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Neutrinoless Double-Beta Decay and Muonium-Antimuonium Transitions

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The effective coupling for muonium-antimuonium transitions is calculated from analogous couplings obtained in recent analyses of nuclear double-beta decay. An effective four-fermion coupling as large as $10G_F$ is found, which should be experimentally accessible with use of muonium in vacuum.

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Recent technical advances in the formation of muonium in vacuum[1] will allow a more exacting search for muonium-antimuonium transitions (M-$\overline{M}$) than has been possible in the past. It is therefore of considerable interest to obtain an estimate of the effective coupling for this process on the basis of very recent analyses of double-beta decay data which argue for the presence of the neutrinoless double-beta decay mode.[2-6] In this note I accept the double-beta decay evidence, which gives a quantitative measure of electron number violation as discussed below, assume that a similar situation obtains in the muon sector, and thereby obtain an estimate of the effective coupling for $\mu^+e^- \to \mu^-e^+$.

Double-beta decay parameters.—Several authors have argued that the latest geochemical measurements in tellurium-isotope decays, namely the ratio $\Gamma(^{130}\mathrm{Te} \to {}^{130}\mathrm{Xe})/\Gamma(^{128}\mathrm{Te} \to {}^{128}\mathrm{Xe})$, are inconsistent with the supposition that these double-beta decay processes are always accompanied by neutrino emission as predicted by the standard model. This discrepancy can be attributed to the presence of an additional neutrinoless mode. Indeed, such an assumption has been used to infer that the electron neutrino is self-conjugate (Majorana). Assuming the absence of a right-handed component leads to a neutrino mass in the tantalizing range of 1–40 eV.[6]

In addition to the mass mechanism of Halprin *et al.*[7,8] employed above to generate the neutrinoless double-beta decay mode, there is also a right-handed current mechanism considered much earlier by Primakoff and Rosen.[9] Analyses incorporating both mechanisms have recently been given based on special cases of the effective charged-current interaction

$$\sqrt{2}H = G_F(j_L J_L^\dagger + \lambda j_R J_R^\dagger + \eta j_R J_L^\dagger + \eta' j_L J_R^\dagger) + \mathrm{H.c.} \qquad (1)$$

Here j and J are, respectively, leptonic and hadronic charged currents. Under the assumption that the electron current neutrino is itself a physical Majorana particle, one analysis ($\lambda = \eta' = 0$) determines a relationship between η and the electron neutrino mass whose extremes are $|\eta| \sim 10^{-4}$ for vanishing mass and, alternatively, a mass of 10–40 eV for $\eta = 0$.[6] Under similar circumstances, a second analysis ($\eta = \eta' = 0$) relates λ and $m_{\nu e}$ with the extreme bounds $|\lambda| \sim 10^{-4}$ for $m_{\nu e} = 0$ and, alternatively, $m_{\nu e} \sim 30$ eV for $\lambda = 0$.[2] These results are also found as special cases of the more general situation treated by Nishiura ($\lambda \neq 0$, $\eta = \eta' \neq 0$).[3]

For later reference, I note that the above interaction with $\eta = \eta'$ can be obtained from the charged-current sector of the $SU_L(2) \otimes SU_R(2)$ interaction,[10]

$$L = g_L(j_L + J_L)W_L + g_R(j_R + J_R)W_R, \quad (2)$$

where the gauge bosons $W_{L,R}$ are related through a mixing angle to the physical mass eigenstates $W_{1,2}$ by

$$W_L = W_1 \cos\theta + W_2 \sin\theta, \qquad W_R = W_1 \sin\theta + W_2 \cos\theta. \quad (3)$$

The left- and right-handed currents are defined through

$$j_{L,R}{}^{\beta} = \tfrac{1}{2}\bar{e}\gamma^{\beta}(1 \mp \gamma_5)\chi_e, \qquad J_{L,R}{}^{\beta} = \tfrac{1}{2}\bar{d}\gamma^{\beta}(1 \mp \gamma^5)u, \quad (4)$$

where χ_e is the Majorana neutrino field. The relations among the low-energy parameters in Eq. (1) and those in Eq. (3) are

$$M_1^2 G_F/\sqrt{2} = (\tfrac{1}{2}g_L)^2(1 + r^2\tan^2\theta)\cos^2\theta, \quad (5)$$

$$\lambda = (g_R/g_L)^2(\tan^2\theta + r^2)/(1 + r^2\tan^2\theta), \quad (6)$$

$$-\eta = (g_R/g_L)(1 - r^2)\tan\theta/(1 + r^2\tan^2\theta), \quad (7)$$

$$r = M_1/M_2. \quad (8)$$

Mohapatra and Vergados[11] have pointed out that there very well may be other neutrinoless double-beta decay mechanisms which do not involve neutrino exchange at all. In particular, they exhibit a Higgs mechanism leading to a breakdown of leptonic U(1) symmetry which employs a triplet with a doubly charged state, Δ^{--}, that couples to electron pairs, $\Delta^{--} \to e^- + e^-$, and which also couples to the conventional Higgs doublet, φ, of Weinberg-Salam theory. In their model the Δee interaction is

$$L = h e_L{}^T \tau_2 \vec{\tau} e_L \cdot \vec{\Delta} + \text{H.c.} \quad (9)$$

and $m_\Delta \sim 100$ GeV with $h \sim$ unity. In Fig. 1 are displayed the three distinct mechanisms for the neutrinoless mode at the quark level.

Extension to muon sector; muonium oscillations.—(A) Double-neutrino exchange mechanism: I base this estimate on the interaction given by Eq. (2) with j extended to include muons as well as electrons, i.e., e-μ universality is maintained. In so doing I ignore any lepton generation mixing and any neutral-current interaction that could directly affect a μ-e transition. The possible diagrams leading to $\mu^- + \mu^- \to e^- + e^-$ are shown in Fig. 2. The diagrams ultimately are to be evaluated in the R gauge with total disregard for any accompanying Higgs contributions that would be present in a proper theory. Those diagrams that employ a mass mechanism will contain a factor of m_ν/M_2 for each neutrino exchange mediated by the mass term in the neutrino propagator (I assume $M_2 \gg M_1$). The diagrams 2(d) and 2(e) will, in contrast, be proportional to η^2 and λ, respectively. Therefore, the most optimistic case arises if the neutrinoless nuclear double-beta decay contributions stem from the λ term. I will subsequently assume that $|\lambda| \gg |\eta|$, which is satisfied, for example, by $g_R/g_L \sim 1$ and $\tan\theta \ll r^2$, so that $\lambda \sim r^2$ and $-\eta = \tan\theta$.

In this context, the transition amplitude for $\mu^+e^- \to \mu^-e^+$ is given by the diagrams in Fig. 3 evaluated with neglect of the external four-mo-

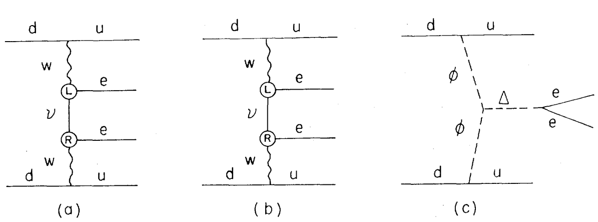

FIG. 1. Mechanisms for neutrinoless double-beta decay at the quark level. Verticles labeled L and R refer to left- and right-handed currents, respectively. (a) Majorana-neutrino exchange, $m_\nu \neq 0$. (b) Majorana-neutrino exchange, $m_\nu = 0$. (c) Higgs Δ-production mechanism.

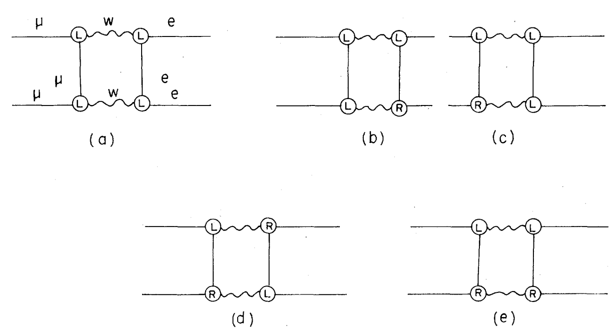

FIG. 2. Diagram for $\mu^-\mu^- \to e^-e^-$ through the various Majorana-neutrino exchange mechanisms. (a) $m_{\nu e} \neq 0$, $m_{\nu\mu} \neq 0$. (b) $m_{\nu e} = 0$, $m_{\nu\mu} \neq 0$. (c) $m_{\nu e} \neq 0$, $m_{\nu\mu} = 0$. (d) and (e) $m_{\nu e} = m_{\nu\mu} = 0$.

menta. The result is

$$T_\nu=(\tfrac{1}{2}g_L)^2(\tfrac{1}{2}g_R)^2 \sum_{b=\pm 1} \bar{\mu}\gamma_\alpha(1-b\gamma_5)\gamma_\varphi\gamma_\beta(1-b\gamma_5)\mu^c\, \bar{e}^c\gamma^\alpha(1+b\gamma_5)\gamma_\sigma\gamma^\beta(1+b\gamma_5)\bar{e}\, I^{\varphi\sigma}, \tag{10}$$

where

$$I^{\varphi\sigma}=(2\pi)^{-4}\int d^4q\, q^\varphi q^\sigma\left(\frac{\cos^2\theta}{-q^2+M_1^{\,2}}+\frac{\sin^2\theta}{-q^2+M_2^{\,2}}\right)\left(\frac{\sin^2\theta}{-q^2+M_1^{\,2}}+\frac{\cos^2\theta}{-q^2+M_2^{\,2}}\right). \tag{11}$$

Evaluation of the integral yields

$$I^{\varphi\sigma}=\tfrac{1}{4}\pi^2 f g^{\varphi\sigma},$$
$$f=(M_1^{\,-2}+M_2^{\,-2})\cos^2\theta\,\sin^2\theta+(M_2^{\,-2}-M_1^{\,-2})(\cos^4\theta+\sin^4\theta)\ln(M_2^{\,2}/M_1^{\,2}). \tag{12}$$

Manipulation of the γ matrices yields

$$T_\nu=CG_{\mathrm{F}}\sum_{b=\pm 1}\bar{\mu}\gamma^\alpha(1+b\gamma_5)\mu^c\bar{e}^c\gamma_\alpha(1-b\gamma_5)e, \tag{13}$$

where

$$C=G_{\mathrm{F}}^{\,-1}(g_L/2)^2(g_R/2)^2 f/\pi^2. \tag{14}$$

In the limit discussed above,

$$C=\lambda(G_{\mathrm{F}}M_1^{\,2}/2\pi^2)\ln(\lambda^{-1}g_R^{\,2}/g_L^{\,2}). \tag{15}$$

For numerical purposes, I use $G_{\mathrm{F}}m_p^{\,2}=10^{-5}$, $M_1=80$ GeV, and $g_R/g_L=1$, and take $\lambda=10^{-4}$ from the nuclear double-beta decay analysis of Doi *et al.*, which yields $C=3\times 10^{-6}$. Because of various uncertainties in both the analysis of the experimental data and the theoretical evaluation of nuclear physics effects, it is not unreasonable to use a value of λ an order of magnitude larger, increasing the amplitude to $C=2\times 10^{-5}$. The present experimental limit on C is of order 5×10^3,[12] so that observation of M-$\overline{M}$ transitions at the level one might reasonably expect from this mechanism would require a truly heroic effort.

(B) Higgs mechanism: Now we shall assume that neutinoless double-beta decay receives an appreciable contribution from the Δ^{--} Higgs mechanism of Mohapatra and Vergados. It is not unreasonable to assume that the same meson also couples to μ^- pairs with equal strength. We then have the M-$\overline{M}$ diagram of Fig. 4. In nuclear double-beta decay, the Δ^{--} contribution is suppressed by mass factors arising from propagation of the Weinberg-Salam Higgs pair. Here no such suppression occurs. The M-$\overline{M}$ transition amplitude is then characterized by an effective C factor of order $h^2/m_\Delta^{\,2}\,G_{\mathrm{F}}$. Using the very reasonable values of Ref. 10, $h\sim 1$ and $m\sim 100$ GeV, we have $C\sim 10$, which is not so very far from the present experimental limit.

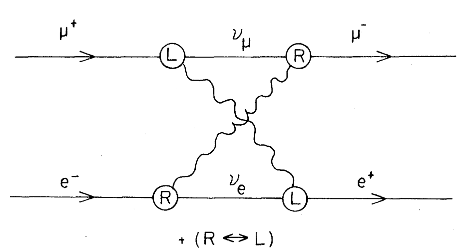

FIG. 3. Dominant Majorana-neutrino exchange diagram for muonium-antimuonium transitions under the assumptions discussed in the text.

In summary, by utilizing recent hints from neutrinoless nuclear double-beta decay, I have examined the similar but wholly leptonic process of muonium-antimuonium transitions. A simple adaptation of the conventional Majorana-neutrino exchange mechanism can reasonably be made to produce an M-$\overline{M}$ transition amplitude characterized by a strength of $10^{-5}G_{\mathrm{F}}$. In contrast, an adaptation of the alternative Δ^{--} mechanism of neutrinoless double-beta decay, which does not involve the exchange of Majorana neutrinos, yields a characteristic strength of $10G_{\mathrm{F}}$ for the M-$\overline{M}$ amplitude. These results provide additional incentive for going forward with the exact-

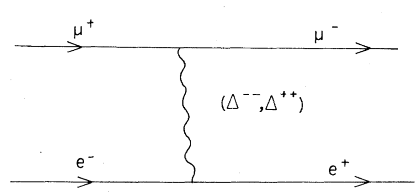

FIG. 4. Higgs Δ-mediated muonium-antimuonium transition.

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Amplitude Analysis of the Reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$ from 1.0 to 2.3 GeV

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An amplitude analysis of $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering, from the reaction $\pi^+p \rightarrow \Delta^{++}\pi^0\pi^0$, has been carried out between 1.0 and 2.3 GeV. The data are consistent with only one of the four π-π phase-shift solutions determined from $\pi^+\pi^-\rightarrow\pi^+\pi^-$ data. A new $J^{PC} = 2^{++}$ state at 1800 MeV with a width of 280 MeV is observed.

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Information on the fundamental process of π-π scattering above 1 GeV has come primarily from analyses[1,2] of the reaction $\pi^+\pi^- \rightarrow \pi^+\pi^-$. These analyses lead to four ambiguous solutions (called A, B, C, and D) for the amplitudes above 1.4 GeV. Preliminary[3] (unextrapolated) $\pi^0\pi^0$ data were used to rule out solutions C and D. Modification[4] of these analyses by means of analyticity and unitarity led to solutions called α, a variant of A, and β and β', variants of solution B. More recent experiments[5,6] studying $\pi^+\pi^- \rightarrow \pi^+\pi^-$ have come to opposite conclusions regarding the resolution of these ambiguities: Becker *et al.*[6] favor the B-type solutions β and β'; Corden *et al.*[5] prefer a solution they call G which is a combination of solution A and solution C. Resolution of the ambiguities is crucial in determining the spectrum of meson states which couple to the π-π system. Solution A requires a new $J^{PC} = 2^{++}$ state at 1550 MeV not present in solution B. In contrast, solution B contains a π-π coupling to the $\rho'(1600)$ not present in solution A. The spectrum is important since the presence or the absence of "unusual states" such as glueballs, $2q$-$2\bar{q}$ states, and q-$\bar{q}$ radial excitations must be accounted for in constructing the definitive form of QCD.

In this paper we report the results of an amplitude analysis of data on $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering. This analysis is considerably simpler than that for $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering since only even partial waves having $I = 0, 2$ contribute. We obtain a unique solution which is consistent with only one of the previous ambiguous solutions. In addition, we observe a $J^{PC} = 2^{++}$ state at 1.8 GeV which has not been previously observed in π-π scattering analyses and which appears to be one of the "unusual states."

The data for this experiment were obtained at the Argonne National Laboratory by use of the 1.5-m streamer chamber in combination with a 68-element lead-glass hodoscope. The outgoing charged tracks were detected in the streamer chamber, and the γ's from π^0 decay were de-

