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Stress Tuning of the Metal-Insulator Transition at Millikelvin Temperatures

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A high-resolution scan of the metal-insulator transition in Si:P at millikelvin temperatures has been obtained by applying uniaxial stress. A sharp, but continuous, metal-insulator transition is resolved, with conductivities below Mott's "minimum" value σ_M. These measurements join smoothly with previous low-resolution experiments, ruling out any discontinuity at σ_M. The reproducible critical behavior disagrees with predictions of existing scaling theories of localization.

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Our understanding of the metal-insulator transition in random systems rests in part on Anderson's demonstration[1] of the absence of electronic diffusion at zero temperature in a sufficiently strong random potential and in part on the work of Mott[2] and Hubbard[3] emphasizing the role of electron correlations in stabilizing the insulating state. This transition has been observed in different materials at electron concentrations n_c varying over eight orders of magnitude,[4] but with a universal scaling for which Mott has argued using simple models:

$$n_c^{1/3} a_B \simeq \tfrac{1}{4}, \tag{1}$$

where a_B is the radius of the isolated impurity wave function. A prototype of such systems is a random array of P donors placed substitutionally into a crystalline Si lattice.

Following these ideas Mott suggested[5] in 1972 that the zero-temperature conductivity $\sigma(0)$ jumped from zero in the insulator to a minimum metallic value

$$\sigma_M = C_M e^2/\hbar n_c^{-1/3}, \tag{2}$$

where $C_M = \frac{1}{20}$ within a factor of 2. For Si:P, $\sigma_M = 20\ (\Omega\ \mathrm{cm})^{-1}$. According to Mott's recent survey,[6] all existing data support this remarkable conclusion.

In contrast, the single-particle scaling theories of localization[7] (neglecting electron-electron interactions) predict a continuous variation through the critical region, i.e., $\sigma(0) < \sigma_M$, of the form

$$\sigma(0) \simeq \sigma_M (n/n_c - 1)^\nu, \tag{3}$$

where $\nu \approx 1$. Although experimental studies[8,9] of Si:P and α-$\mathrm{Ge}_{1-x}\mathrm{Au}_x$ have found $\sigma(0)$ values significantly less than σ_M, Mott has argued[6] correctly that macroscopic inhomogeneities could have played a dominant role for $\sigma(0) < \sigma_M$. Furthermore, in Si:P, the sharpness of the transition[8] precluded resolution of this critical region. Thus, Eqs. (2) and (3) have escaped a definitive test.

We report here a high-resolution, zero-temperature study of the metal-insulator transition, using uniaxial compressive stress S to tune n_c through n of uncompensated, slightly insulating Si:P samples. We find reproducible behavior for $n/n_c - 1 > 10^{-3}$ of the form $\sigma(0) \propto (S - S_c)^\nu$, with $\nu = 0.48 \pm 0.07$, which is distinguishable from apparent rounding of the transition at lower n/n_c. The reproducible behavior is consistent with previous findings[8] for $\sigma(0) > \sigma_M$ which gave $\sigma(0) = 13\sigma_M(n/n_c - 1)^\nu$, with $\nu = 0.55 \pm 0.10$. We claim that the reproducibility and consistency identify intrinsic behavior; the data thus rule out Mott's minimum metallic conductivity and are inconsistent with existing scaling theories of localization.

The capability of significantly varying n_c with modest stresses[10] relies both on the small energy scale of the donor band (~1 meV, rather than typical electronic energies ~1 eV) and on the multivalley nature of the Si conduction band. The latter allows a direct coupling of the donor wave function[10] and thus the width of the donor band to uniaxial deformations. For effective-mass donors with a degenerate ground state, the variation of n_c can be reduced[11] to a Mott criterion [Eq. (1)] with different effective Bohr radii in the high-stress (single valley) and stress-free (many valley) cases.

In Si:P the valley degeneracy of the donor wave function is lifted at zero stress as a result of the short-range central-cell potential, which also causes a shrinkage of the ground-state wave-function radius. Application of stress mixes in the relatively more extended excited states and reduces n_c as per Eq. (1): An insulator is thus transformed into a metal at $T = 0$ K. [This qualitative conclusion of Eq. (1) is supported by a detailed calculation[12] of the effect of stress on the donor bands.] We approximate the (in general,

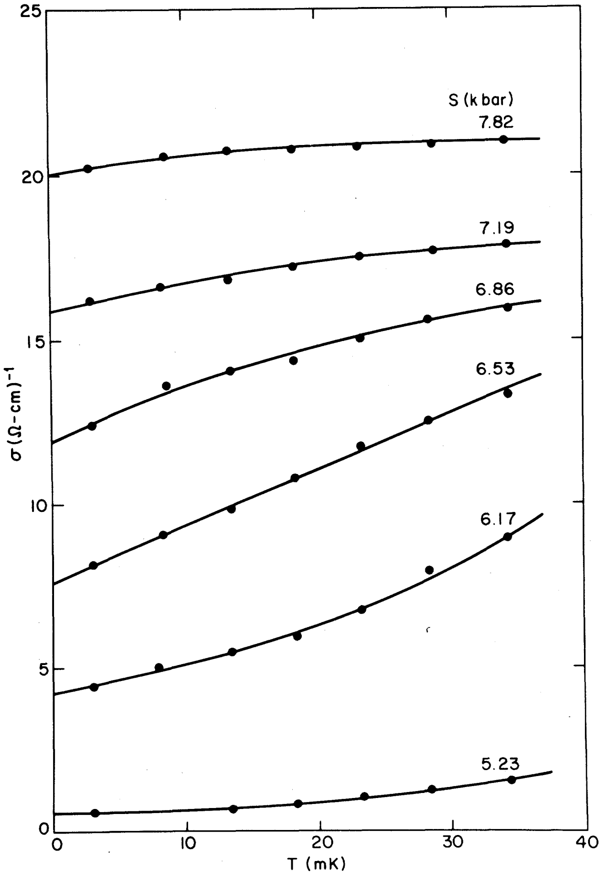

FIG. 1. Conductivity σ of a sample of Si:P as a function of temperature T for a series of values of uniaxial stress S near the metal-insulator transition. The solid lines are fits by the form $\sigma(T) = \sigma(0) + BT^\beta$ where $\sigma(0)$, B, and β are fitting parameters.

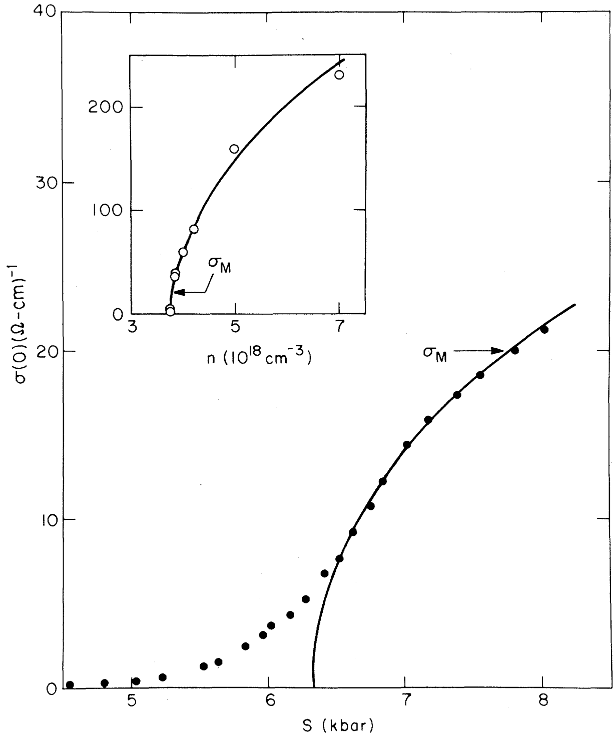

FIG. 2. Extrapolated values of zero-temperature conductivity $\sigma(0)$, obtained as illustrated in Fig. 1, as a function of uniaxial stress S. The solid line is fit to the region of $\sigma(0)$ which reproduces in three samples and has the form $\sigma(0) \propto (S - S_c)^\nu$ with $\nu = 0.49$. The inset shows $\sigma(0)$ vs n from Ref. 8 with the curve $\sigma(0) = 13\sigma_M(n/n_c - 1)^\nu$ with $\nu = 0.55$.

nonlinear) variation of n_c with stress S by a linear relation over the critical region (7.3 ± 0.8 kbar for our three samples), since it is a narrow range around a moderate value of S. At 4.2 K, our measurements of conductivity in fact show an essentially linear dependence on S between 4 and 12 kbar.

Our Si:P samples, with dimensions $0.3 \times 0.8 \times 7.0\ \text{mm}^3$, were prepared as in Ref. 8. Two were parallel cuts from the same wafer (perpendicular to the [111] axis), and in one of the slices two sections (samples 2 and 3) were measured with the same current leads. For sample 3, the voltage leads were not collinear with the current flow. The samples were mounted in a pressure device operated by ^{4}He as illustrated in the inset to Fig. 3. S (applied approximately along $[12\bar{3}]$) was measured capacitively at the upper end of the sample. This device was thermally anchored to a Cu-nuclear cooling system, with T measured by using a ^{3}He melting curve thermometer. The conductivity measurements were done at 11 Hz frequency and at power levels below 10^{-15} W.

Sample 1 was cooled twice from room temperature with quantitatively the same results. Samples 2 and 3 also showed the same critical behavior, and only differed from each other by 15% at low S. This reproducibility rules out large inhomogeneities in current and S. After initial stress cycling, the samples showed no hysteresis in S at constant T to within the accuracy of our measurements [0.5% in $\sigma(T)$].

The variation of $\sigma(T)$ was similar for all our samples. In Fig. 1 we have plotted $\sigma(T)$ for sample 1 at values of S close to the transition. The extrapolations shown (based on least-squares fits by the form in the caption) indicate metallic, i.e., finite $\sigma(0)$ values below σ_M. Above 5 $(\Omega\ \text{cm})^{-1}$ the form of $\sigma(T)$ varies with S, but below all the samples had $\sigma(T) = \sigma(0) + AT^2$, as found in Ref. 8 for $\sigma(0) \lesssim \sigma_M/10$. The T^2 term is not understood but a similar contribution has been shown to be related to surface conditions in insulating samples.[13] For small S, the variation of $\sigma(0)$ (although sample dependent) is roughly exponential in S, inconsistent with the classical percolation[14] form, $(n - n_c)^{1.6}$.

Figure 2 shows $\sigma(0)$ as a function of S for sample 1, while the inset exhibits the corresponding variation with n. By fitting the data in Fig. 2 above 6.5 kbar and the (similar) 3-mK data for samples 2 and 3 with the form $(S - S_c)^\nu$, we get S_c(kbar) = 6.3, 6.5, and 6.5, all ± 0.2, and $\nu = 0.49$, 0.41, and 0.51, all ± 0.07. The quoted errors include not only small statistical errors, but also systematic errors inherent in the $T = 0$ K extrapolations and in the choice of the lowest S value to include in the fit. We argue that the agreement among samples indicates universal behavior above S_c in the region fitted by the solid line in Fig. 2.

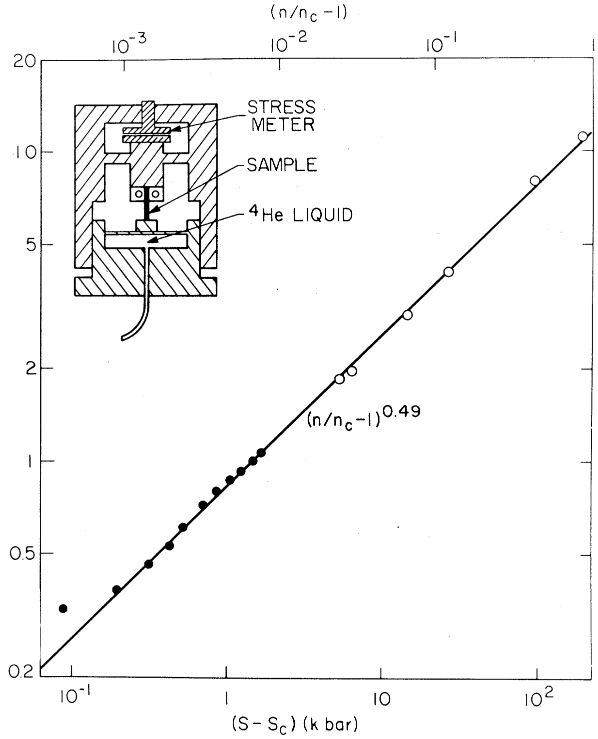

FIG. 3. A combination of the data from Fig. 2 and its inset to illustrate the smooth variations of $\sigma(0)$ through σ_M. Values of $n/n_c - 1$ are shown on the upper scale based on the open circles from Ref. 8. Values of $S - S_c$ are shown on the lower scale based on the solid circles from this work. A combination of all the results gives $\nu = 0.48 \pm 0.07$.

If one assumes that n_c varies linearly with S, our values of ν agree with the previous $\nu = 0.55 \pm 0.10$. To emphasize this agreement, we combine both sets of Fig. 2 on a log-log scale in Fig. 3. The scaling of stress and density shown implies[15] $(n/n_c - 1)/(S - S_c) = 5.4 \times 10^{-3}\ \text{kbar}^{-1}$. The smooth variation rules out a significant change in the behavior of $\sigma(0)$ as it passes through σ_M. The solid line is a fit by the scaling form of Eq. (3) for $\frac{1}{4}\sigma_M \lesssim \sigma(0) \lesssim 13\sigma_M$ and $10^{-3} \lesssim n/n_c - 1 \lesssim 1$. The wide range of this fit suggests that the characteristic conductivity of the transition region may be significantly larger than σ_M. This conclusion is supported by the fact that the measured conductivity deviates markedly from either that calculated[8] for free electrons or that deduced[6] from the specific heat[16] below a conductivity ~ $10\sigma_M$, close to the Ioffe-Regal value[17] $\sigma_{IR} \simeq e^2/3\hbar n_c^{-1/3}$.

Even for $\sigma(0) < \sigma_M$, estimated as the critical region by current scaling theories of localization,[7,18] our results give a $\nu = 0.48 \pm 0.07$, inconsistent with the theoretical $\nu \approx 1$. We speculate that the more rapid variation of $\sigma(0)$ with $n - n_c$ arises from Coulomb effects whose importance has been emphasized by Mott.[2,5,6] Additional evidence, attributed to Coulomb interactions in bulk systems, has come from the temperature,[19,20] magnetic field,[20] and compensation[21] dependence of the conductivity, and also from the tunneling conductance.[22]

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