MeV. The slow component agrees, as shown in Fig. 3(a), with the energy spectrum of the neutrons decaying from the non-GQR continuum region with 20 MeV excitation energy, where no strong giant resonances are excited in the present (α, α') reaction. The fast component corresponds to the population of the excited levels around 3.5 MeV in ^{118}Sn. Direct escape of the unbound neutron particle out of the GQR leads to such one-hole states as $(1g_{7/2})^{-1}$, $(2d_{5/2})^{-1}$, $(2d_{3/2})^{-1}$, and $(3s_{1/2})^{-1}$ in ^{118}Sn. The population strength for the fast neutrons following the GQR in ^{119}Sn is in accord with the spectroscopic strength distribution for the single-hole states populated by the reaction ^{119}Sn$(p, d)^{118}$Sn,[19] as shown in Fig. 3(b).

In short, the present $(\alpha, \alpha' n)$ coincidence work demonstrates that the GQR in a medium-heavy nucleus shows up selectively above the continuum by measurement of inelastically scattered particles in coincidence with decaying fast neutrons at backward angles with respect to the recoil axis. Although the fast-neutron branch is a small fraction of the major nondirect spreading process, such direct-escape neutrons carry indeed a microscopic particle-hole feature of the GQR.

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Revised Formulation of the Phenomenological Interacting Boson Approximation

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Previous interacting-boson-approximation studies of deformed nuclei have used different forms of the quadrupole operator in the Hamiltonian and $E2$ operators. A revised formalism is proposed which employs consistent operators and embodies a simpler Hamiltonian. It yields improved agreement with the data for deformed nuclei, a number of parameter-free predictions for transition regions, a specific form of the O(6) limit which agrees with that found empirically, and a closer relation to the neutron-proton version of the model, IBA-2.

PACS numbers: 21.60.Fw, 21.60.Ev, 23.20.Lv

The interacting boson approximation[1] (IBA) represents a significant step forward in our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group-theoretical techniques which have also found recent application to problems in atomic, molecular, and high-energy physics.[2,3] The application of the phenomenological version of this model (IBA-1) to deformed nuclei is currently a subject of considerable interest and controversy.[4] Recent

studies[5,6] have revealed some of the global characteristics which must result from the application of the model to such nuclei, and it has been shown that these correspond well to the available empirical information. Nevertheless, these studies have also revealed a seeming inconsistency in the formulation of the model. The deduced parametrization of the quadrupole operator, necessary to reproduce empirical $B(E2)$ values, implies a form for this operator which is considerably different from that assumed in the Hamiltonian. It is the purpose of this Letter to propose a revised formulation which removes this inconsistency and results in equivalent or improved agreement with the data with fewer free parameters. In addition, it still yields the three limiting symmetries of the model, allows a simpler treatment of transition regions, and is more closely related to the neutron-proton version of the model, IBA-2.

The IBA-1 Hamiltonian used for deformed nuclei to date has been

$$H = -\kappa Q\cdot Q - \kappa' L\cdot L + \kappa'' P\cdot P, \qquad (1)$$

where the first two terms provide the rigorous SU(3) limit, while the third breaks this symmetry and raises the energy of the first excited $K^\pi = 0^+$ band above the first $K^\pi = 2^+$ (γ) band. This is indeed the empirical situation found in the majority of deformed nuclei. The quadrupole operator has the form

$$Q = (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + (\chi_Q/\sqrt{5})(d^\dagger \tilde{d})^{(2)}, \qquad (2)$$

where χ_Q, the ratio of the $\Delta n_d = \pm 1$ and 0 terms, can in principle vary between the values 0 and $-(35)^{1/2}/2$, associated with the O(6) and SU(3) limits, respectively. However, in IBA-1 calculations to date, the parameter χ_Q in the quadrupole operator of the *Hamiltonian* has always been *defined* to take the SU(3) value of $-(35)^{1/2}/2$. In contrast, in the $E2$ *transition operator*, which is given by $T(E2) = \alpha Q$, where α determines the absolute scale of the predicted $B(E2)$ values, χ_Q has been allowed to *vary* to fit the empirical relative $B(E2)$ values. In fact, it has recently been shown[6] that these data restrict the value of χ_Q in the $E2$ operator to the range $-1.2 \leq \chi_Q \leq -0.5$ across all deformed rare-earth nuclei. Thus, in the existing framework, different forms of the quadrupole operators in H and $T(E2)$ must be used. It is, however, clearly desirable to use consistent forms in both cases and the $B(E2)$ results just cited show that this constraint implies a reduction in the magnitude of χ_Q in the $Q\cdot Q$ term of the Hamiltonian. The effect of such a reduction will be to decrease the size of the $\Delta n_d = \pm 1$ term relative to the $\Delta n_d = \pm 2$ term, arising from $Q\cdot Q$. Since the effects of the $P\cdot P$ term in Eq. (1) arise from a similar mechanism, it is reasonable to propose a new Hamiltonian which is simply

$$H = -\kappa Q'\cdot Q' - \kappa' L\cdot L, \qquad (3)$$

where Q' is defined by Eq. (2), but χ_Q is now treated as a free parameter within the range implied by the limiting symmetries, and $T(E2)$ is defined by the identical form of Q'. Before comparing the results of this revised formalism with earlier IBA-1 calculations, it is worth noting that the form of the Hamiltonian of Eq. (3) results in a particularly simple parametrization. Since the $L\cdot L$ term is diagonal, the relative off-diagonal interactions are totally specified by $Q'\cdot Q'$. Given that the basis states are initially degenerate ($\epsilon_d = 0$), the final wave functions can depend only on χ_Q, for a given boson number N. Moreover, all relative $B(E2)$ values are also uniquely specified by the same χ_Q. The strengths κ and α then act only as scale factors on the energies and $B(E2)$ values, while κ' determines the effective moment of inertia in the bands. Thus for a given nucleus, χ_Q can be uniquely determined from a single $B(E2)$ ratio, and the study of Ref. 6 indicates that a suitable one, frequently known empirically, is $B(E2;2_\gamma^+ \rightarrow 0_g^+)/B(E2;2_g^+ \rightarrow 0_g^+)$. All other relative $B(E2)$ values, and the ordering and relative separation of the predicted bands, then follow unambiguously, and without further parametrization. This coupling of the $E2$ operator and the Hamiltonian thus differs essentially from the earlier formalism in which variations in χ_Q to fit $E2$ transition strengths left energies and wave functions unaffected. When values of χ_Q are now extracted in the manner just described, it is found that χ_Q is again restricted to a narrow range of values: in this case $-1.5 \leq \chi_Q \leq -1.0$. This range then *automatically predicts* the first $K^\pi = 0^+$ band to be above the γ band in energy, and, as pointed out in Ref. 5, this is indeed the case in the majority of deformed nuclei.

It is now instructive to compare the predictions of the two frameworks in more detail. The most thorough test to date of the original IBA-1 Hamiltonian in deformed nuclei is that of ^{168}Er,[5] and these earlier calculations are compared with those arising from the modified Hamiltonian in Fig. 1. The χ_Q parameter was determined as -1.1, and the parameters κ and κ' were chosen

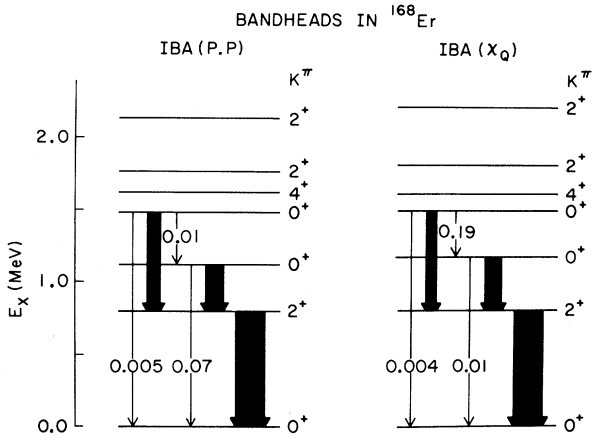

FIG. 1. Earlier (P·P) and current (χ_Q) IBA-1 calculations for ^{168}Er. The widths of, or numbers on, the arrows are the approximate squared intrinsic $E2$ matrix elements between the bands, normalized to 1 for each $\gamma \rightarrow g$ branch.

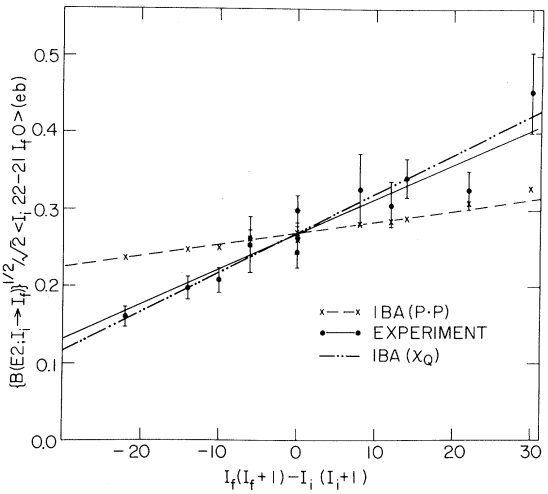

FIG. 2. Mikhailov plots for $\gamma \rightarrow g$ transitions in ^{168}Er.

to give the empirical moment of inertia in the ground (g) band, and the γ band energy. It can be seen that the bandhead energies in the two calculations are effectively identical. In addition, the relative $B(E2)$ strengths between the bands are, in most cases, equivalent in the two calculations. The two differences that emerge in the new formalism are the prediction of an even weaker $0_2^+ \rightarrow 0_g^+$ branch, and a stronger $0_3^+ \rightarrow 0_2^+$ branch. The former is actually in better agreement with the data, while no conclusion can be reached concerning the latter since the experimental limits for these transitions are consistent with either result. One of the major criticisms[4] of the earlier study[5] was that the effective $\Delta K = 2$ mixing between γ and ground band was underestimated by a factor of 3. In response, it was argued[5] that the matrix element involved (≈ 0.5 keV) was so small that this discrepancy could not be considered significant. Nevertheless, this feature can be reexamined in the new framework. Figure 2 shows the Mikhailov plot, where the slope of a given line is simply related to the $\Delta K = 2$ mixing matrix element, while its intercept at $I_i = I_f$ is related to the intrinsic $\gamma \rightarrow g$ $E2$ matrix element. While the difference of a factor of 3 in the slopes of the lines from the old calculation and the data is evident, the line representing the predictions of the revised Hamiltonian now shows excellent agreement with the data. Note that this improvement results from a formalism involving *one less parameter* than before.

As pointed out earlier, it is straightforward to extract χ_Q values for various nuclei and hence to extract the effective $\gamma \rightarrow g$ $\Delta K = 2$ mixing matrix elements in each case. The results of this procedure are shown in Fig. 3, using the alternative formalism of the band mixing parameter z_γ. The agreement with the data is remarkable, and in fact, none of the discrepancies is as large as a factor of 3. The earlier predictions of Bes *et al.*[7] are also shown, which result from a microscopic random-phase-approximation calculation based on Nilsson orbits. The trends of the two sets of predictions are evidently very similar.

While the focus here so far has been on deformed nuclei, it is interesting to inquire whether the present simplified formalism retains the

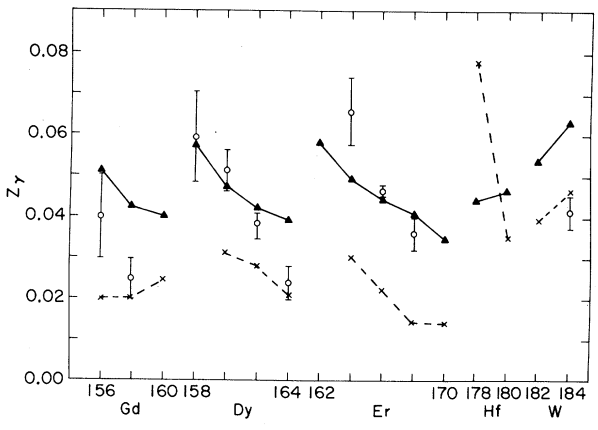

FIG. 3. The $\gamma \rightarrow g$ band mixing parameter z_γ. Solid lines and triangles, current study; dashed lines and crosses, Bes *et al.*, Ref. 7; circles, experimental values.

generality which was an essential and attractive feature of the earlier one, and thus whether it contains the other two limiting symmetries. For SU(5) nuclei, as before, a term in $\epsilon_d n_d$ can be added, while if χ_Q is chosen as zero, the quadrupole operator in H becomes a generator of O(6), and hence a level scheme with O(6) symmetry results. However, there are now only two terms in the Hamiltonian while the original formalism contained three, the $L\cdot L$ and $P\cdot P$ terms, and an "octupole" term $[\approx (d^\dagger \tilde{d})^{(3)}]$. The question therefore arises as to whether the results for a specific O(6) nucleus, such as ^{196}Pt,[8] can be reproduced. In the original formalism, the eigenvalue expression is

$$E = \tfrac{1}{4}A(N-\sigma)(N+\sigma+4)+B\tau(\tau+3)+CL(L+1),$$

where A, B, and C are constants related to the strengths of the three interactions employed. In the revised formalism, the O(6) eigenvalues are

$$E = A'\{(N-\sigma)(N+\sigma+4)+\tau(\tau+3)\}+C'L(L+1),$$

where $A' = 2\kappa$ and $C' = -\kappa'$. Thus the current formalism produces a *special case* of the O(6) limit, in which $A = 4B$. It is remarkable that the parameters used in Ref. 8 to fit the ^{196}Pt levels were $A = 185$ keV and $B = 42$ keV, and thus *almost in the 4:1 ratio.* Since the wave functions and $E2$ operators in the two calculations are the same, the $B(E2)$ values will be also. An interesting open question is whether this specific form of the O(6) symmetry corresponds to that manifested empirically in other O(6) nuclei.

In describing the transition from the O(6) to deformed regions, previous calculations[9] have introduced a progressively larger $Q\cdot Q$ term into the O(6) Hamiltonian. Clearly, with the new Hamiltonian, it can be hoped that such a transition can be simulated simply by increasing the magnitude of χ_Q. In fact, it is easy to study the effects of such a procedure on the $B(E2)$ values since contour plots of *any* $B(E2)$ ratio can be constructed, as a function of χ_Q and N. An example of such a plot is given in Fig. 4, for the ratio $B(E2;2_\gamma^+ \to 0_g^+)/B(E2;2_g^+ \to 0_g^+)$, which takes the value zero in the rigorous SU(3) and O(6) limits. In deformed nuclei, N is typically 12–16, and this $B(E2)$ ratio ranges from 0.01 to 0.04, corresponding to χ_Q values between -1.0 and -1.5. In ^{196}Pt ($N = 6$) it is effectively zero, corresponding to $\chi_Q = 0$, which is the O(6) value. Thus, an essentially *parameter-independent* prediction of Fig. 4 is the existence of a "peak" in this ratio between these two regions. In fact, in the Os nuclei this ratio has empirical values of ≈ 0.1.

To summarize, it has been shown that the proposed revision to the IBA-1 formalism provides a description of both deformed and O(6) nuclei, comparable to earlier calculations and with fewer free parameters. Contour plots of the type shown in Fig. 4 offer the intriguing possibility to predict, without specific parameter fitting, the overall behavior of any $B(E2)$ ratio across the transition from deformed to O(6)-like nuclei. Nevertheless, it should be emphasized that the simple Hamiltonian of Eq. (3) may not prove adequate in providing detailed fits to specific nuclei throughout these regions. Rather, the additional terms of the original Hamiltonian may have to be reintroduced as, albeit, smaller perturbations in any specific calculation. However, given the improved agreement of many features of the calcu-

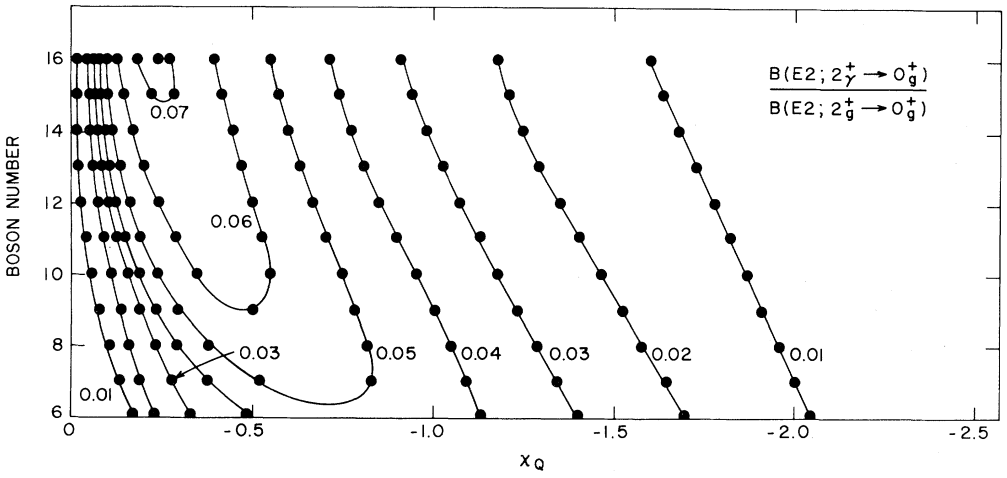

FIG. 4. Calculated contour plot of a $B(E2)$ ratio.

lated spectrum of ^{168}Er, without any such additional terms, it seems clear that this Hamiltonian represents a far better starting point for a calculation in the IBA-1 basis.

Finally, one additional point can be made. The use of a $Q \cdot Q$ term with varying χ_Q is, of course, directly analogous to the procedure employed in the IBA-2 Hamiltonian. Thus the proposed revision brings the two Hamiltonians much closer together in form, and may therefore lead to an improved ability to project the microscopic description available for the IBA-2 parameters into the IBA-1 basis. Such an outcome would be highly desirable, since it would provide an effective microscopic basis for the model while maintaining the simplicity and underlying symmetries inherent in the IBA-1 formalism.

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Angular Distributions for a Model System of Nonadiabatic Molecular Collisions: The Quenching of Na*($3p$) by H_2 and D_2

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Angular distributions for the electronic to vibrational-rotational and translational energy transfer process $Na^*(3p) + H_2, D_2 \rightarrow Na(3s) + H_2(v', j')$ with product energy analysis have been measured for the first time. The differential cross sections are forward peaked, constant but small between 35° and 160°, and very slightly increasing at 180°. The observations can be qualitatively understood by a simple model for the particle motion on the attractive $\tilde{A}^2B_2$ excited-state surface with a hop to the repulsive $\tilde{X}^2A_1$ ground state.

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The quenching of atomic resonance excitation by small molecules is one of the most elementary processes in nonadiabatic molecular collision physics. Powerful modern theoretical and experimental methods[1] justify the hope for a fully quantitative understanding of such processes, and this transfer of electronic excitation energy into heavy-particle motion is one of the key questions of photochemistry. The process

$$Na^*(3^2P) + H_2(v=0,j) \rightarrow Na(3^2S) + H_2(v',j') \quad (1)$$

at initially thermal relative kinetic energies $E_{c.m.}$ may be considered as a model system, tractable by both theory and experiment. *Ab initio* potential surfaces for this and other systems are now available,[2,3] and semiclassical[4] as well as rather rigorous quantum mechanical scattering calculations are possible.[5] Pioneering state-selective experiments in cells[6-8] were restricted to integrated cross sections and more complicated systems. The latter is also true of the first angular-resolved experiment by Silver, Blais, and Kwei.[9]

We want to report here the first experimental angular distributions with product energy analysis for the model process (1). The experimental technique is essentially that used previously[10] for the study of forward-angle averaged energy-transfer spectra. For the present differential-cross-section measurements the energy and angular resolution has been drastically improved by

