$2mc^2$ in superheavy systems with $Z > Z_{cr} \sim 170$.

This work was supported by the Bundesministerium für Forschung und Technologie and in part by the Deutsche Forschungsgemeinschaft (Heisenberg-Programm).

[1]J. Reinhardt, B. Müller, and W. Greiner, Phys. Rev. A 24, 103 (1981).

[2]F. G. Werner and J. A. Wheeler, Phys. Rev. 109, 126 (1958).

[3]W. Pieper and W. Greiner, Z. Phys. 218, 327 (1969).

[4]G. Soff, B. Müller, and J. Rafelski, Z. Naturforsch. Teil A 29, 1267 (1974).

[5]M. Gyulassy, Phys. Rev. Lett. 33, 921 (1974).

[6]M. Gyulassy, Nucl. Phys. A244, 497 (1975).

[7]G. A. Rinker and L. Wilets, Phys. Rev. A 12, 748 (1975).

[8]E. H. Wichmann and N. M. Kroll, Phys. Rev. 101, 843 (1956).

[9]G. E. Brown and G. W. Schaefer, Proc. Roy. Soc. London, Ser. A 233, 527 (1956).

[10]G. E. Brown, J. S. Langer, and G. W. Schaefer, Proc. Roy. Soc. London, Ser. A 251, 92 (1959).

[11]G. E. Brown and D. F. Mayers, Proc. Roy. Soc. London, Ser. A 251, 105 (1959).

[12]G. W. Erickson and D. R. Jennie, Ann. Phys. (N.Y.) 35, 271, 447 (1965).

[13]A. M. Desiderio and W. R. Johnson, Phys. Rev. A 3, 1267 (1971).

[14]P. J. Mohr, Ann. Phys. (N.Y.) 88, 26, 52 (1974).

[15]K. T. Cheng and W. R. Johnson, Phys. Rev. A 14, 1943 (1976).

[16]D. Liesen, P. Armbruster, F. Bosch, S. Hagmann, P. H. Mokler, H. J. Wollersheim, H. Schmidt-Böcking, R. Schuch, and J. B. Wilhelmy, Phys. Rev. Lett. 44, 983 (1980).

[17]L. F. Shampine and M. K. Gordon, *Computer Solution of Ordinary Differential Equations: The Initial Value Problem* (Freeman, San Francisco, 1975).

[18]P. Schlüter *et al.*, to be published.

Large-Scale Collision-Free Instability of Two-Dimensional Plasma Sheets

H. Goldstein[(a)] and K. Schindler
Theoretische Physik IV, Ruhr-Universität Bochum, D-4630 Bochum 1, West Germany

(Received 10 November 1981)

The stability of a two-dimensional plasma sheet with a small field component parallel to the normal direction of the sheet is studied by means of the energy principle of collision-free kinetic theory. Numerical computations show that, depending on the parameter regime, unstable perturbations exist. The mode resembles a tearing mode. The typical wavelength along the main magnetic field direction is comparable with the scale length of the equilibrium.

PACS numbers: 52.35.+g, 52.30.+r

Sudden changes in the topology of magnetic fields (reconnection) has been suggested to provide, under suitable conditions, a powerful mechanism to release free energy from magnetized plasma in the laboratory,[1] in space,[2-6] and near stars.[7] Although a substantial amount of knowledge has been accumulated on this topic, a number of basic questions have remained open.

The present paper deals with generalized (two dimensional) collision-free plasma sheets. Since the strictly one-dimensional case, in which electron Landau resonance dominates, is reasonably understood,[2,8] we concentrate on the two-dimensional case where a magnetic field component B_z normal to the current sheet (Fig. 1) is sufficiently large to suppress electron Landau resonances. In addition, we are motivated to study this case by the fact that the near-Earth geomagnetic tail, where space craft provide *in situ* observations, falls into that regime. Here, one of the important problems is to identify the elementary plasma processes that govern the observed dynamic characteristics occurring in connection with geomagnetic activity.

In the literature two main branches of approach can be distinguished: (a) The undisturbed equilibrium is quiet in the sense that noise from microinstabilities does not play a significant role: The plasma is collision free even on large space and time scales,[3,9,10] (b) Microinstabilities, e.g., the lower-hybrid drift instability, lead to sufficiently pronounced transport phenomena, such that the plasma behaves as a dissipative fluid on large scales.[3,11]

Since the respective domains of applicability of these two types of processes are not yet clear,

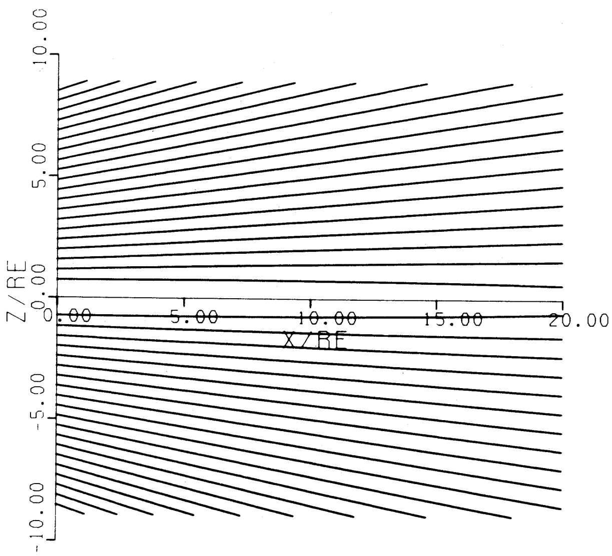

FIG. 1. Magnetic field lines of an equilibrium ($b = \frac{1}{45}$, $\epsilon_i = 0.35$, $\epsilon_e = 0.66\times 10^{-3}$). This configuration can be visualized as representing a piece of the geomagnetic tail with z axis parallel to the south-north direction, and x axis oriented along the plasma sheet away from the sun. (Lengths are expressed in units of the Earth's radius R_E.)

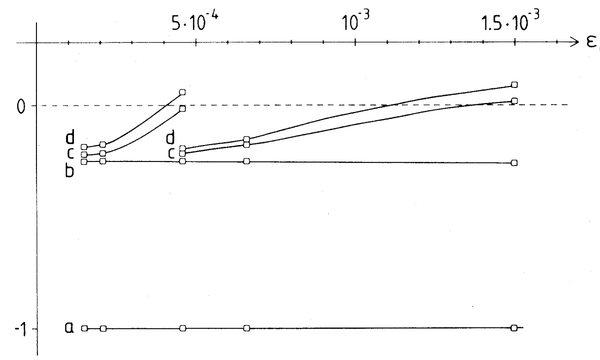

FIG. 2. Stability diagram for $b = \frac{1}{45}$, $\epsilon_i = 0.35$. Computed values of $\inf W_2$, marked by squares, are connected by curves to guide the eye; $\inf F_2$ is normalized to -1. Negative values on curve d correspond to unstable equilibria. The meaning of a–d is explained in the text.

it seems appropriate to follow each line separately. (A convincing combination of both aspects seems at present unfeasible.) In this paper we assume standpoint (a).

The earlier approaches to the present problem show rather severe deficiencies: Either charge-separation effects due to the electrons[12] were ignored[3] (possibly ionospheric currents reduce charge densities in the magnetotail[13]; however, it is not yet fully clear to what extent), or the analysis was based on a WKB approximation where the wavelength of the mode along the plasma sheet had to be restricted to values much smaller than the characteristic length of the equilibrium in that direction.[9,10] Furthermore, in the latter group of papers the new effect, i.e., the normal magnetic field component B_z, was introduced inconsistently. Since the two methods gave opposite answers regarding stability, a more refined approach seemed necessary.

In the present study electron effects are fully taken into account, the WKB method is abandoned, and the equilibrium is self-consistent, at least to lowest significant order in B_z. The main restriction, that could not be relaxed, is the two dimensionality in the sense that any y dependence (Fig. 1) is ignored.

The equilibrium distribution function for each particle species s was chosen to correspond to drifting Maxwellians distributions,[9,10]

$$F_{0s}(H_{0s}, P_{0s}) = a_s \exp\left(-\frac{H_{0s} - u_s P_{0s}}{T_s}\right);$$

$$H_{0s} = \tfrac{1}{2} m_s v^2 + e_s \varphi_0, \quad P_{0s} = m_s v_y + (e_s/c) A_0.$$

H_{0s}, P_{0s}, $\vec{v}$, m_s, e_s, u_s, a_s, and T_s are the particle energy, y component of the generalized momentum, individual velocity, mass, charge, bulk velocity, a constant, and temperature in energy units for each particle species s; A_0 and φ_0 denote the y component of the vector potential and the scalar potential, respectively. The subscript zero refers to time-independent (equilibrium) quantities. The charge density ρ_0 and the (y component) current density j_0 are functions of the equilibrium potentials A_0 and φ_0, which are determined by Ampere's law and the condition of quasineutrality, appropriate for nonrelativistic phenomena on scales much larger than the Debye length.

Suitable solutions, representing the quiet magnetotail of the Earth were found by Birn, Sommer, and Schindler.[14] The nonvanishing components of the magnetic field are

$$B_{0x}(x, z) = -\hat{B}(x) \tanh\left(\frac{\hat{B}(x) \cdot z}{A_c}\right),$$

$$B_{0z}(x, z) = -\frac{1}{\hat{B}(x)} \frac{d\hat{B}(x)}{dx} \left[A_c + B_{0x}(x, z) \cdot z\right].$$

The function $\hat{B}(x)$ is arbitrary. Here we choose

$$\hat{B}(x) = B_c (1 + x/x_c)^{-1}.$$

The equilibrium depends on the nondimensional parameters $b = L_z/L_x$ and $\epsilon_s = \rho_s/L_x$, where L_z and L_x are the plasma sheet thickness and the length scale along the tail, respectively, and ρ_s denotes the gyroradius of a particle with thermal velocity outside the sheet ($z \gg L_z$). An example for a tail equilibrium is illustrated in Fig. 1 showing the magnetic field lines.

The stability can be analyzed in two dimensions by an energy principle,[15,16] in the sense that there exists a quadratic functional $W_2(A_1)$ of the perturbation A_1 of the vector potential with the following property:

$$\inf_{A_1} W_2(A_1) > 0 \ (<0) \Rightarrow \text{stability (instability)}.$$

W_2 may be case in the following form[16]:

$$W_2(A_1) = F_2 + W_2{}^i + W_2{}^e + W_2{}^\varphi;$$

$$F_2(A_1) = \int d^2r\left(\frac{(\nabla A_1)^2}{8\pi} - \frac{1}{2c}\frac{\partial j_0}{\partial A_0}A_1{}^2\right),$$

$$W_2{}^s(A_1) = \tfrac{1}{2}e^2\int d\Omega\left|\frac{\partial F_{0s}}{\partial H_{0s}}\right|\left\langle\frac{1}{m_s c}\left(p_{0s} - \frac{e_s}{c}A_0\right)A_1\right\rangle^2,$$

$$W_2{}^\varphi(A_1) = \sum_{s=e,\,i}\tfrac{1}{2}e_s{}^2\int d\Omega\left|\frac{\partial F_{0s}}{\partial H_{0s}}\right|(\varphi_1 - \langle\varphi_1\rangle)^2,$$

$$d^2r = dx\,dz, \quad d\Omega = d^2r\,d^3v.$$

For simplicity we confine the discussion to one singly charged ion species ($s = i$) and electrons ($s = e$); $\langle\cdots\rangle$ denotes a spatial average carried out over that domain to which a particle of a given set of constants of motion has access on equilibrium orbits. In the case of adiabatic particles the magnetic moment μ_{0s} is taken into account in addition to H_{0s} and P_{0s}.[16]

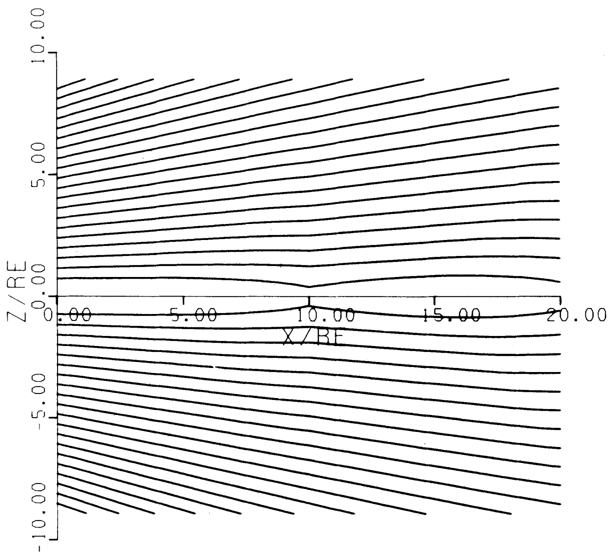

FIG. 3. Perturbed state of the equilibrium of Fig. 1, using the minimizing mode A_1.

$\varphi_1(A_1)$ is determined by the quasineutrality condition

$$\frac{\partial \rho_0}{\partial \varphi_0}\varphi_1 + \frac{\partial \rho_0}{\partial A_0}A_1 + \sum_{s=e,\,i}\int\left|\frac{\partial F_{0s}}{\partial H_{0s}}\right|\left\langle -\frac{1}{m_s c}\left(P_{0s} - \frac{e_s}{c}\right)A_0 A_1 + \varphi_1\right\rangle d\Omega = 0.$$

This energy principle has the advantage that it allows a straightforward numerical stability analysis. Moreover no assumption about the unstable mode appearing at a point of marginal stability is necessary; it is obtained as a result of our computation, via the minimizing function A_1. We use the method of finite elements,[17] where a minimum of 12 and a maximum of 84 grid points are located in the upper half-plane ($z \geq 0$). (The most unstable modes have the same symmetry with respect to the x axis as the equilibrium, see Fig. 1.)

In Fig. 2 we present numerical results (12 grid points) for a class of equilibrium configurations with $b = 1/45$ and $\epsilon_i = 0.35$, which includes the example of Fig. 1. We use ϵ_e as a free parameter, which may be regarded as a measure of electron temperature because $\epsilon_e \sim T_e{}^{1/2}$. The four curves a–d illustrate the following different stabilizing effects, respectively:

$$\inf_{A_1} F_2(A_1),$$

$$\inf_{A_1}(F_2 + W_2{}^i),$$

$$\inf_{A_1}(F_2 + W_2{}^i + W_2{}^e),$$

$$\inf_{A_1}(F_2 + W_2{}^i + W_2{}^e + W_2{}^\varphi) = \inf_{A_1} W_2.$$

First we note that $\inf_{A_1} F_2$ is negative for all values of ϵ_e. Since $W_2{}^i$, $W_2{}^e$, and $W_2{}^\varphi$ are positive this is a basic requirement for instability. Comparing the differences among the quantities in Fig. 2 we find that it is largest between a and b. This shows that the stabilizing effect by the ions via $W_2{}^i$ dominates. The contributions by the electrons and the scalar potential are smaller.

The corresponding curves c and d are discontinuous at $\epsilon_e = b^2$. At this value the electron gyroradius in the plasma sheet is equal to the radius of curvature of the magnetic field lines. Therefore the magnetic moment is conserved for $\epsilon_e < b^2$ only, which leads to a rapid change of stability properties at $\epsilon_e = b^2$. (A more exact treatment would replace the jumps at $\epsilon_e = b^2$ by a smooth transition.) In each regime, W_2^e and W_2^φ are seen to become less stabilizing with decreasing ϵ_e. This behavior was expected by earlier estimates using the energy principle.[18]

The answer to the principal question, whether the equilibrium considered is unstable, is obtained from curve d. Negative values corresponding to unstable equilibria are found in both the adiabatic and in the nonadiabatic electron regime. (Positive values do not safely prove stability since the number of grid points is small.)

The computed minimizing modes have the shape of tearing modes[19] as can be seen from Fig. 3. The characteristic length in the x direction is an order of magnitude larger than the length scale for the magnetic field variation along x. Similar results were obtained for the equilibrium parameter (e.g., with a ratio of ion and electron temperatures more realistic for the magnetosphere).

We emphasize that self-consistency of the equilibrium requires that the z dependence of B_z be taken into account. This is particularly important, because the stability properties crucially depend on the dependence of B_z on z. For instance, if B_z has no zero in the entire domain considered, then F_2 is positive for all trial functions such that the equilibrium is stable.[12] (This fact illustrates that it is problematic to approximate B_z by a constant, as done in some of the earlier approaches.[9,10])

Our method, although it provides a correct stability criterion, does not allow detailed conclusions about the instability mechanism. However, because of the geometry of the mode and from the various runs, we have the impression that the instability process requires nonadiabatic ions (whereas electrons may be adiabatic as evident from Fig. 2). Therefore we feel that it is appropriate to label the instability as an ion-tearing mode. We are, of course, at present not able to decide what type of resonance process (if any) dominates.

This work was supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich Plasmaphysik Bochum/Jülich. The numerical computations were sponsored by the Association EURATOM-Kernforschungsanlage.

[a]Present address: Max-Planck-Institut für Aeronomie, D-3411 Katlenburg-Lindau 3, West Germany.

[1]A recent review of reconnection experiments was given by P. J. Baum and A. Bratenahl, in *Advances in Electronics and Electron Physics*, edited by C. Morton (Academic, London, 1980), p. 1.

[2]B. Coppi, G. Laval, and R. Pellat, Phys. Rev. Lett. 16, 1207 (1966).

[3]K. Schindler, J. Geophys. Res. 79, 2803 (1974).

[4]E. W. Hones, Jr., J. Geophys. Res. 82, 5633 (1977).

[5]G. L. Siscoe, N. F. Ness, and C. M. Yeates, J. Geophys. Res. 80, 4359 (1975).

[6]J. B. Greenly and B. U. Ö. Sonnerup, J. Geophys. Res. 86, 1305 (1981).

[7]Overviews on solar flare models are given in *Solar Flare*, edited by P. A. Sturrock (Colorado Associated Univ. Press, Boulder, 1980), and in *Solar Flare Magnetohydrodynamics*, edited by E. R. Priest (Gordon and Breach, New York, 1981).

[8]K. Schindler, in *Proceedings of the Seventh International Conference on Phenomena in Ionized Gases, Belgrade, 1965*, edited by B. Perovic and D. Tosić (Gradjevinska Knjiga Publishing House, Belgrade, Yugoslavia, 1966), Vol. II, p. 736; G. Laval, R. Pellat, and M. Vuillemin, in *Proceedings of the Conference on Plasma Physics and Controlled Nuclear Fusion, Culham, 1965* (International Atomic Energy Agency, Vienna, 1968), Vol. II, p. 259.

[9]A. A. Galeev and L. M. Zelenyĭ, Zh. Eksp. Teor. Fiz. 70, 2133 (1976) [Sov. Phys. JETP 43, 1113 (1976)].

[10]B. Lembege, These du Doctorat 3e cycle, Universite de Paris XI, 1976 (unpublished).

[11]K. Papadopoulos, in *Dynamics of the Magnetosphere*, edited by S.-I. Akasofu (Reidel, Dordrecht, 1979), p. 289; J. F. Drake, N. T. Gladd, and J. D. Huba, Phys. Fluids 24, 78 (1981); J. D. Huba, N. T. Gladd, and J. F. Drake, J. Geophys. Res. 86, 5881 (1981); L. Janicke, Phys. Fluids 23, 1843 (1980); J. Birn, J. Geophys. Res. 85, 1214 (1980); F. V. Coroniti, J. Geophys. Res. 85, 6719 (1980).

[12]The importance of charge-separation effects was first pointed out by R. Pellat (see footnote 4 in Ref. 9).

[13]H. Goldstein and K. Schindler, J. Geophys. Res. 83, 2574 (1978).

[14]J. Birn, R. Sommer, and K. Schindler, Astrophys. Space Sci. 35, 389 (1975).

[15]K. Schindler, D. Pfirsch, and H. Wobig, Plasma Phys. 15, 1165 (1973).

[16]H. Goldstein, Thesis, Ruhr-Universität Bochum, West Germany, 1981 (unpublished).

[17]A. Mitchell and R. Wait, *The Finite Element Method in Partial Differential Equations* (Wiley, New York, 1976).

[18]K. Schindler and J. Birn, Phys. Rep. 47, 109 (1978).

[19]H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids 6, 459 (1963).