories, Albuquerque, N. Mex. 87185.

[1]G. H. Wannier, Phys. Rev. 90, 817–825 (1953).

[2]A. Temkin, J. Phys. B 7, L450–453 (1974).

[3]A. R. P. Rau, Phys. Rev. A 4, 207–220 (1971).

[4]J. William McGowan and E. M. Clarke, Phys. Rev. 167, 43–51 (1968).

[5]C. E. Brion and G. E. Thomas, Phys. Rev. Lett. 20, 241–243 (1968).

[6]P. Marchand, C. Paquet, and P. Marmet, Phys. Rev. 180, 123–132 (1969).

[7]S. Cvejanovic and F. H. Read, J. Phys. B 7, 1841–1852 (1974).

[8]F. Pichou, A. Huetz, G. Joyez, and M. Landau, J. Phys. B 11, 3683–3692 (1978).

[9]H. Klar and W. Schlecht, J. Phys. B 9, 1699–1711 (1976).

[10]Chris H. Greene and A. R. P. Rau, Phys. Rev. Lett. 48, 533–537 (1982).

[11]H. C. Bryant *et al.*, Phys. Rev. Lett. 38, 228 (1977).

[12]H. C. Bryant *et al.*, At. Phys. 7, 29–63 (1981).

[13]H. C. Bryant *et al.*, to be published.

[14]Charles Alan Frost, Los Alamos National Laboratory Report No. LA-8976-T, 1981 (unpublished).

[15]D. A. Clark *et al.*, Bull. Am. Phys. Soc. 25, 1137 (1980).

[16]John T. Broad and William P. Reinhardt, Phys. Rev. A 14, 2159–2173 (1976).

[17]F. James and M. Roos, Comput. Phys. Commun. 10, 343–367 (1975).

Instabilities, Self-Oscillation, and Chaos in a Simple Nonlinear Optical Interaction

Y. Silberberg and I. Bar Joseph

Department of Electronics, The Weizmann Institute of Science, Rehovot 76100, Israel

(Received 17 March 1982)

It is shown that two light beams interacting in a third-order nonlinear medium undergo transition from a stationary to periodic and chaotic states, as their intensities are increased. A threshold for the onset of instabilities is calculated and verified by computer simulations. It is therefore proved that external feedback is not necessary for self-oscillations in nonlinear optical systems.

PACS numbers: 42.65.Bp

The stability of nonlinear optical systems has been the subject of an intense study lately, especially since Ikeda[1] has shown that a Fabry-Perot ring resonator containing a saturable absorber can show instability in regions considered stable before. Later Ikeda, Daido, and Akimoto[2] proved that a ring resonator which contains a third-order dispersive nonlinear medium undergoes successive bifurcations as the incident power is increased, leading to chaos or "optical turbulence." Similar behavior was later predicted for standing-wave resonators[3] and for distributed-feedback resonators.[4] Gibbs *et al.*[5] recently demonstrated the main features of these predictions in a synthesized Fabry-Perot resonator.

In all these works the external feedback, supplied by the mirrors of the resonators, was necessary for the onset of self-oscillations. In this Letter we show that self-oscillations and chaos can be obtained in an optical system without any external feedback. Specifically, we consider two monochromatic waves interacting in a third-order dispersive nonlinear medium. A steady-state solution always exists, according to which the two waves travel through the medium without any exchange of power. However, above a certain threshold intensity this steady-state solution is unstable. The continuous output is replaced by strong oscillations in the outgoing waves, and eventually by chaos. The fact that all these phenomena are present in such a simple system suggests that instabilities, self-oscillations, and chaos are fundamental processes in nonlinear optics. Moreover, since the configuration of two waves interacting in a nonlinear material is one of the most frequent in laser physics and nonlinear wave mixing experiments the importance of this finding is evident.

We consider the counterpropagating wave geometry of Fig. 1(a), although our treatment will trivially hold for the geometry of Fig. 1(b) by redefining $z \to z/\cos\theta$. The two fields are taken to be of the form

$$E_1 = A_1(z,t)\exp[i(\omega t - kz)] + \text{c.c.}$$

and

$$E_2 = A_2(z,t)\exp[i(\omega t + kz)] + \text{c.c.},$$

with constant input amplitudes of $I_1^{1/2}$ and $I_2^{1/2}$, respectively. The third-order nonlinear susceptibility is assumed to be real and to obey a Debye relaxation equation:

$$\tau\dot{\chi}_{\rm NL} + \chi_{\rm NL} = \alpha\langle E^2\rangle, \qquad (1)$$

where $\langle\,\rangle$ denotes a time average, α is a constant, and τ is the medium response time. Equation (1) applies directly to most Kerr media, and approximately to a two-level atom far from resonance. With use of the slowly varying amplitude approximation, Maxwell equations yield

$$\frac{\partial A_1}{\partial z} + \frac{n_0}{c}\frac{\partial A_1}{\partial t} = -i\beta(H_{11}+H_{22})A_1 - i\beta H_{12}A_2,$$
$$-\frac{\partial A_2}{\partial z} + \frac{n_0}{c}\frac{\partial A_2}{\partial t} = -i\beta(H_{11}+H_{22})A_2 - i\beta H_{12}{}^*A_1, \qquad (2)$$

where $\beta = 2\pi\omega\alpha/cn_0$ and

$$H_{ij} \equiv \frac{1}{\tau}\int_{-\infty}^{t} A_i(z,t')A_j{}^*(z,t')\exp\left(\frac{t'-t}{\tau}\right)dt'. \qquad (3)$$

A time-independent solution for the set (2) is easily obtained:

$$A_1{}^0(z) = I_1{}^{1/2}\exp[-i\beta(I_1+2I_2)z + i\varphi_1],$$
$$A_2{}^0(z) = I_2{}^{1/2}\exp[+i\beta(I_2+2I_1)z + i\varphi_2]. \qquad (4)$$

The phases φ_1 and φ_2 are determined by the initial conditions. Note that the intensities $I_i = A_iA_i{}^*$ are constant and the two waves do not interchange energy. The mutual influence of the two fields is only through the phase term which is identical for equal inputs. Application of this phenomenon has been recently proposed for enhancing the sensitivity of Sagnac interferometers.[6]

We now claim that the steady-state solution (4) is instable for large input fields. To prove this point we look for a perturbed time-dependent amplitude of the form

$$A_i(z,t) = A_i{}^0(z)[1 + \epsilon F_i(z)e^{\lambda t} + \epsilon G_i{}^*(z)e^{\lambda^* t}], \quad i=1,2, \qquad (5)$$

with the boundary conditions $F_1(0) = G_1(0) = F_2(L) = G_2(L) = 0$. The form of (5) is necessary because the interaction mixes conjugate terms. Substituting Eq. (5) into Eq. (2) and linearizing, we get a system of equations for the perturbations F_i, G_i. These equations, together with the boundary condition, set a condition on λ. Obviously, $\text{Re}(\lambda) > 0$ describes a diverging perturbation and therefore unstable solution. Figure 2 depicts the threshold intensity at which $\text{Re}(\lambda) = 0$, as a func- of the relaxation time τ. Equal input intensities were assumed. The region below the line is characterized by a stable steady-state solution, while that in the shaded region is self-oscillating

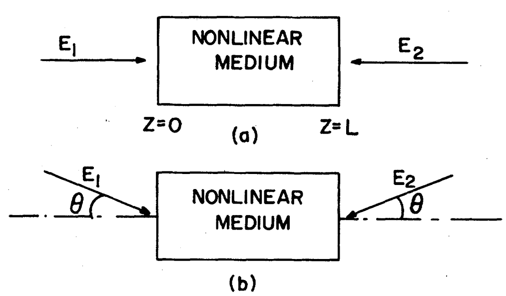

FIG. 1. Two waves interacting in a nonlinear medium. (a) Counterpropagating beams. (b) Noncollinear configuration.

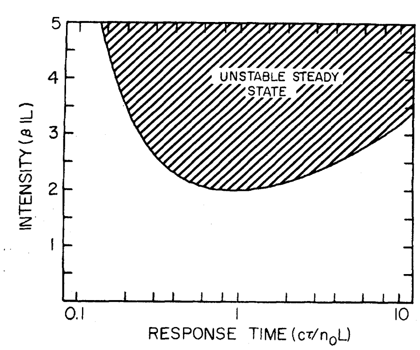

FIG. 2. Instability threshold intensity vs response time of the medium. The stationary solution is unstable in the shaded region.

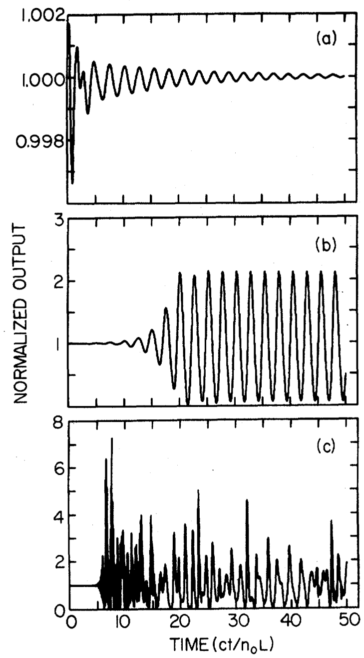

FIG. 3. Intensity vs time for one of the outgoing waves in a medium with $\tau = n_0L/c$, for (a) $\beta IL = 1.9$, (b) $\beta IL = 3$, and (c) $\beta IL = 10$. The initial conditions were the steady-state solution with a small random noise. Note the different vertical scales.

in spite of the constant input. Instabilities are easier to get when the relaxation time is equal to the transit in the medium. In order to gain a better understanding of the time-dependent solution, we have solved Eq. (2) numerically. The solutions for three different input intensities are given in Fig. 3 for the case $\tau = n_0L/c$. In all cases the initial condition was the steady-state solution (4), to which a small random noise of the order of 10^{-3} or less was added. It is clear that for intensities below the instability threshold, the perturbation is damped (note the scale). Above it, stable strong oscillations are formed, and, eventually, chaos prevails.

The physical source of this phenomenon can be traced to the combined effect of two processes: (i) When two light beams of different frequencies interact, the lower-frequency beam is enhanced, and its gain is maximal for a frequency which is shifted down by the reciprocal of the response time. Thus a sideband of one beam can experience gain by interacting with the counterpropagating beam. (ii) Sidebands can be reflected by the grating formed by the main frequency components if they are within the bandwidth of about c/n_0L which characterizes that grating. These processes provide the necessary gain and feedback which explain the buildup of oscillations at a sideband. The observed optimal conditions for oscillations, i.e., transit time equal to the response time, as well as the oscillation period, can thus be described qualitatively by these processes. Preliminary studies show that as the intensity is increased, bifurcations and period doubling are encountered just as in a device having an external feedback.[2]

Experimental observation of the phenomena described above can be performed in systems of atomic vapors. It is evident from Fig. 2 that a system with a transit time approximately equal to its atomic lifetime is optimal. An experiment can be most conveniently done with only one input beam, which is back reflected using a mirror. The system is then fully equivalent to a two-input configuration with twice its length. The intensity required for the onset of instability, i.e., $\beta IL > 2$, is of the same order of magnitude as that required for obtaining gain in degenerate four-wave mixing ($\beta IL > \pi/4$), an experiment which has been demonstrated several times in Na vapor.[7]

Discussions with S. Shtrikman, A. A. Friesem, and D. Bensimon are gratefully acknowledged.

[1] K. Ikeda, Opt. Commun. 30, 257 (1979).
[2] K. Ikeda, H. Daido, and O. Akimoto, Phys. Rev. Lett. 45, 709 (1980).
[3] W. J. Firth, Opt. Commun. 39, 343 (1981).
[4] H. G. Winful and G. D. Cooperman, Appl. Phys. Lett. 40, 298 (1982).
[5] H. M. Gibbs, F. A. Hopf, D. L. Kaplan, and R. L. Shoemaker, Phys. Rev. Lett. 46, 474 (1981).
[6] A. E. Kaplan and P. Meystre, Opt. Lett. 6, 591 (1981).
[7] D. M. Bloom, P. F. Liao, and N. P. Economou, Opt. Lett. 2, 58 (1978); R. C. Lind and D. G. Steel, Opt. Lett. 6, 554 (1981).