Collective Focusing of an Intense Ion Beam

Scott Robertson
Department of Physics University of California, Irvine, California 92717
(Received 10 November 1981)

The space-charge coupling between the electrons and ions composing an intense, charge-neutralized ion beam is shown theoretically and experimentally to result in both species being brought to a common focus by a solenoidal magnetic lens. The collective focal length is the geometric mean of the focal lengths of the two species calculated independently. Upper and lower limits on the electron density allowed with the lens are derived.

PACS numbers: 52.90.+z, 07.77.+p, 29.25.Fb, 41.80.Gg

Advances in technology have led to the creation of intense beams of ions having charge densities significantly above the usual space-charge limit.[1] These ion beams carry with them an equal number of electrons and therefore may be described as moving plasmas which satisfy the quasineutrality condition. In this Letter it is shown that, as a result of quasineutrality, a solenoidal magnetic lens brings both species in such a beam to a common focus although their charge-to-mass ratios differ in sign and magnitude. The field required for a given focal length is shown to be significantly less than the field required for the lens to focus ions traveling alone.

This collectively focusing lens is similar to the lens proposed by Gabor[2-4] in which a cloud of electrons confined in a magnetic mirror creates an electrostatic field which focuses an ion beam. Gabor showed that the electrostatic focusing obtainable from the maximum allowed electron density was significantly greater than the focusing obtained from the magnetic field acting alone. The lens described here differs in that (1) the incident ion beam is space-charge neutralized and (2) the required surplus of negative charge at the center of the lens is due to the "squeezing" of the neutralizing electrons by the magnetic field rather than due to prefilling. A novel high-current ion accelerator has recently been proposed which uses electron space charge to focus the ions.[5,6] The lens described here might also be used in the design of high-current accelerators and may have applications in the area of inertial confinement fusion.

Consider a short solenoidal field B_z which is uniform except for a short distance at each end where B_z falls rapidly to zero and where there is also a radial field component. An electron entering the field parallel to the axis will be given an angular velocity $r\dot{\theta}$, where r and θ are cylindrical coordinates aligned with the beam axis. Conservation of canonical angular momentum requires that $\dot{\theta} = -\frac{1}{2}\Omega_e = -\frac{1}{2}eB_z/m_e$, where e is the electron charge and m_e is the electron mass.[7] The radial equation of motion inside the region of uniform field is

$$m_e(\ddot{r} - r\dot{\theta}^2) + eE_r + er\dot{\theta}B_z = 0 \qquad (1)$$

or

$$\ddot{r} + \tfrac{1}{4}r\Omega_e^{\,2} + (e/m_e)E_r = 0\,, \qquad (2)$$

where E_r is the radial electric field due to the charge of the beam. For a tenuous beam ($E_r \approx 0$) the solution of Eq. (2) is $r(t) = r_0\cos(\frac{1}{2}\Omega_e t)$ and $\dot{r}(t) = -\frac{1}{2}r_0\Omega_e\sin(\frac{1}{2}\Omega_e t)$, where r_0 is the initial electron radius. In a typical magnetic lens for electrons, the time within the lens is much shorter than $\Omega_e^{\,-1}$ so that the electrons are not focused within the lens but instead drift to a focus outside the lens as a result of the radially inward velocity which they acquire.

The equation of motion for ions is

$$\ddot{r} + \tfrac{1}{4}r\Omega_i^{\,2} - (e/m_i)E_r = 0. \qquad (3)$$

If a beam composed equally of electrons and ions is sufficiently dense, the maintenance of charge neutrality will require that the radial motions of the electrons and ions be identical. In this case the motion is simply given by the simultaneous solutions of Eqs. (2) and (3) which are

$$\ddot{r} + \tfrac{1}{4}r\Omega_i\Omega_e = 0, \qquad (4)$$

$$eE_r = -\tfrac{1}{4}(m_i - m_e)r\Omega_i\Omega_e. \qquad (5)$$

Note that Eq. (4) is the same as the equation of motion of a tenuous beam of particles having a charge-to-mass ratio of $e/(m_i m_e)^{1/2}$. Thus the collective focal length is the geometric mean of the focal lengths for the two species calculated independently. For a weak lens in which the radial position of the particles does not vary significantly within the lens, this collective focal length is $F \approx -v_z r_0/\dot{r}(t=L/v_z) \approx 4v_z^2/\Omega_i \Omega_e L$, where v_z is the axial beam velocity and L is the length of the lens.

The assumption that the two species travel together is equivalent to the inequality $n_i - n_e \ll n_e$, where n_i and n_e are the ion and electron charge densities, respectively. From Eq. (5) and Poisson's equation it is easily shown that this relation is equivalent to $\omega_{pe}^2 \gg \frac{1}{2}\Omega_e^2$. This places a lower limit on the allowed beam density which depends on the magnetic field strength. It has also been assumed that the applied magnetic field is not perturbed by the beam. This requires that $\Delta B_z/B_z \ll 1$, where ΔB_z is the perturbation in the applied field due to the azimuthal current density $J = \frac{1}{2} n_e e r \Omega_e$. This relation is easily shown to be equivalent to $(c/\omega_{pe})^2 \gg (\frac{1}{2}r_0)^2$, or to the obvious statement that the magnetic skin depth must be much longer than the beam radius. The last inequality places an upper limit on the charge density allowed within the lens. This limit can, of course, be exceeded at the focus if this point lies outside the lens.

The apparatus in which this collective focusing effect was verified is shown schematically in Fig. 1. A charge-neutralized proton beam is generated by an annular (11.25 cm i.d.×15 cm o.d.) magnetically insulated ion diode connected to a Marx generator. The diode voltage [Fig. 2(a)] rises initially to 180 kV and then decays approximately linearly to zero in 700 nsec. The peak ion current density at the diode [Fig. 2(b)] is 10–20 A/cm^2. The beam propagates initially in a 15 cm diam×80 cm aluminum drift tube where the current density is reduced to $\lesssim 0.5$ A/cm^2 because of divergence and velocity dispersion. At this lower current density, the condition $c/\omega_{pe} > \frac{1}{2} r_0$ is marginally satisfied. This aluminum tube is connected to a 30 cm×50 cm Lucite tube in which a longitudinal magnetic field is generated by a circular coil. The pulsed magnetic field is confined to the Lucite tube by aluminum flanges at each end. The beam current density is measured by a Faraday cup which can be moved axially and radially in additional lengths of Lucite tube placed downstream.

The peak current density was measured as a function of the applied magnetic field with the Faraday cup positioned on axis 20 cm downstream of the region containing the magnetic field. The current density increased as the peak field was increased to 75 G, and then decreased at higher fields [Fig. 3 and for other conditions Figs. 2(c) and 2(d)]. This was interpreted as being due to the focal point moving from behind to in front of the Faraday cup as the field was increased, with 75 G corresponding to a focal length of 20 cm measured from the downstream field boundary. It was found by moving the Faraday cup radially that the beam diameter at the focal point was reduced from 18 cm with no field to 3 cm (full width at half maximum) at 75 G. At this field, the beam density exceeds the minimum required for the quasineutrality condition to hold ($\omega_{pe}^2/\Omega_e^2 \approx 10$).

The optimum fields determined with the Faraday cup positioned 10, 20, 50, and 100 cm downstream of the lens are plotted in Fig. 4. Also shown is the focal length of the lens determined by numerical integration of Eq. (4) with $\Omega_i \Omega_e$ determined from the experimentally measured

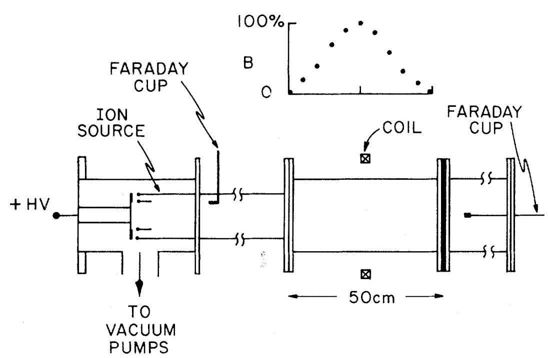

FIG. 1. Schematic diagram of the experimental apparatus.

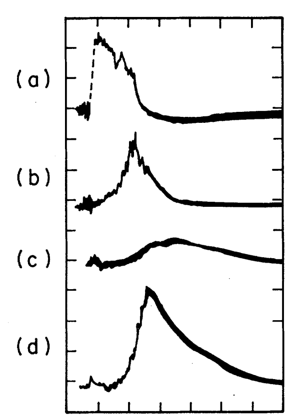

FIG. 2. Oscillograms. (a) Diode voltage, 70 kV/div; (b) ion current density near the diode, 5 A/cm^2/div; (c) ion current density 10 cm beyond the field region with no field, 0.5 A/cm^2/div; (d) same as (c) with the optimum focusing field of 88 G, 0.5 A/cm^2/div. The horizontal scale is 500 nsec/div.

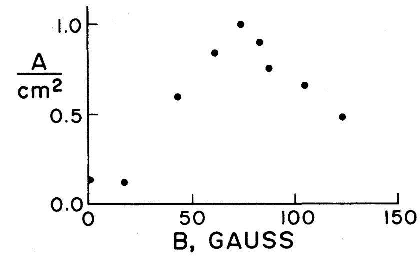

FIG. 3. Peak current density as a function of applied field measured on axis 20 cm beyond the field. Each point corresponds to one firing of the beam.

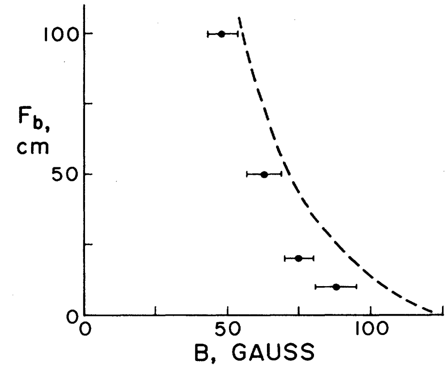

FIG. 4. The experimentally determined focal length as a function of magnetic field (points) and the calculated focal length (dotted line). The error bars give only the statistical uncertainty.

field profile. The relation $dz = v_z dt$ was used to convert the time derivatives to spatial derivatives with $v_z = 5.2\times10^6$ m/sec corresponding to a mean ion energy of 149 kV. The small difference between the experimental data and theoretical curve can be accounted for by spherical aberration and a small (< 10%) error in the mean beam velocity. In the absence of space-charge coupling between the electrons and ions, a field of 2–3 kG would have been required to obtain the observed focal lengths.

Within the lens, a positive potential at the surface of the beam was observed by means of an electrostatic probe. The data agreed qualitatively with the potential implied by Eq. (5). The potential was reduced by an order of magnitude and collective focusing was not observed when a conducting metal liner was placed within the Lucite tube. The experiment described here is therefore not inconsistent with that of Maenchen *et al.*,[8] in which collective effects were not observed for a lens having an aluminum screen liner which focused a beam not satisfying $c/\omega_{pe} > \frac{1}{2} r_0$.

The condition $c/\omega_{pe} > \frac{1}{2} r_0$ limits the beam density to 10^{10} cm^{-3} for a beam of 10 cm radius. This limit scales as r_0^{-2} so that the total ion current which can pass through the lens is independent of its radius. This limit is 1700 A when $v_z = 0.1c$. For applications in inertial fusion, the use of the lens would be limited to schemes employing the heaviest ions where the required energy per particle is very high resulting in lower requirements on the current.[9]

The author acknowledges valuable discussions with Gadi Barak and Norman Rostoker. This research was supported by the U. S. Department of Energy.

[1]S. Humphries, Jr., Nucl. Fusion 20, 1549 (1980).

[2]D. Gabor, Nature (London) 160, 90 (1947).

[3]R. Booth and H. W. Lefevre, Nucl. Instrum. Methods 151, 143 (1978), and references therein.

[4]R. M. Mobley, G. Gammel, and A. W. Maschke, IEEE Trans. Nucl. Sci. 26, 3112 (1979).

[5]A. Irani and N. Rostoker, Part. Accel. 8, 107 (1978).

[6]A. Fisher, P. Gilad, F. Goldin, and N. Rostoker, Appl. Phys. Lett. 37, 531 (1980).

[7]This result is a special case of Busch's theorem. See, for instance, J. R. Pierce, *Theory and Design of Electron Beams* (Van Nostrand, New York, 1954), p. 35.

[8]J. Maenchen, L. Wiley, S. Humphries, Jr., E. Peleg, R. N. Sudan, and D. A. Hammer, Phys. Fluids 22, 555 (1979).

[9]See, for instance, L. C. Teng, IEEE Trans. Nucl. Sci. 28, 3110 (1981).