able mass M, has a peak that varies with M as  $n_0 \cong 2M^{1/2}$ , and a peak-to-width ratio  $n_0/D \cong 2$ .

We wish to thank Gregory Snow for his participation in the running of this experiment. We are indebted to Guenter Prokesch, Donald Humbert, and Ruth Snyder for their technical assistance.

<sup>(a)</sup>Present address: Fermi National Accelerator Laboratory, Batavia, Ill. 60510.

<sup>1</sup>R. L. Cool *et al.*, Phys. Rev. Lett. 47, 701 (1981).

<sup>2</sup>K. Goulianos, H. Sticker, and S. N. White, following Letter [ Phys. Rev. Lett. <u>48</u>, 1454 (1982)].

<sup>3</sup>R. L. Cool *et al.*, Phys. Rev. D <u>24</u>, 2821 (1981).

 ${}^{4}E$ . Albini *et al.*, Nuovo Cimento <u>32</u>, 101 (1976), and references therein.

<sup>5</sup>Z. Koba, H. B. Nielson, and P. Olesen, Nucl. Phys. B40, 317 (1972).

<sup>6</sup>P. Slattery, Phys. Rev. D  $\underline{7}$ , 2073 (1973). For a compilation of the data, see Ref. 4 and E. DeWolf *et al.*, Nucl. Phys. B87, 325 (1975).

 $^{7}\pi p$  data: F. C. Winkelmann *et al.*, Phys. Rev. Lett. <u>32</u>, 121 (1974). *pp* data: F. T. Dao *et al.*, Phys. Lett. <u>45B</u>, 399 (1973); S. J. Barish *et al.*, Phys. Rev. Lett.

31, 1080 (1973); J. W. Chapman et al., Phys. Rev.

Lett. 32, 257 (1974); M. G. Albrow et al., Nucl. Phys.

B102, 275 (1976). pp data: F. Grard et al., Phys.

Lett. <u>59B</u>, 409 (1975); C. P. Ward *et al.*, Nucl. Phys. <u>B153</u>, 299 (1979).

<sup>8</sup>S. Barshay *et al.*, Phys. Rev. Lett. <u>32</u>, 1390 (1974).

## Universality of Charged Multiplicity Distributions

K. Goulianos, H. Sticker, and S. N. White The Rockefeller University, New York, New York 10021 (Received 28 January 1982)

It is shown that the charged multiplicity distributions of the diffractive and nondiffractive components of hadronic interactions, as well as those of hadronic states produced in a variety of other reactions, follow a universal function which depends only on the mass available for pionization. This function is Gaussian, peaks at  $n_0 \cong 2M^{1/2}$ , where M is the available mass in gigaelectronvolts, and has a width  $D = (\langle n^2 \rangle - n_0^2)^{1/2}$  such that  $n_0/D \cong 2$ .

PACS numbers: 13.85.Hd

The preceding Letter<sup>1</sup> reported that the charged multiplicity distributions of high-mass pion, kaon, and nucleon states produced diffractively in the reaction

$$h + p \rightarrow X + p \quad (h = \pi^{\pm}, K^{\pm}, p^{\pm}) \tag{1}$$

are described well by a Gaussian function which depends only on the mass available for pion production,  $M = M_x - M_h$ , as follows:

$$P_{n} = \frac{2}{(2\pi)^{1/2}D} \exp\left(-\frac{(n-n_{0})^{2}}{2D^{2}}\right),$$
  

$$n_{0} = 2M^{1/2}, \quad n_{0}/D = 2,$$
(2)

where  $D = (\langle n^2 \rangle - n_0^2)^{1/2}$ .

For singly charged hadrons, at any given mass M, the sum of  $P_n$  over odd values of n should be normalized to unity. The nominal normalization constant given above is good to ~1%. Koba-Nielson-Oleson (KNO) scaling,<sup>2</sup> which states that the product  $P_n \langle n \rangle$  is a function of  $n/\langle n \rangle$  only, is

manifestly satisfied by Eq. (2), as can be seen by rewriting it as  $(2) \sqrt{2}$ 

$$P_n n_0 = \left(\frac{8}{\pi}\right)^{1/2} \exp\left[-2(1 - n/n_0)^2\right].$$
 (3)

The diffractive data of Ref. 1 agree well with this formula. In comparing experimentally measured multiplicity moments with those calculated using Eq. (2), care must be taken to sum only over the allowed positive values of n. This procedure increases the calculated average multiplicity and decreases the width of its distribution so that  $\langle n \rangle$  is generally somewhat larger than  $n_0$ and  $\langle n \rangle / D$  greater than 2.

It is well known<sup>3-5</sup> that charged multiplicity data of inclusive hadronic reactions do not agree with Eq. (2). For example, for  $pp \rightarrow$  anything,<sup>3-6</sup> the increase of the average multiplicity with energy is slower than  $2s^{1/4}$  [see Fig. 1(a)], the  $\langle n \rangle / D$  ratio decreases as the energy increases [see Fig. 1(b)], and the KNO distribution of the data is asymmetric about  $n/\langle n \rangle = 1$  (see Fig. 2), in disagreement with Eq. (3). On the other hand, data on  $e^+e^- \rightarrow$  anything are consistent<sup>7</sup> with  $\langle n \rangle$  $= 2s^{1/4}$  (see Fig. 3) and their KNO scaling curve is symmetric.<sup>8</sup>

In this paper, we analyze the inclusive pp - anything charged multiplicities, recognizing that they derive from two distinct parts of the inelas-

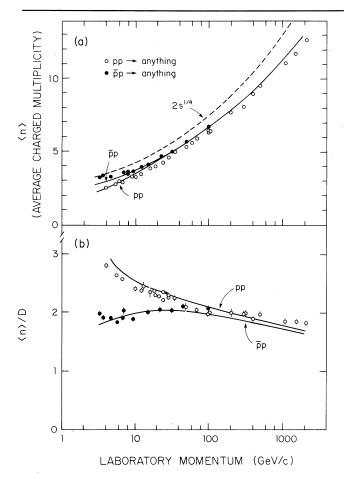


FIG. 1. The average charged multiplicity and the ratio of the average to the width as a function of laboratory momentum for  $pp \rightarrow$  anything and  $\overline{p}p \rightarrow$  anything. The solid curves were calculated using Eqs. (5) and (6) in the text.

tic cross section; the diffractive component, for which the available mass is  $M = M_x - M_p$ , and the nondiffractive "hard core" for which  $M = (s)^{1/2}$  $-2M_{p}$ . We then find that Eq. (2), which describes well the diffractive multiplicities, also provides a good description of the multiplicities of the hard core. After an examination of data from several other reactions, we come to the conclusion that the distribution represented by Eq. (2) is universal, describing to a good approximation all known hadronic charged multiplicities up to  $\sqrt{s} \sim 60$  GeV. This suggests that gluons may play an important role in the hadronization process. Small differences in the multiplicities between certain reactions, such as between pp and  $\overline{b}p$ , which cannot be explained by our procedure of applying Eq. (2) at the appropriate available mass of each identifiable component of the in-

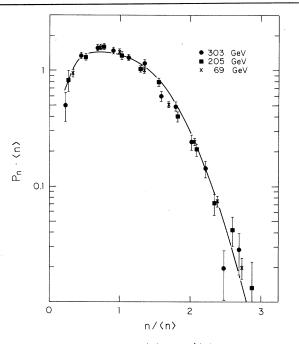


FIG. 2. The product  $P_n \langle n \rangle$  vs  $n / \langle n \rangle$  for charged particles in  $pp \rightarrow$  anything. The curve was calculated using Eq. (5) in the text. The data are from Ref. 6: 69 GeV, V. V. Ammosov *et al.*; 205 GeV, S. Barish *et al.*; 303 GeV, A. Firestone *et al.* 

elastic cross section, may then be attributed to the difference in the quark content of the initial states.

The total, elastic, and inelastic pp cross sections are shown in Fig. 4. The inelastic cross section is consistent<sup>9</sup> with being composed of a hard core,  $\sigma_0 = 26.3$  mb, and a diffractive compo-

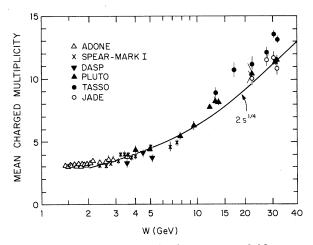


FIG. 3. Mean charged multiplicity vs available energy for  $e^+e^- \rightarrow$  anything.

nent,  $\sigma_D$ , which consists of the contribution of single-diffraction dissociation,  $2\sigma_{SD}$ , and that of double-diffraction dissociation,  $\sigma_{DD}$ :

$$\sigma_i = \sigma_0 + 2\sigma_{SD} + \sigma_{DD}. \tag{4}$$

In calculating charged multiplicity distributions, we assume that the inelastic cross section in excess of  $\sigma_0$  is all due to single-diffraction disso-

ciation with a  $1/M_x^2$  mass dependence. The double-diffractive process, which becomes important only at very high energies, is approximated well by this assumption, since there is only a small probability for both diffractive masses to be large. Multiplicity distributions for pp are then generated using Eq. (2) separately for the diffractive and nondiffractive components at the appropriate available mass:

$$P_{n+2} = \frac{1}{\sigma_i} \left\{ \sigma_0 P_{n+2} \left[ (s)^{1/2} - 2M_p \right] + \frac{\sigma_L}{\ln(0.1s)} \int_1^{0.1s} \frac{1}{M_x^2} P_{n+1} (M_x - M_p) dM_x^2 \right\}, \quad n = 0, 2, 4, \dots$$
(5)

The average multiplicity, the  $\langle n \rangle / D$  ratio, and the KNO distribution calculated using this equation are in good agreement with the data, as shown in Figs. 1 and 2. For  $\bar{p}p$ , we add the annihilation cross section,  $\bar{\sigma} = \sigma_T(\bar{p}p) - \sigma_T(pp)$ , to the pp inelastic cross section and use the distribution

$$P_{n} = \frac{1}{\sigma_{i}} \left\{ \sigma_{0} P_{n} \left[ (s)^{1/2} - 2M_{p} \right] + \frac{\sigma_{D}}{\ln(0.1s)} \int_{1}^{0.1s} \frac{1}{M_{x}^{2}} P_{n-1} (M_{x} - M_{p}) dM_{x}^{2} + \overline{\sigma} P_{n} (s^{1/2}) \right\},$$
(6)

where the second term is not included for n = 0. The  $\langle n \rangle$  and  $\langle n \rangle / D$  calculated using this distribution function agree well with the data<sup>4,10</sup> (see Fig. 1). However, at high energies, there remains a small but significant difference between the average pp and  $\bar{p}p$  multiplicities which is not predicted by the equations given above. If this difference were due to the annihilation cross section, its average multiplicity would have to be larger than  $2s^{1/4}$  by more than 50%. We prefer the interpretation that this disparity arises from a small difference in the pp and  $\bar{p}p$  hard cores and that the annihilation multiplicities follow the distribution given by Eq. (2).

A similar analysis of pp – anything in terms of diffractive and nondiffractive components was performed previously<sup>11</sup> and led to the same conclusions about the multiplicities of the hard core. More recently, an experiment<sup>12</sup> which removed leading particle effects obtained similar results. Our contribution in this area is that we have demonstrated that the same function that describes the multiplicity distribution of diffractive high-mass states also describes the pp and  $\bar{p}p$ hard cores and  $\bar{p}p$  annihilation.

By applying the same ideas to  $\pi^* p$  reactions, treating the nondiffractive inelastic cross section as a hard core with available mass  $M = (s)^{1/2} - M_p$ , we have obtained good agreement of our calculated  $\langle n \rangle$ ,  $\langle n \rangle / D$ , and KNO scaling distributions with existing data. We have also investigated the reactions  $\nu_{\mu} + p \rightarrow \mu^- + X^{**}$  and  $\overline{\nu}_{\mu} + p \rightarrow \mu^+ + X^0$ . Again, the multiplicity distributions reported for these reactions<sup>13</sup> are in good agreement with our predictions. In particular, the higher average multiplicity and  $\langle n \rangle / D$  ratio of  $X^{**}$  relative to that of  $X^0$  arise naturally as a consequence of summing over n = 2, 4, 6, ... for  $X^{++}$  and n = 0, 2, 4, ... for  $X^0$ . A determination<sup>14</sup> of the average multiplicity of the state X in  $pp \rightarrow \mu^+\mu^-X$  is also consistent with  $2s^{1/4}$  behavior.

Recently, two  $\overline{p}p$  experiments being performed at  $\sqrt{s} = 540$  GeV reported the values  $3.9 \pm 0.3$ ,<sup>15</sup> and  $3.0 \pm 0.1$ ,<sup>16</sup> for  $d\langle n \rangle / d\eta |_{\eta=0}$ , the average charged multiplicity per unit rapidity in the cen-

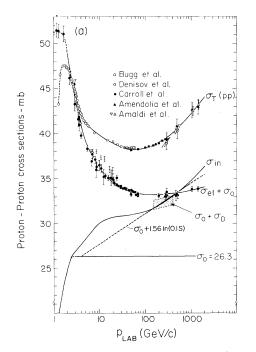


FIG. 4. The total, the elastic, and the inelastic pp cross sections (from Ref. 9).

tral region. At lower energies, the values of the rapidity plateau are lower. Typically, at  $\sqrt{s}$ = 30.8 and 62.8 GeV,  $d\langle n \rangle / d\eta |_{\eta=0}$  = 1.67 and 2.03, respectively.<sup>5</sup> Since the rapidity interval increases with energy as lns and the charged multiplicity as  $s^{1/4}$ , we expect  $d\langle n \rangle / d\eta$  to vary as  $Cs^{1/4}/\ln s$ . Setting C = 2.1 gives 1.71, 2.00, and 3.91 at  $\sqrt{s} = 30.8$ , 62.8, and 540 GeV, in excellent agreement with the measurements of Ref. 5 and Ref. 15.

In conclusion, we find that the distribution represented by Eq. (2) provides a good description of the charged multiplicities not only of highmass diffractive states but also of a wide variety of other hadronic states where the available mass for pionization can be identified. This universality of the multiplicity distribution implies that the guark content of the dissociating state does not play an important role in determination of the multiplicity and therefore it must be that the gluons dominate the process of hadronization.

<sup>1</sup>R. L. Cool et al., preceding Letter [ Phys. Rev. Lett. 48, 1451 (1981)].

<sup>2</sup>Z. Koba, H. B. Nielson, and P. Olesen, Nucl. Phys.

B40, 317 (1972).

<sup>3</sup>P. Slattery, Phys. Rev. D 7, 2073 (1973). <sup>4</sup>E. DeWolf et al., Nucl. Phys. <u>B87</u>, 325 (1975); E. Albini et al., Nuovo Cimento 32, 101 (1976). <sup>5</sup>W. Thomé et al., Nucl. Phys. B129, 365 (1977). <sup>6</sup>C. Bromberg et al., Phys. Rev. D 15, 64 (1977); W. M. Morse et al., Phys. Rev. D 15, 66 (1977); A. Firestone et al., Phys. Rev. D 10, 2080 (1974); S. Barish et al., Phys. Rev. D 9, 2689 (1974); V. V. Ammosov et al., Nucl. Phys. B58, 77 (1973). <sup>7</sup>C. Bacci et al., Phys. Lett. 86B, 234 (1979); J. L. Siegrist, Ph.D. thesis, SLAC Report No. SLAC-225 UC-34d, 1980 (unpublished); R. Brandelik et al., Nucl. Phys. B148, 189 (1979); Ch. Berger et al., Phys. Lett. 78B, 176 (1978); R. Brandelik et al., Phys. Lett. 89B, 418 (1980), and 94B, 444 (1980); W. Bartel et al., Phys. Lett. 88B, 171 (1979). <sup>8</sup>Ch. Berger et al., Phys. Lett. <u>95B</u>, 313 (1980). <sup>9</sup>K. Goulianos, Phys. Rev. D <u>14</u>, 1445 (1976). <sup>10</sup>D. E. Zissa *et al.*, Phys. Rev. D <u>21</u>, 3059 (1979); R. E. Ansorge et al., Phys. Lett. <u>59B</u>, 299 (1975). <sup>11</sup>K. Fialkowski and H. I. Miettinen, Phys. Lett. 43B, 61 (1973). <sup>12</sup>M. Basile *et al.*, Phys. Lett. <u>95B</u>, 311 (1980). <sup>13</sup>P. Allen et al., Nucl. Phys. B181, 385 (1981);

M. Derrick et al., Phys. Rev. D 17, 17 (1978).

<sup>14</sup>D. Antreasyan et al., CERN Report No. EP/81-116, to be published.

- <sup>15</sup>G. Arnison *et al.*, Phys. Lett. 107B, 320 (1981).
- <sup>16</sup>K. Alpgård *et al.*, Phys. Lett. 107B, 310, 315 (1981).

## Two-Phonon Octupole Excitation in <sup>147</sup>Gd

P. Kleinheinz, J. Styczen,<sup>(a)</sup> and M. Piiparinen<sup>(b)</sup>

Institut für Kernphysik, Kernforschungsanlage Jülich, D-5170 Jülich, Federal Republic of Germany

and

J. Blomqvist

Research Institute of Physics, S-10405 Stockholm 50, Sweden

and

## M. Kortelahti

Department of Physics, University of Jyväskylä, SF-40720 Jyväskylä, Finland (Received 28 December 1981)

A half-life measurement gives  $(57 \pm 13)B_W$  for the 1575-keV E3 transition in <sup>147</sup>Gd, which characterized the 2.572-MeV (19/2)<sup>-</sup> level as  $\nu f_{1/2} \times 3^- \times 3^-$  two-phonon octupole state. This is the first identification of a nuclear two-phonon octupole excitation. The observed departures from harmonic vibration (twice the one-phonon energy and twice the transition rate) can be quantitatively understood in terms of the microscopic composition of the states, and can be derived from other observed features of the octupole vibrations in this region.

PACS numbers: 23.20.Ck. 21.10.Re, 25.60.-t, 27.60.+j

Many attempts have been made to identify twophonon octupole states in nuclei. Such excitations could, in principle, be most clearly observed in

Coulomb excitation with heavy ions. But even in the simple case of <sup>208</sup>Pb, which was studied through Coulomb excitation with <sup>208</sup>Pb beams<sup>1</sup> as

© 1982 The American Physical Society