Reheating an Inflationary Universe

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A numerical analysis of the evolution of the Higgs expectation value and the temperature of the universe during the symmetry-breaking phase transition in an SU(5) theory with radiatively induced symmetry breaking is presented. It is shown that there is sufficient inflation (exponential expansion) to explain the cosmological homogeneity, isotropy, flatness, and monopole puzzles, and also that the universe reheats to a temperature $O(10^{14}$ GeV) so that the usual scheme for baryogenesis can proceed.

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Although the hot big bang model has proven to be a remarkably simple and reliable framework for understanding the evolution of the universe, there are several observational facts which to date it has failed to elucidate. These cosmological "conundrums" include[1-3] (1) the present high degree of isotropy of the universe; (2) the large-scale homogeneity of the universe; and (3) the nearly critical energy density of the universe. These problems are compounded when grand unified theories (GUTs) are incorporated into the model. The simplest unified models predict a relic abundance of superheavy magnetic monopoles which is at least 10^{12} times greater than the observational limit.[4]

Guth[2] has suggested that all of the above-mentioned puzzles might be explained if the phase transition associated with the spontaneous symmetry breaking (SSB) of the GUT is first order. During a first-order phase transition the universe can become "trapped" in a metastable symmetric phase even after the temperature drops below the critical temperature for the transition, $T_c \sim O(10^{14}$ GeV)—the phenomenon known as supercooling. In the standard model, the evolution of the scale factor of the universe $R(t)$ is given by

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3m_{\mathrm{Pl}}{}^2}\rho - \frac{k}{R^2}, \tag{1}$$

where $m_{\mathrm{Pl}} = 1.22\times10^{19}$GeV is the Planck mass, and $k = \pm1,0$ is the signature of the curvature. The energy density ρ includes matter, radiation, and vacuum energy (cosmological term). The vacuum-energy term today is known to be less than 10^{-46} GeV^4. Therefore the value of the scalar potential at the $T=0$ SSB minimum (the vacuum energy) must be nearly zero. If the universe supercools much below T_c, the vacuum energy of the symmetric minimum which is $O(T_c{}^4)$ dominates ρ, and $R(t)$ increases exponentially until the transition is complete. The size and entropy of the universe are increased by a factor of $O((R/R_0)^3)$, where R/R_0 is the expansion during the exponential growth (deSitter) phase. Guth[2] argues that a growth factor of $O(10^{28})$ is sufficient to solve the three cosmological conundrums, and to dilute the relic monopole abundance to an acceptable level. Unfortunately, for models in which there is extreme supercooling, exponential expansion prevents completion of the phase transition and the "graceful" return to a radiation-dominated universe.[2]

Recently, a new inflation scenario involving GUTs which undergo radiatively induced SSB (Coleman-Weinberg SSB[5]) has been proposed.[6,7] This scenario appears to preserve the desirable features of the original scenario while overcoming the troublesome features. It was shown that in an SU(5) model with Coleman-Weinberg SSB, after the universe supercools to a temperature of about 10^8 GeV, the barrier between the symmetric and asymmetrical vacua becomes small and the metastability limit of the transition is reached[6] (see Fig. 1). The symmetric phase, which has been *metastable*, becomes effectively

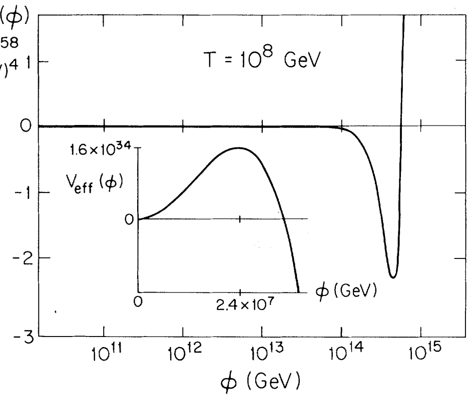

FIG. 1. The finite-temperature effective potential as a function of φ for $T = 10^8$ GeV (see Ref. 6). An enlargement of the region near $\varphi = 0$ is shown in the inset. Note that for this figure $V(0) = 0$.

unstable; thermal fluctuations then drive the universe away from the unstable symmetric phase.

In Coleman-Weinberg SSB there are no dimensional coupling constants, and the only parameter with dimensions of length that can affect the universe when φ (the adjoint Higgs field) is small is the inverse temperature (T^{-1}). Thus one expects the size of a typical fluctuation region to be $O(T_1^{-1})$ and the value of the φ field to be $O(T_1)$, where $T_1 \cong 10^8$ GeV is the metastability limit. The potential $V(\varphi)$ is very flat for $\varphi \lesssim 0.1$ (see Fig. 1) and so the time required for φ to increase to σ, its value at the SSB minimum, should be long compared to the expansion timescale, $t_{\rm exp} = (\dot{R}/R)^{-1}$. This accounts for the key feature of the new scenario: While the universe is dominated by the cosmological term (i.e., until $\varphi \simeq \sigma$), a *single* fluctuation region can grow to sufficient size to encompass the present observable universe. Provided the temperature rises to a value $O(T_c)$ after φ grows from $O(T_1)$ to $O(\sigma)$, the cosmological isotropy, flatness, and homogeneity problems are solved. Since the observable universe is within one fluctuation in which the Higgs field has the same orientation, the only monopoles within the present horizon are those which are subsequently produced by particle collisions. This should be a small, but possibly detectable, number.

In this Letter we report the results of numerical calculations of the evolution of $\langle\varphi\rangle$ and of the temperature of the universe T after the fluctuation region forms in the Georgi-Glashow SU(5) model with Coleman-Weinberg SSB.[8] The effective ($T = 0$) scalar potential for the adjoint Higgs field is

$$V_0(\varphi) = B\varphi^4\left(\ln\frac{\varphi^2}{\sigma^2} - \frac{1}{2}\right) + \frac{1}{2}B\sigma^4, \qquad (2)$$

where the adjoint Higgs field $\Phi = \varphi\,\mathrm{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$, $B = (5625/1024\pi^2)g^4$, g is the gauge coupling constant, and $\sigma = 4.5\times10^{14}$ GeV. The time evolution of φ is given by

$$\frac{d}{dt}\left(\frac{15}{4}\dot{\varphi}^2 + V(\varphi)\right) = -3\frac{\dot{R}}{R}\left(\frac{15}{2}\dot{\varphi}^2\right) - \delta. \qquad (3)$$

Finite-temperature corrections to $V_0(\varphi)$ are negligible in our calculations (see Ref. 6). The $\dot{R}/R$ term represents the energy-density loss caused by the expansion of the universe, and the δ term represents the energy density per unit time which is drained from the Higgs field through radiation of particles. Physically, such a term is expected since all the quantum fields which obtain a mass due to $\langle\varphi\rangle$ are coupled to a time-varying classical scalar field. It is difficult to calculate the precise form of such a term from first principles. However, it should depend upon φ and $\dot{\varphi}$, and the most general, dimensionally correct term involving just those two quantities is

$$\delta = ag^2\dot{\varphi}^d\varphi^{5-2d}. \qquad (4)$$

We have included the factor of g^2 since δ is likely to depend upon the gauge coupling strength. One might argue that the dominant energy-loss processes should involve Higgs self-couplings instead of gauge couplings, but these are expected to be within a few orders of magnitude of g. We have considered a variety of values for d and for a and find that our results are extremely insensitive to both. In what follows we shall take $\delta = ag^2\dot{\varphi}^2\varphi$ since that allows us to elucidate our numerical results by solving the equation for φ approximately in two regimes.

The equation for the evolution of the energy density in radiation (particles) is then given by

$$\frac{d\rho_r}{dt} = -4\frac{\dot{R}}{R}\rho_r + \delta, \qquad (5)$$

where we have assumed that $\rho_r \propto T^4$. Equations (3) and (5) must be supplemented by Eq. (1) for $\dot{R}/R$, with

$$\rho = \tfrac{15}{4}\dot{\varphi}^2 + V(\varphi) + \rho_r. \qquad (6)$$

In addition, we have also evolved g^2 with $\langle\varphi\rangle$.[9]

Equations (1)–(6) form a set of coupled equations for a semiclassical approximation to the evolution of φ, R, and T. We have numerically integrated them subject to the following initial conditions: $\varphi(0)=T(0)=\beta\times10^8$ GeV, $\dot\varphi(0)=0$. The complete time evolution of φ and $\rho_r^{1/4}$ is shown in Fig. 2 for $a=1.0$ and $\varphi(0)=3\times10^8$ GeV. [$\varphi(0)$ is the initial value of $\langle\varphi\rangle$ in the interior of the fluctuation region.] There are two interesting regimes which we will now discuss in more detail. (i) $\varphi\simeq\varphi(0)$. Because of the flatness of $V(\varphi)$, φ grows very slowly and essentially all the inflation occurs here. (ii) $\varphi\simeq\sigma$. φ is changing very rapidly ($\dot\varphi\simeq\sigma^2$) and the energy in the Higgs field is converted to radiation, reheating the universe and damping the motion of φ.

When $\varphi\simeq\varphi(0)$ the energy density of the universe is dominated by the vacuum energy density and $R=R_0\exp(t/t_{\rm exp})$, where $t_{\rm exp}{}^{-1}=(4\pi B/3)^{1/2}\sigma^2/m_{\rm Pl}\simeq7.2\times10^9$ GeV. For early times, Eq. (3) implies $\varphi(t)=\varphi(0)\exp(\lambda t)$. For $a\lesssim1700\beta^{-1}$, the "friction" slowing the evolution of φ is dominated by the expansion of the universe [the $\dot R/R$ term in Eq. (3)] and $\lambda^{-1}\simeq1817\beta^{-1.6}$. For $a>1700\beta^{-1}$ the radiation-damping term controls the growth of φ. Since the potential steepens rapidly [$V'(\varphi)\propto-\varphi^3$ for $\varphi<\sigma$], most of the time required for φ to reach $\varphi=\sigma$ elapses while φ is $\ll\sigma$ and the universe is expanding exponentially (see Fig. 2). Therefore, the inflation factor $R/R_0=\exp(t/t_{\rm exp})$, where t is approximately the time it takes for φ to reach $\varphi\simeq\sigma$. The growth factor R/R_0 is shown in Fig. 3 as a function of a for $\varphi(0)=\beta\times10^8$. For $\beta\lesssim7$, a single fluctuation inflates to a size greater than that of the observable universe today ($R/R_0>10^{25}$); if a is sufficiently large, enough inflation can result even for $\beta\simeq10$. Recall that the metastability limit is $T_1\simeq10^8$ GeV and that a "typical" fluctuation should have $\varphi(0)\simeq O(T_1)$.

The other regime of interest is when $\varphi\simeq\sigma$. Here the damping due to the expansion of the universe is negligible, and to a good approximation the equation for $\langle\varphi\rangle$ [Eq. (3)] is just that of a damped harmonic oscillator. For $a\lesssim12$, the oscillation period is $\tau_{\rm osc}\simeq4.8\times10^{-4}t_{\rm exp}\sim O(\sigma^{-1})$, and the damping timescale is $\tau_{\rm damp}\simeq1.9\tau_{\rm osc}a^{-1}$. For $a\gtrsim12$, particle radiation drains energy from the Higgs field so rapidly that $\langle\varphi\rangle$ is critically damped.

As $\langle\varphi\rangle$ changes with time, particle species which couple to φ (e.g., X, Y bosons, the $\underline{5}$ of Higgs, and φ itself) should be radiated. Thermalizing interactions among these species (two-body scatterings, decays, etc.) should populate the other particle species. The energy density in radiation and temperature are related by $\rho_r=(\pi^2/30)g_*T^4$. Here $g_*(\equiv\sum g_b+\frac{7}{8}\sum g_f)$ counts the total number degrees of freedom of all the relativistic particle species present, and is $O(10^2)$, so $T\simeq\rho_r{}^{1/4}/2$ (Fig. 2). Initially, ρ_r drops precipitously due to the exponential expansion of the universe; after a few $t_{\rm exp}$ the rate that energy is being "pumped in" by φ and drained by the expansion reaches a balance and $\rho_r{}^{1/4}$ stabilizes at a value $\rho_r{}^{1/4}\simeq a^{1/4}\beta^{1.5}(3\times10^6)$ GeV. As φ and $\dot\varphi$ increase dramatically, so does ρ_r.

Most of the energy in the φ field is converted into radiation when $\varphi\simeq\sigma$, over the course of just a few oscillation periods. One might have worried that the energy of the φ field would not be efficiently turned into radiation, because it would get red-shifted away as rapidly as it was converted into radiation. However, this does not happen over a wide range of a because the coherent energy of the adjoint Higgs is released in much less than an expansion time. For $10^3\gtrsim a\gtrsim10^{-3}$ more than 60% of the available vacuum energy ($\frac{1}{2}B\sigma^4$) is converted into radiation. If all the vacuum energy were converted to radiation, the final temperature would be $T_0=(1.73$

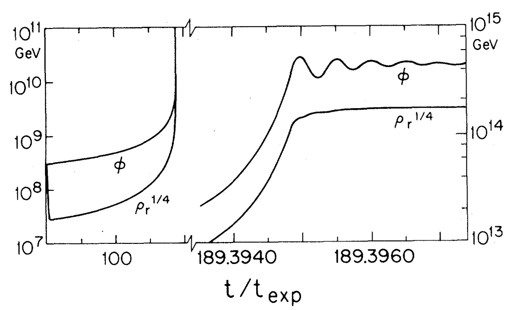

FIG. 2. The time evolution of φ and $\rho_r{}^{1/4}(\simeq2T)$ for $\varphi(0)=T(0)=3\times10^8$ GeV and $a=1$. Time is given in units of $t_{\rm exp}\simeq(7.2\times10^9\text{ GeV})^{-1}\simeq10^{-34}$ sec.

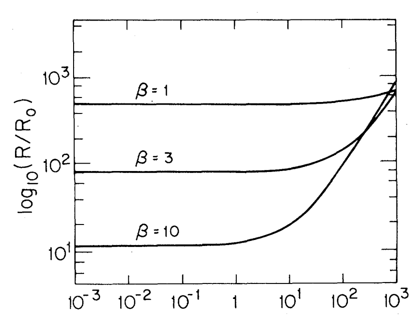

FIG. 3. The growth factor of the universe for $\varphi(0)=T(0)=\beta\times10^8$ GeV.

$\times 10^{14}$ GeV)$(\pi^2 g_*/30)^{-1/4} \simeq 10^{14}$ GeV.

In the standard scenario for producing the baryon asymmetry,[10] all the important processes occur for $T < M$, where $M \approx O(10^{14}$ GeV) is the mass of the superheavy boson whose out-of-equilibrium decays produce a net baryon number. Since the universe is reheated to a temperature of $O(10^{14}$ GeV), baryogenesis can proceed in the usual way. The details and the final asymmetry produced may be slightly different since the superheavy bosons may be initially under- or overabundant depending upon precisely which particle species are produced by the time varying φ.[11] It is also possible that the time-varying φ while far from equilibrium would directly produce an excess of baryons over antibaryons. If the requisite C and CP violation is spontaneous rather than intrinsic, the usual problems of matter and antimatter domains and of the domain walls is avoided, since the observable universe is contained with one domain.

To summarize, we find that if the value of φ in a fluctuation region is $\lesssim 7 \times 10^8$ GeV after the metastability limit ($T_1 \simeq 10^8$ GeV) is reached, then sufficient inflation occurs to solve the usual cosmological conundrums confirming the conjectures and approximations of Refs. 6 and 7. The time variation of φ results in sufficient radiation of particles to reheat the universe to $O(10^{14}$ GeV). Such efficient reheating probably insures that baryogenesis, one of the most attractive features of unification, proceeds in the usual way. However, since any initial spectrum of density fluctuations is erased during inflation, a new spectrum must be created during or after the reheating of the universe.

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