Vol. I, p. 1971; R. Cowsik and J. McClelland, Phys. Rev. Lett. 29, 669 (1972); Ap. J. 180, 7 (1973)], or heavier than about 2 GeV (see Ref. 7) *and* lighter than the temperature of the universe after the gravitinos decay. The last proviso is added here to guarantee that the gauge fermions have time to annihilate; Eq. (4) shows that it can be satisfied only if $\sqrt{F} > 10^{13}\xi^{-1/6}$ GeV. This proviso can be somewhat relaxed if the gauge-fermion mass is larger than a few gigaelectronvolts. A more thorough discussion of gauge-fermion annihilation in a specific model is given by S. Dimopoulos and S. Raby, to be published.

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[14]This may raise the early baryon-entropy ratio above levels which can be accounted for by baryon-nonconserving processes in the very early universe. For instance, if $\sqrt{F} = 10^{15}$ GeV so that $m_g = 10^{11}$ GeV, then (4) gives an entropy increase by a factor $> 10^4$; a present value of 10^{-10} for the baryon-entropy ratio would thus correspond to an early value of $> 10^{-6}$, which seems rather high. Of course, those would be no problem here if one were to suppose that baryon number is conserved, and that a large baryon-entropy ratio was put in at the beginning, or that $\sqrt{F} > 10^{18}$ GeV, in which case baryon production could occur after the gravitinos decay.

Determination of the Fundamental Parameters of the K^0-$\bar{K}^0$ System in the Energy Range 30–110 GeV

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K^0 regeneration data in the energy range 30–110 GeV have been analyzed to determine the values of the K_L-K_S mass difference, Δm, K_S lifetime, τ_S, and CP-nonconservation parameter η_{+-}. We find $\Delta m = 0.482(14)\times 10^{10}\hbar\ \mathrm{s}^{-1}$, $\tau_S = 0.905(7)\times 10^{-10}$ s, $|\eta_{+-}| = 2.09(2)\times 10^{-3}$, and $\tan\varphi_{+-} = 0.709(102)$, corresponding to $\varphi_{+-} = 35(4)°$. The data suggest that these parameters may have an anomalous energy dependence.

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In this Letter we describe an analysis of data from three K^0 regeneration experiments[1-3] to determine for the first time the fundamental parameters of the K^0-$\bar{K}^0$ system at Fermilab energies. The details of the work summarized here are to be published elsewhere.[4]

In the experiments analyzed here, a K_L beam impinged on a material target (hydrogen,[1] carbon,[2] or lead[3]) and the $K^0 \to \pi^+\pi^-$ decay distribution in the forward direction behind the targets was studied. The proper time distribution of the decays is given by[5]

$$N_{+-}(t) = N_L\{|\rho|^2\exp(-t/\tau_S) + |\eta_{+-}|^2\exp(-t/\tau_L) + 2|\rho||\eta_{+-}|\exp(-t/2\tau_S - t/2\tau_L)\cos(\Delta m t + \Phi)\}, \quad (1)$$

where N_L is the flux of K_L's, $t = 0$ at the exit face of the target, and $\Phi = \arg\rho - \arg\eta_{+-} \equiv \varphi_\rho - \varphi_{+-}$. The coherent regeneration amplitude ρ for a target of length L with N scatterers per unit volume is given by

$$\rho = i\pi N\Lambda_S\,\alpha(L/\Lambda_S)[f(0)-\bar{f}(0)]/k. \tag{2}$$

In Eq. (2) Λ_S is the K_S mean decay length, k is the K^0 wave number, and $f(0)$ $[\bar{f}(0)]$ is the forward elastic scattering amplitude of K^0 ($\bar{K}^0$) for the target used. The function α contains the effects of integrating the regeneration amplitude over a finite length target:

$$\alpha(L/\Lambda_S) = \frac{1-\exp(-\beta L/\Lambda_S)}{\beta}; \quad \beta \equiv \tfrac{1}{2} - i\Delta m\tau_S. \tag{3}$$

In Refs. 1–3 the $\pi^+\pi^-$ decay distributions in 10-GeV kaon energy bins were fitted with Eq. (1) to determine $|\rho|$ and φ_ρ, and in Refs. 1 and 2 N_L was measured with $K_L \to \pi\mu\nu$ decays collected simultaneously. The parameters Δm, τ_S, τ_L, $|\eta_{+-}|$, and φ_{+-} were set equal to their accepted values[6] as determined at low energy.

Here we have analyzed the data, treating Δm, τ_S, and $|\eta_{+-}|$ as variables in the fits and letting them attain their "best" values. Because the data cannot sustain a fit to this many free parameters, we have constrained $|\rho|$ and φ_ρ to lie near their published values.[7] A systematic study of the effects of these "soft" constraints is presented by Aronson *et al.*[4]

The phase φ_{+-} of η_{+-} cannot, however, be extracted as directly from these data, and must be handled differently. As inspection of Eq. (1) indicates, the measured quantity is $\Phi = \varphi_\rho - \varphi_{+-} = \pi/2 + \varphi_\alpha + \varphi_{21} - \varphi_{+-}$, where $\varphi_\alpha = \arg\alpha(L/\Lambda_S)$ and $\varphi_{21} = \arg[f(0)-\bar{f}(0)]$. Thus, it is necessary to supply external information on φ_{21} to obtain φ_{+-}. We have done this by computing φ_{21} with several Regge-pole and absorption models. The results of these calculations[4] are consistent in the amount of energy dependence of φ_{21} predicted in our energy range.[8] The phase results quoted here use φ_{21} from the recent model of Diu and Ferraz de Camargo.[9]

The fit to Eq. (1) was carried out in two different ways, denoted by A and B.

Method A.—The entire 30–110-GeV energy range was used to determine the best values of each of the parameters; these are given in Table I. If the parameters are taken to be constants, then the best values are

$$\Delta m = 0.482(14)\times10^{10}\hbar\ \mathrm{s}^{-1},\quad \tau_S = 0.905(7)\times10^{-10}\ \mathrm{s},$$
$$|\eta_{+-}| = 2.09(2)\times10^{-3},\quad \tan\varphi_{+-} = 0.709(102). \tag{4}$$

Note that our result for $|\eta_{+-}|$ is in excellent agreement with the value $2.15(14)\times10^{-3}$ obtained earlier by Birulev *et al.*[10] in the range 14–50 GeV. Note also that the results in Eq. (4) differ from the corresponding low-energy values[6] ($E_K \simeq 5$ GeV) by 4, 2, 9, and 3 standard deviations for Δm, τ_S, $|\eta_{+-}|$, and $\tan\varphi_{+-}$, respectively. This has led us to investigate a possible energy dependence of the K^0-$\bar{K}^0$ parameters. Accordingly,

TABLE I. Results of the direct, one-step analysis A.

Parameter		Energy-independent fit x_0	χ^2/d.f.	Fits of the form $x = x_0(1+b_x\gamma^2)$ x_0	$10^6 b_x$	χ^2/d.f.
$10^{-10}\Delta m$	a	0.482 ± 0.014	536/488	0.557 ± 0.036	-8.48 ± 2.89	521/484
($\hbar$ sec^{-1})	b	0.534 ± 0.002	604/492	0.535 ± 0.002	-7.43 ± 1.48	533/488
	c	0.532 ± 0.002	573/492	0.534 ± 0.002	-6.30 ± 1.46	550/488
$10^{10}\tau_S$	a	0.905 ± 0.007	536/488	0.880 ± 0.015	$+1.77\pm0.90$	521/484
(sec)	b	0.895 ± 0.002	604/492	0.892 ± 0.002	$+1.27\pm0.38$	533/488
	c	0.893 ± 0.002	573/492	0.892 ± 0.002	$+0.99\pm0.38$	550/488
$\|\eta_{+-}\|$	a	2.09 ± 0.02	536/488	2.14 ± 0.04	-2.01 ± 0.86	521/484
(10^{-3})	b	2.14 ± 0.01	604/492	2.23 ± 0.02	-3.60 ± 0.52	533/488
	c	2.07 ± 0.01	573/492	2.07 ± 0.02	-0.20 ± 0.62	550/488
$\tan\varphi_{+-}$	a	0.709 ± 0.102	536/488	1.276 ± 0.499	-33.7 ± 12.3	521/484
	b	1.009 ± 0.036	604/492	0.954 ± 0.048	-21.5 ± 7.0	533/488
	c	1.081 ± 0.040	573/492	1.033 ± 0.052	-22.3 ± 6.7	550/488

[a] Internal fit.
[b] External fit, with low-energy values at $E_K \cong 5$ GeV: $\Delta m = 0.5349\pm0.0022$, $\tau_S = 0.8923\pm0.0022$, $|\eta_{+-}| = 2.274\pm0.022$, and $\tan\varphi_{+-} = 0.986\pm0.055$.
[c] As in b, except $|\eta_{+-}| = 1.95\pm0.03$.

we allowed each parameter in Eq. (1) to vary as

$$x = x_0[1 + b_x\gamma^N]; \; N = 1, 2; \; \gamma = E_K/m_K, \qquad (5)$$

where $x = \Delta m$, τ_S, $|\eta_{+-}|$, or $\tan\varphi_{+-}$. The case $N = 2$ is the one given in the table; the $N = 1$ results[4] are similar. We performed fits to x_0 and the slopes b_x first using only the data of Refs. 1–3 ("internal" fits), and then including in addition a world average value[6] at $E_K \simeq 5$ GeV ("external" fits). We see from the *internal* fits [entries (a) in the table] that the b_x differ from zero by 3, 2, 2, and 3 standard deviations for Δm, τ_S, $|\eta_{+-}|$, and $\tan\varphi_{+-}$, respectively.[11]

Method B.—The data of Refs. 1–3 were fitted by Eq. (1) to determine Δm, τ_S, $|\eta_{+-}|$, and $\tan\varphi_{+-}$ in each of eight energy bins (10 GeV wide) from 30 to 110 GeV. The resulting values of each parameter were then fitted to Eq. (5) to determine x_0 and b_x. This two-step procedure is perhaps easier to grasp, but has a potential drawback in that correlations among the slope parameters, if they exist, are partially obscured. Figure 1 shows an example of this analysis for $\tan\varphi_{+-}$, and similar graphs for the other parameters are given in Ref. 4. The results obtained by Methods A and B are in good qualitative agreement, the principal difference being that Method B gives slope parameters which are somewhat smaller in magnitude and also in statistical significance.

Notwithstanding the agreement between the results of these two different methods, the possibility remains that the apparent energy dependence of the K^0-$\bar{K}^0$ parameters is due to some unknown

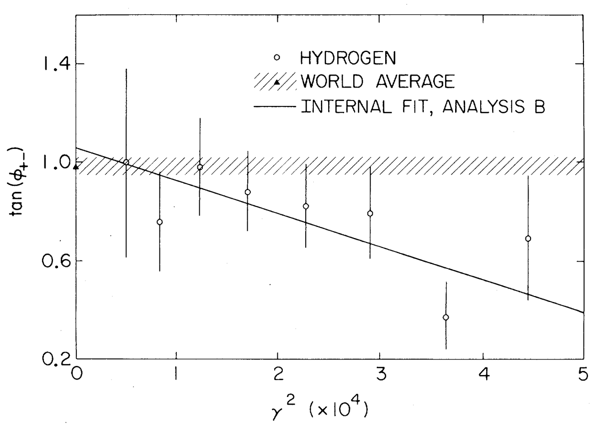

FIG. 1. A plot of $\tan\varphi_{+-}$ vs γ^2 using the results for φ_{+-} from each energy bin in analysis B. The line is the best fit to the eight Fermilab data points, and is not constrained by the low-energy world average value. The shaded band gives the error on the low-energy value, which is denoted by a triangle.

systematic effect in the data. To test for this we carried out an extensive series of auxiliary tests[1–4] on the data to search for various systematic effects. No effect was found which could account for the present results. We stress that *the internal fit results are completely independent of the low-energy determinations of any of the parameters*, and hence of any possible systematic uncertainties arising from a comparison of high- and low-energy data. When the low-energy values *are* included [entries (b) and (c) in the table], slope parameters of greater statistical significance (typically 3–5 standard deviations) emerge.

Effects of the type reported here, namely a dependence of quantities determined in the K^0 rest frame on the K^0 laboratory energy, could arise from the motion of the kaons with respect to some external field or medium. A phenomenological analysis of such effects in terms of external fields is treated in detail in Ref. 4. The conclusion of this analysis is that the present results cannot be explained in terms of a conventional hypercharge, electromagnetic, or gravitational field, or from the scattering of K^0 and $\bar{K}^0$ from stray charges or neutrinos. This suggests that these effects, if real, may arise from some new interaction. One candidate is a tensor field of finite range, a possibility which has been considered by a number of authors.[12] Such a field could at the same time resolve a number of other problems, including the missing mass of the universe and possible discrepancies in determination of neutrino oscillation parameters. These and other related questions are discussed in more detail in Ref. 4.

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[7]We "softly" constrained each parameter p ($p = |\rho|$, φ_ρ) to the value p_0 by adding to χ^2 terms of the form $(p - p_0)^2/\sigma_p{}^2$, where σ_p governs the softness of the constraint. We took p_0 to be the published values (see Refs. 1–3) and σ_p to be 1, 3, or 6 times the published errors. We found that for all kaon energies below 110 GeV the data in fact determine *all* parameters rather well, the average contribution to χ^2 from either the $|\rho|$ or φ_ρ term being ≤ 0.04.

[8]All models predict a change in φ_{21} of $\leq 2°$ for hydrogen in the energy range 30–110 GeV. By contrast, our hydrogen data show that in this range $\varphi_{21} - \varphi_{+-}$ changes by $(19.3 \pm 6.4)°$.

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[11]As can be seen from the table, the four constraints $b_x = 0$ lead to an increase in χ^2 of 15. This means that the data exclude the hypothesis of no energy dependence at a confidence level > 95%. Note also that in the energy-dependent fits the low-energy intercepts, x_0, are in better agreement with the accepted low-energy values than are the energy-independent results in Eq. (4).

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Conformally Invariant Quantization of the Liouville Theory

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The Liouville theory is quantized with use of Fock-space methods, an infinite set of charges L_n, $n = 0, \pm 1, \ldots$, is constructed which represents the conformal algebra in two dimensions, and consequences of this algebra are discussed. It is then argued, with use of variational methods in Fock space, that the spectrum of the Liouville Hamiltonian is continuous, and that there exist energy eigenstates obeying the constraints $L_n|E\rangle = 0$, $n > 0$.

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The (1 +1)-dimensional Liouville quantum field theory,[1] described by

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu \varphi)^2 - \mu^2 \exp(2\pi^{1/2}\varphi/\beta), \tag{1}$$

appears in a manifestly covariant quantization of the relativistic string.[2] In this Letter we exhibit a quantization scheme for the Liouville model which maintains the conformal symmetry of the classical theory. Within this scheme the energy spectrum and certain correlation functions can be calculated exactly.

Let us first review the classical Liouville field theory.[3] Since we eventually wish to apply our results to the quantum string problem, we shall study (1) on a finite space of length L: $0 \leq x \leq L$. For simplicity we shall also choose periodic boundary conditions appropriate to closed string theories. It is convenient to introduce dimensionless coordinates $2\pi x = L\sigma$, $2\pi t = L\tau$, $0 \leq \sigma \leq 2\pi$, and to employ the Fourier series expansions[4]

$$\varphi(\sigma) = \frac{i}{2\sqrt{\pi}}\left[a_0 - a_0{}^\dagger + \sum_{\substack{n\neq 0\\ -\infty}}^{\infty} \frac{1}{n}(a_n e^{-in\sigma} + b_n e^{in\sigma})\right], \tag{2a}$$

$$\pi(\sigma) \equiv \dot{\varphi}(\sigma) = \frac{1}{2\sqrt{\pi}}\left[a_0 + a_0{}^\dagger + \sum_{\substack{n\neq 0\\ -\infty}}^{\infty} (a_n e^{-in\sigma} + b_n e^{in\sigma})\right], \tag{2b}$$

where

$$a_n{}^\dagger = a_{-n},\ b_n{}^\dagger = b_{-n} \text{ for } n \neq 0. \tag{3}$$

