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Theory of Runaway-Current Sustainment by Lower-Hybrid Waves

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A mechanism is proposed whereby high-phase-velocity lower-hybrid waves can interact with lower-parallel-velocity electrons through nonlinearly excited plasma waves. Significant steady-state current can be sustained by the rf after the Ohmic field is turned off in a tokamak provided the initial electron distribution is in the runaway regime with density below a critical value.

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Recent experiments on Versator and Princeton Large Torus (PLT) plasmas[1,2] have shown that lower-hybrid waves can be very effective in sustaining a current in a tokamak, believed to be carried primarily by runaway electrons with an efficiency of about 1 A of current per 1 W of rf power in the low-density region of operation where substantial numbers of such runaways are present. In fact, a very sharp density threshold is observed above which there is no current drive. Similar current-drive results with lower-hybrid waves have also been reported in other tokamaks.[3-6] One distinguishing feature of the PLT experiment is that the toroidal electric field measured at the plasma edge is held zero (or negative) during the rf pulse, thus achieving current drive with the transformer induction effectively shut off for a period of up to 1 s, approximately the classical skin time; and the launched lower-hybrid (LH) waves have very high phase velocity along the magnetic field, close to $c/2$. An interesting theoretical question is "How can such high-phase-velocity waves interact with the low-energy part of the tail electrons to sustain the tail distribution, which is presumably very flat in $v_\parallel$?" Although the recent theory of Fisch[7] has provided much of the impetus for the experimental activities, there is considerable discrepancy between the much larger current observed and Fisch's prediction because his theory did not take into account the runaway electrons.

We propose here a mechanism which allows these fast waves to interact with lower-parallel-velocity runaways through the nonlinearly excited plasma waves. The launched LH waves are in Cherenkov resonance with fast electrons in the tail whose parallel velocity is $v_\parallel = v_{\rm ph} \equiv \omega_k/k_\parallel$. This resonance interaction flattens the parallel velocity distribution of the electrons, thus effectively raising their parallel temperatures and producing anisotropy with $T_\parallel > T_\perp$. The anisotropy in the fast-electron distribution with $T_\parallel > T_\perp$ can in turn destabilize the low-phase-velocity magnetized plasma waves with $\omega_k/k_\parallel = \omega_p/k$ through the anomalous Doppler resonance: $(\omega_k+\Omega)/k_\parallel = v_\parallel$ for $\omega_k \ll \Omega = eB/mc$, the electron gyrofrequency. Plasma waves can then interact with slower electrons in the tail through Cherenkov resonance, quasilinearly maintaining their flat distribution by depositing momentum to the tail, thus sustaining the tail current.

Consider first the preformed runaway tail when the electric field is on.[8-10] For an electric field exceeding a few percent of the Dreicer field $E_{\rm D} = e\ln\Lambda/\lambda_{\rm D}^2$ where $\lambda_{\rm D}^2 = T/4\pi ne^2$, which is typical in the low-density region since $E/E_{\rm D} \propto n^{-1}$, a substantial part of the plasma current is carried by runaways. The runaway distribution is very flat in $v_\parallel$ for $v_\parallel \geq v_c = (E_{\rm D}/E)^{1/2} v_e$ with more parallel energy than perpendicular because the electrons are accelerated by an electric field along the magnetic field.

Let us suppose that the Ohmic electric field is now switched off, but before the tail distribution has been significantly relaxed the lower-hybrid waves of very high parallel phase velocity $v_{max} \geqslant v_{ph} \geqslant v_{min}$ are launched. The waves can only be in Cherenkov resonance with the very energetic electrons with $v_{\parallel} \geqslant v_{min}$ (typically 100 keV) and quasilinearly diffuse the energetic distribution in $v_{\parallel}$, $f(v_{\parallel})$, thus effectively raising the parallel temperature $T_{\parallel}$ at the very end of the tail. With this increment of $T_{\parallel}$ due to the LH waves, the original distribution for $v_{\parallel} > v_{min}$ becomes more anisotropic with $T_{\parallel} > T_{\perp}$. This anisotropy in the fast particles can again destabilize the slow plasma waves with $\omega_k/k_{\parallel} = \omega_p/k$ between v_c and $v_0 = (\omega_p/\Omega)v_{max}$ through the anomalous Doppler resonance: $\omega_k + \Omega = k_{\parallel}v_{\parallel}$ as $\Omega \gg \omega_p \geqslant \omega_k$, where v_{max} is the maximum parallel phase velocity. Thus for particles with $v_{\parallel} > v_{min}$, the effect of the lower-hybrid waves is the same as that of an electric field—to increase their parallel temperature by flattening their parallel distribution. The unstable plasma waves can interact with slower particles with $v_0 \gtrsim v_{\parallel} \geqslant v_c$ through Landau damping, depositing energy and momentum to sustain this slower part of the tail by maintaining a flat but asymmetric distribution.

For this mechanism to be most effective, the range of parallel phase velocities of the LH waves should be broad with $v_{max}/v_{min} \gtrsim \Omega/\omega_p$, and v_{min} of the LH waves should be $v_D = v_c\Omega/\omega_p$. If $v_{min} > v_D + (T_{\perp}/m)^{1/2}$, there are no particles to interact with and the mechanism fails. More importantly, the fraction of runaway electrons $\Delta n/n$ is exponentially dependent on $-E_D/4E$ or n^{-1} as $\Delta n = S\tau$, where

$$S = 0.35 n\nu_0(E_D/E)^{3/8}\exp[-E_D/4E - (2E_D/E)^{1/2}]$$

is the runaway production rate, τ their confinement time, and ν_0 the collision frequency.[11] Because the growth rate due to anisotropy of the slow plasma wave is proportional to $\Delta n/n$, there is a critical density below which the growth rate can overcome the collisional damping rate. Conversely, for

$$E_D/E(E_D/E)_c \approx 4\{[\ln(\tau\omega_p) + 4] - 4[1 + \ln(\tau\omega_p)^{1/2}]^{1/2}\},$$

the plasma wave remains stable. Furthermore, as the density increases, the wave frequency approaches the lower-hybrid frequency and ion tail heating becomes important and wave energy is channeled to the ions.

To analytically obtain this rf-sustained steady-state current, we seek a quasistationary state as a result of the process described above by using the quasilinear equations including a model collision term,[12] normalized according to $k \to k/\lambda_D$, $v \to vv_e$, $t \to t/\omega_p$, $f \to fn/v_e^3$, $W_k \to W_k 4\pi nT\lambda_D^3$:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_{\parallel}}D_{LH}\frac{\partial f}{\partial v_{\parallel}} + \frac{\partial}{\partial v_{\parallel}}D_0\frac{\partial f}{\partial v_{\parallel}} + \left(\frac{\partial}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}}\frac{\partial}{\partial v_{\perp}}\right)D_1\left(\frac{\partial}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}}\frac{\partial}{\partial v_{\perp}}\right)f + \frac{\partial}{\partial v_{\parallel}}\nu(v)\left(v_{\parallel}f + \frac{\partial f}{\partial v_{\parallel}}\right), \quad (1)$$

where $\nu = \nu_0(v_{\parallel}^2 + v_{\perp}^2)^{-3/2}$ is the collision frequency,

$$D_{LH}(v_{\parallel}) = (\pi/2)\int d^3k\, W_{LH}(k)\delta(\omega_k - k_{\parallel}v_{\parallel})(2\pi)^{-3}$$

for $v_{max} > v_{\parallel} > v_{min}$ and vanishes otherwise, $D_{LH}(v_{\parallel})$ being the quasilinear diffusion coefficient due to the lower-hybrid wave intensity $W_{LH} = |E_k|_{LH}^2$. Also

$$D_1(v_{\parallel}, v_{\perp}) = (\pi/2)\int d^3k (k_{\parallel}/k)^2(k_{\perp}v_{\perp}/\Omega_c)^2 W_k\,\delta(\Omega - k_{\parallel}v_{\parallel})(2\pi)^{-3}$$

for $v_{\parallel} > v_D$, and

$$D_0(v_{\parallel}) = 2\pi\int d^3k(k_{\parallel}/k)^2 W_k\,\delta(\omega_k - k_{\parallel}v_{\parallel})(2\pi)^{-3}$$

for $v_c < v_{\parallel} < v_D$, are the diffusion coefficients due to the nonlinearly excited slow waves for quasilinear pitch-angle scattering on the fast particles and parallel diffusion on the slow particles, respectively.

The wave intensity $W_k = |E_k|^2$ evolves according to

$$\dot{W}_k = (2\gamma_1 + 2\gamma_0 - \nu_0)W_k, \quad (2)$$

where the growth rate due to the anomalous Doppler resonance $v_{\parallel} = \Omega/k_{\parallel}$ is given by

$$\gamma_1 = (\pi/8)(\omega_k/k^2)\int d^3v(k_{\perp}v_{\perp}/\Omega)^2 k_{\parallel}\{(\partial f/\partial v_{\parallel}) - [(v_{\parallel}/v_{\perp})(\partial f/\partial v_{\perp})]\}\,\delta(\Omega - k_{\parallel}v_{\parallel}).$$

The waves will be unstable if this growth rate exceeds the sum of the Landau damping rate

$$\gamma_0 = (\pi/2)(\omega_k/k^2)\int d^3v\, k_\parallel(\partial f/\partial v_\parallel)\delta(\omega_k - k_\parallel v_\parallel),$$

due to the local negative slope and the collisional damping rate. Thus after the rf is turned on, there is an initial unstable phase of wave growth. However, a new quasistationary state will eventually be reached.

To describe such a quasisteady state analytically, we seek a time-independent solution of Eqs. (1) and (2). For simplicity, we make the following *Ansatz*:

$$f(v_\parallel, v_\perp) = F(v_\parallel)\exp(-v_\perp^2/2T)/2\pi T.$$

The quantity T is indeterminate in this calculation and can be obtained only when loss processes like synchrotron radiation are included. We have not considered such processes; however, our calculation does not require knowledge of this quantity. Setting $\partial f/\partial t = 0$ in Eq. (1) and dropping the subscript in $v_\parallel$, we obtain

$$D_{\mathrm{LH}}\frac{\partial F}{\partial v} + \overline{D}_1\left(T\frac{\partial F}{\partial v} + vF\right) = 0, \quad \text{for } v > v_D, \tag{3}$$

which is the balance between quasilinear tail raising (flattening) by LH waves and tail shrinking due to pitch-angle scattering by unstable plasma waves. The ensemble-averaged pitch-angle diffusion is $\langle D\rangle = \overline{D}_1 T$ with

$$\overline{D}_1 = (1/4\pi)\int_0^{\pi/2} d\theta \int dk\, k^4 \sin^3\theta \cos^2\theta\, W(\vec{k})\delta(\Omega - kv\cos\theta)\Omega^{-2}.$$

Because the growth rate is peaked on θ we may further assume that $W(\vec{k}) = W(k)\delta(\theta - \theta_0)$ and find

$$\overline{D}_1 = [\Omega^2 \tan^3\theta_0 W(k = \Omega/v\cos\theta)]/(v^5 4\pi).$$

In the region $v_D > v_\parallel \geq v_c$, we have quasilinear diffusion by plasma waves balanced by collisions:

$$\nu(v)(vF + \partial F/\partial v) + D_0 \partial F/\partial v = 0, \tag{4}$$

where

$$D_0 = (1/2\pi)\int_0^{\pi/2} d\theta \int_0^\infty dk\, k \sin\theta \cos\theta W(k)\delta(\theta - \theta_0)\delta(1/k - v) = \sin\theta_0 \cos\theta_0 W(k = 1/v)/v^3 2\pi.$$

Since $D_0 \gg \nu v_e^2$ in the region, we find from Eq. (4)

$$\partial F/\partial v = -[\nu(v)v/D_0]F(v_c). \tag{5}$$

With Eq. (5) we can find the Landau damping rate γ_0. Setting $\gamma_1 = -\gamma_0 + \nu_0/2$ for the steady-state plasma waves, we obtain a relation between the anisotropy and collision frequency,

$$T\frac{\partial F}{\partial v} + vF = \frac{2\nu_0\Omega^3}{\sin^2\theta_0}\left[\frac{F(v_c)}{D_0} + \frac{1}{\pi\cos\theta_0}\right]. \tag{6}$$

Substituting Eq. (3) into Eq. (6), we therefore obtain

$$D_{\mathrm{LH}}\frac{\partial F}{\partial v} = -\frac{2\overline{D}_1}{D_0}\frac{\nu_0\Omega^2}{\sin^2\theta_0}\left[F(v_c) + \frac{D_0}{\pi\cos\theta_0}\right]. \tag{7}$$

From the assumed spectrum $W(\vec{k}) = W(k)\delta(\theta - \theta_0)$, we find

$$\frac{\overline{D}_1(v)}{D_0(v\cos\theta_0/\Omega)} = \frac{1}{2}\frac{\sin^2\theta_0}{\cos\theta_0}\frac{1}{\Omega v^2}. \tag{8}$$

Substituting Eq. (8) into Eq. (7), we can evaluate the power requirement for the lower-hybrid waves to sustain this distribution

$$P_{\mathrm{LH}} = \int_{v_D}^{v_{\max}} dv\, \frac{v^2}{2}\frac{\partial}{\partial v}D_{\mathrm{LH}}\frac{\partial F}{\partial v} = \frac{\nu_0\Omega}{\cos\theta_0}F(v_c)\ln\left(\frac{v_{\max}}{v_D}\right),$$

$$= 7.5\times 10^9 n_{14}^2 T_{10}^{-1/2} F(v_c)\ln\left(\frac{v_{\max}}{v_D}\right)\frac{\Omega}{\omega_p\cos\theta_0}\ \mathrm{W/m^3}, \tag{9}$$

where we have neglected the collisional damping compared with Landau damping in Eq. (8) valid for $D_0/D_{\text{class}} < (\Delta n/n)(\omega_p/\nu_0)$, $F(v_c)$ is the value of the distribution at v_c, and the density is normalized to 10^{14} cm^{-3} and temperature to 10 keV.

The electron current can be evaluated after determining F for $v_c < v < v_{\max}$. We note that $F \simeq F(v_c)$ for $v_c < v < v_D$. Using Eq. (7), it can be shown that

$$F(v) \simeq F(v_c)\{1 - \nu_0\Omega/\cos\theta_0\bar{D}_{\text{LH}}[(v^2 - v_0^2)/2]\}$$

for $v_D < v < v_{\max}$ assuming $D_{\text{LH}} = \bar{D}_{\text{LH}}/v^3$. However, since F cannot be negative, it has to be set equal to zero for $v^2 > v_B^2 \equiv (2\cos\theta_0\bar{D}_{\text{LH}}/\nu_0\Omega) + v_D^2$ in the case $v_B < v_{\max}$. The current is hence given by

$$J = \int_{v_c}^{v_{\max}} dv\, vF \simeq F(v_c)\frac{v_{\max}^2}{2}\left[1 - \frac{\nu_0\Omega v_{\max}^2}{4\cos\theta_0\bar{D}_{\text{LH}}}\right],$$

$$= 6.5\times 10^8 n_{14} T_{10}^{1/2} F(v_c)\frac{v_{\max}^2}{2v_e^2}\left[1 - \frac{\Omega D_{\text{class}} v_{\max}^2}{4\cos\theta_0\omega_p\bar{D}_{\text{LH}} v_e^2}\right] \text{A/m}^2,$$

for $v_B > v_{\max} \gg v_D$. Taking $n_{14} = 0.07$, $T_{10} = 0.2$, $v_c = 3v_e$, $v_{\max}/v_D = 3$, $\Omega/\omega_p = 3$, $R_0 = 1$ m, $\cos\theta_0 \simeq 1$, and $D_{\text{class}}/\bar{D}_{\text{LH}} = 10^{-3}$, which are typical PLT parameters, the current drive efficiency given by $J/P_{\text{LH}} 2\pi R_0 \simeq 2$ A/W appears to be consistent with experimental observations.

In conclusion, a mechanism is proposed which allows particles outside the resonance region to interact with the LH waves. This effectively spreads out the wave energy over a wider velocity space, thereby sustaining significantly more current. Other effects, such as toroidal effects on ray trajectories, and parametric instabilities, may also lead to broadening of the lower-hybrid wave spectrum. However, these other effects remain to be fully examined. Finally, as the excited plasma waves propagate to higher densities, the spectrum can extend below v_c leading to enhanced runaway production when there is a small residual Ohmic field.[13]

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