Upper-Hybrid Wave Collapse

Wave collapse in a magnetized plasma was recently considered by Giles.[1] Starting from the usual fluid equations and introducing the velocity potential $\frac{1}{2}[\Phi\exp(-i\omega t)+\mathrm{c.c.}]$, with ω equal to the upper-hybrid frequency ω_{UH}, he derived a nonlinear Schrödinger equation [Eq. (13) in Ref. 1]

$$\left(2i\omega_{\mathrm{UH}}\frac{\partial}{\partial t}+a^2\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\right)\Phi_r+\frac{1}{4\lambda_{\mathrm{D}}^2}\left(1+\frac{\Omega_e^2}{\omega_{\mathrm{UH}}^2}\right)|\Phi_r|^2\Phi_r=0, \qquad (1)$$

where $\omega_{\mathrm{UH}}=(\omega_p^2+\Omega_e^2)^{1/2}$, ω_p is the electron plasma frequency, Ω_e the gyrofrequency, λ_{D} the Debye length, and a^2 the square of the electron thermal velocity. Equations which are comparatively similar to (1) have appeared in many works on weakly magnetized[2] ($\Omega_e \ll \omega_p$) plasmas.

The pressure term in the equation of momentum [Eq. (2) in Ref. 1] is, however, not reliable if Ω_e is comparable to or larger than ω_p. The main purpose of the present Comment is to demonstrate that the structure of Eq. (1) above is changed significantly if results of a refined equation of momentum are included.

Considering wave propagation almost perpendicular ($k_z \ll k_\perp$) to the external magnetic field and assuming that $k_\perp^2a^2 \ll \omega_p^2-3\Omega_e^2$, I adopt kinetic theory to derive the approximate dispersion relation[3,4] for the upper-hybrid wave,

$$\omega^2 \approx \omega_{\mathrm{UH}}^2-\frac{k_z^2}{k_\perp^2}\frac{\omega_p^2\Omega_e^2}{\omega_{\mathrm{UH}}^2}\frac{k_\perp^2a^2}{1-3\Omega_e^2/\omega_p^2}. \qquad (2)$$

Equation (2) corresponds to the upper-hybrid-wave equation[3,4]

$$\left(2i\omega\frac{\partial}{\partial t}+\omega^2-\omega_{\mathrm{UH}}^2\right)\nabla_\perp^2\Phi+\frac{\omega_p^2\Omega_e^2}{\omega^2}\frac{\partial^2}{\partial z^2}\Phi+\left(1-\frac{3\Omega_e^2}{\omega_p^2}\right)^{-1}a^2\nabla_\perp^4\Phi \approx \omega_p^2\nabla_\perp\cdot\left(\frac{n_s}{n_0}\nabla_\perp\Phi\right), \qquad (3)$$

where n_s is the ion density perturbation which has been derived by Giles [Eq. (12) in Ref. 1]. Using his simplified version of (12) I insert $n_s/n_0 = -(1/4\omega_p^2\lambda_{\mathrm{D}}^2)(1+\Omega_e^2/\omega^2)|\nabla_\perp\Phi|^2$ in the right-hand side of (3).

By neglecting the $\partial/\partial t$ term in (3), I have thus found an equation which can have stationary, but unstable, cigar-shaped solutions.[5] Alternatively, by choosing[1] $\omega \equiv \omega_{\mathrm{UH}}$ and neglecting[1] the $\partial^2/\partial z^2$ term, I recover Eq. (1) if a^2 is replaced by $(1-3\Omega_e^2/\omega_p^2)^{-1}a^2$. As $\Omega_e \approx \omega_p$ in the experimental work[6] which is discussed by Giles, we realize that the sign in front of a^2 in (1) ought to be *negative*[3] instead of positive.

It is plausible to imagine that the mechanism described above is important for the generation of the auroral kilometric radiation[7] as well as for the nonthermal continuum radiation[8] in the magnetosphere.

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