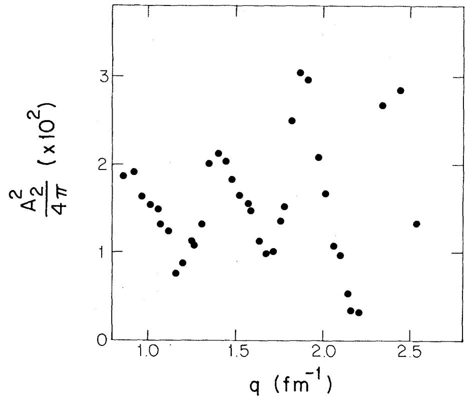

FIG. 3. $A_2{}^2/4\pi$ for the 2^+ (0.081) state of ^{154}Sm excited by 800-MeV protons (Ref. 11) plotted as a function of momentum transfer q. The wild oscillations indicate the non-Tassie nature of the excitation.

coupling strength, A_L, and presented a simple procedure for extracting it from the data in a model-insensitive way using large-momentum-transfer scattering of strongly interacting probes. The independent measures A_L and $B(EL)$ can be used to constrain the functional form of the transition density. The method is easily generalized to non-Tassie forms, and should provide a simple, model-insensitive way to extract new nuclear structure information from large-momentum-transfer inelastic hadron scattering.

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$$\beta_L = [(2L+1)/4\pi]^{1/2}A_L.$$

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Antineutrino Spectrum from the Fission Products of ^{239}Pu

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The electron and antineutrino spectra from the products of thermal fission of ^{239}Pu were calculated. A microscopic calculation of the β-decay schemes of the neutron-rich fission products led to an accuracy in absolute and relative shapes of the predicted e^- and $\bar{\nu}$ spectra of <5% in the range $2.5 \leq E \leq 7.5$ MeV. The calculations yield, together with the recently calculated $\bar{\nu}$ spectrum of the fission products of ^{235}U, the first reliable theoretical basis for the interpretation of reactor antineutrino-oscillation experiments.

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Lepton-number nonconservation would manifest itself in neutrino oscillations and these would be the most sensitive tool to search for a lepton mixing.[1,2] The weak-interaction eigenstates ν_e, ν_μ, and ν_τ would not be stationary states but superpositions of pure states ν_1, ν_2, and ν_3 with finite rest masses. Gauge model descriptions[2] of the pure states ν_i predict flavor oscillations $\nu_e \rightarrow \nu_\mu$, $\nu_\mu \rightarrow \nu_\tau$, $\nu_\tau \rightarrow \nu_e$, and particle-antiparticle oscillations $\nu_e \rightarrow \bar{\nu}_{eL}$, etc., where $\bar{\nu}_{eL}$ is a wrong- (left-) handed antineutrino.

The search for neutrino oscillations thus is of extreme interest in connection with the fundamental structure of leptonic currents. It is also of large interest because of its consequences for the particle physics in the very early universe.[3,4]

A nonvanishing neutrino rest mass further could significantly affect the total mass of the universe, and also the apparent lack of solar neutrinos could be due to a neutrino mixing.

In the case of a two-neutrino system (which might well approximate the general case), the oscillations are described by the oscillation length $L = 2.5p/M^2$ which is, by $M^2 = |m_1^2 - m_2^2|$, dependent on the difference of the mass eigenvalues m_i (L in meters, the neutrino momentum p in megaelectronvolts, and M^2 in electronvolts squared), and a mixing parameter θ. The probability of finding a $\nu_{l'}$ at a distance R from a source of ν_l is

$$w_{\nu_{l'},\nu_l} = \tfrac{1}{2}\sin^2\theta[1 - \cos(2\pi R/L)], \tag{1}$$

while

$$w_{\nu_l,\nu_l} = 1 - w_{\nu_{l'},\nu_l}. \tag{2}$$

Thus, oscillation effects will be observable only when $L \lesssim R$, and the neutrino mass ranges accessible by various types of experiments are $M^2 > M_{\min}^2 = 4\pi p_{\min}/R_{\max}$.

Reactor experiments are suitable to study mass differences in the range $0.01 < M^2 < 5$ eV^2 ($E_{\bar{\nu}} < 10$ MeV, $d < 100$ m) which covers a range of particular theoretical interest. In the "minimal form" of the O(10) grand unification theory the neutrino mass matrix is found to be proportional to the u-quark mass matrix, and the neutrino masses are there in the range $10^{0\pm 2}$ eV depending on the strength of the O(10) symmetry breaking.[5] Cosmological theories give upper limits for the neutrino masses of $m_i < 50$ eV (Refs. 6 and 7) [and a possible mass window 10 GeV $< m_i <$ 60 GeV (Ref. 4)].

The range accessible by reactor experiments covers further the range of contradictory results from other experiments: The high-energy accelerator experiments yield, with the assumption of full mixing, for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations $M^2 < 0.7-1.2$ eV^2 (Refs. 8–10) and $M^2 = 1.4 \pm 0.4$ eV^2 (Ref. 11). Reactor experiments could therefore be of decisive importance for the solution of the question of neutrino oscillations.

A serious problem in interpreting the results of reactor antineutrino-oscillation experiments is the uncertainty in the spectrum of antineutrinos emerging from the reactor. The reason is that the procedure used to determine the oscillation length is to extract it from the difference of the $\bar{\nu}$ spectra at a distance R from the reactor core and at $R = 0$ [when using the charged-current reactions $\bar{\nu}_e + p \to n + e^+$ (Ref. 12) or $\bar{\nu}_e + d \to n + n + e^+$ (Ref. 13)] or from the ratio of the total cross sections of the charged- and neutral-current reactions $\bar{\nu}_e + d \to n + n + e^+$ and $\bar{\nu}_e + d \to n + p + \bar{\nu}_e$ (Ref. 13). Consequently, the first type of experiment, in particular, depends totally on the calculated $\bar{\nu}$ spectrum at $R = 0$. This spectrum is characteristic of the β^- decay of the fission products.

The accuracy in any calculation of the $\bar{\nu}$ spectrum is a direct reflection of the accuracy of the three main sets of input data entering into Eq. (3), which describes the reactor electron spectrum (for secular equilibrium),

$$N(E_\beta) = \sum_{Z,A} Y(Z,A) \sum_i b_i(Z,A,E_0^{(i)}) P_i(E_\beta, E_0^{(i)}, Z). \tag{3}$$

These input data are (1) the cumulative fission yields $Y(Z,A)$, (2) the β branching ratios $b_i(E_0^{(i)})$, and (3) the β end-point energies $E_0^{(i)}$. $P(E_\beta, E_0^{(i)}, Z)$ is the normalized electron spectrum shape for end-point energy $E_0^{(i)}$. The $\bar{\nu}$ spectrum is obtained from the same equation by replacing $P(E_\beta, E_0^{(i)}, Z)$ by $P(E_0^{(i)} - E_{\bar{\nu}}, E_0^{(i)}, Z)$.

The fission yields are relatively well known. The main source of uncertainty in all calculations performed until now[14-18] was—as pointed out previously[19]—the assumptions on the shape of the β strength function for the fission products with unknown decay schemes, which determine the $b_i(E_0^{(i)})$ and $E_0^{(i)}$. Experimental decay schemes exist only for ~270 of the 740 fission products.

The β spectra of the remaining fission products were calculated in Refs. 14–18 with the assumption $S_\beta(E) \sim \rho(E)$ or with even rougher assumptions. From the definition of $S_\beta(E)$,

$$S_\beta(E) = \sum\nolimits_J \rho_J \bar{B}_J(E)/D,$$

with ρ denoting the level density in the daughter nucleus and $B(E)$ the reduced β transition probability to a state at excitation energy E in the daughter nucleus, it is already apparent that this assumption does not take into account any nuclear structure effects and does not fulfill any sum rule (for further discussion see Refs. 19 and 20). It is thus not surprising that with the treatment hitherto of the shape of the β strength function S_{β^-}, none of the existing calculations (Refs. 14–18) reproduces the recently measured[21] β spectrum of ^{235}U fission products to the extent that they could be considered as a safe basis for interpretation of $\bar{\nu}$-oscillation experiments (see Table I).

For a reliable extraction of neutrino mass differences and mixing angles from reactor $\bar{\nu}$ experiments the $\bar{\nu}$ spectrum in the reactor core has to be known with an uncertainty in shape of a few percent (see, for example, Ref. 12). We have recently calculated the e^- and $\bar{\nu}$ spectra of the fission products of ^{235}U with this required accuracy,[22] and present in this Letter a corresponding calculation for ^{239}Pu.

As in Ref. 22 the recommended fission yields of Rider[23] were used. The unphysical assumptions on $S_\beta(E)$ used in Refs. 14–18 were replaced by the results of microscopic calculations of $S_\beta(E)$ (for all fission products with unknown decay schemes) using a generalized Brown-Bolsterli model with a Gamow-Teller residual interaction. The strength functions were calculated in Ref. 20 for *all* nuclei between the β stability line and the neutron drip line in order to predict β-decay halflives, and beta-delayed neutron and fission rates for applications in astrophysics and for reactor kinetics and reactor decay heat calculations. Since the position of the states fed by β decay is calculated relative to the ground state of the parent nucleus, the calculated $S_\beta(E)$ are independent of Q_β values or mass formulas (see the corresponding discussion in Ref. 15). For the β branching ratios of known fission products the decay data of the evaluated nuclear structure data file (ENSDF, 1981), completed by some more recent data,[24] have been used. The correct shape factors for unique first forbidden transitions were taken into account.

Table I shows the calculated e^- and $\bar{\nu}$ spectra for ^{239}Pu. The results of Refs. 15–17 which are given for comparison deviate considerably (up to a factor of 3 or more) in the range important for the analysis of the $\bar{\nu}$-oscillation experiments, $2.5 \leq E_{\bar{\nu}} \leq 7.5$ MeV (see also Fig. 1).

In Table I the results of our calculation for ^{235}U are also given, together with the results of the recent high-precision measurement[21] of the corresponding e^- spectrum. The same set of $S_\beta(E)$ as for ^{239}Pu had been used in these calculations.

TABLE I. Calculated e^- and $\bar{\nu}$ spectra from ^{239}Pu fission and comparison with the results from recent calculations (Refs. 15–17). Also listed is the e^- spectrum, calculated with the same set of $S_\beta(E)$ as the ^{239}Pu spectra, for ^{235}U, and the experimental result for the latter (from Ref. 21).

E_{kin} (MeV)	N_β' (per fission per MeV)						$N_{\bar{\nu}}$ (per fission per MeV)			
	^{239}Pu			^{235}U			^{239}Pu			
	Present calc.[a]	Vogel[a] Ref.15	Kopeikin[a] Ref.17	Calc.with microsc. S_β[a] Ref.22	Experim.[b] Ref.21	Rel. uncertainty (%)	Present calc.[a]	Vogel[a] Ref.15	Kopeikin[a] Ref.17	Avignone[a] Ref.16
1.0	1.72	1.77		1.98	1.91	5.0	2.25	2.34		1.89
1.5	1.14	1.14	1.18	1.36	1.31	3.0	1.46	1.50	1.45	1.31
2.0	7.53(-1)	7.35(-1)	8.01(-1)	9.23(-1)	8.8(-1)	2.5	1.09	1.08	1.15	1.05
2.5	4.82(-1)	4.72(-1)		6.12(-1)	5.87(-1)	2.2	7.28(-1)	7.10(-1)		7.47(-1)
3.0	2.97(-1)	3.03(-1)	3.46(-1)	4.00(-1)	3.99(-1)	2.0	4.85(-1)	4.77(-1)	5.57(-1)	5.43(-1)
3.5	1.78(-1)	1.89(-1)		2.53(-1)	2.52(-1)	1.5	3.06(-1)	3.14(-1)		3.70(-1)
4.0	1.04(-1)	1.15(-1)	1.21(-1)	1.55(-1)	1.54(-1)	<1.5	1.90(-1)	2.03(-1)	2.35(-1)	2.36(-1)
4.5	5.91(-2)	6.84(-2)		9.25(-2)	9.1(-2)	<1.5	1.07(-1)	1.21(-1)		1.40(-1)
5.0	3.44(-2)	3.95(-2)	4.13(-1)	5.61(-2)	5.50(-2)	<1.5	6.26(-2)	7.46(-2)	7.89(-2)	8.39(-2)
5.5	1.81(-2)	2.10(-2)		3.21(-2)	3.14(-2)	<1.5	3.70(-2)	4.37(-2)		5.14(-2)
6.0	8.90(-3)	1.01(-2)	1.24(-2)	1.75(-2)	1.72(-2)	<1.5	1.98(-2)	2.37(-2)	2.70(-2)	3.00(-2)
6.5	4.16(-3)	4.55(-3)		9.09(-3)	8.8(-3)	<1.5	9.61(-3)	1.01(-2)		1.84(-2)
7.0	1.58(-3)	2.02(-3)	3.38(-3)	3.89(-3)	3.80(-3)	1.5	4.61(-3)	4.93(-3)	7.58(-3)	9.96(-3)
7.5	4.92(-4)	7.79(-4)		1.32(-3)	1.32(-3)	2.5	1.62(-3)	2.20(-3)		4.83(-3)
8.0	7.46(-5)		6.87(-4)	2.96(-4)	2.60(-4)	12	4.90(-4)	7.78(-4)	2.07(-3)	2.26(-3)
8.5	3.18(-5)			1.31(-4)	4.3(-5)	70	7.49(-5)			4.84(-4)
9.0	9.55(-6)		8.48(-5)	3.65(-5)	<3(-5)		3.36(-5)		3.03(-4)	2.22(-4)

[a] Infinite exposure time (secular equilibrium).

[b] Exposure time 1.5 days.

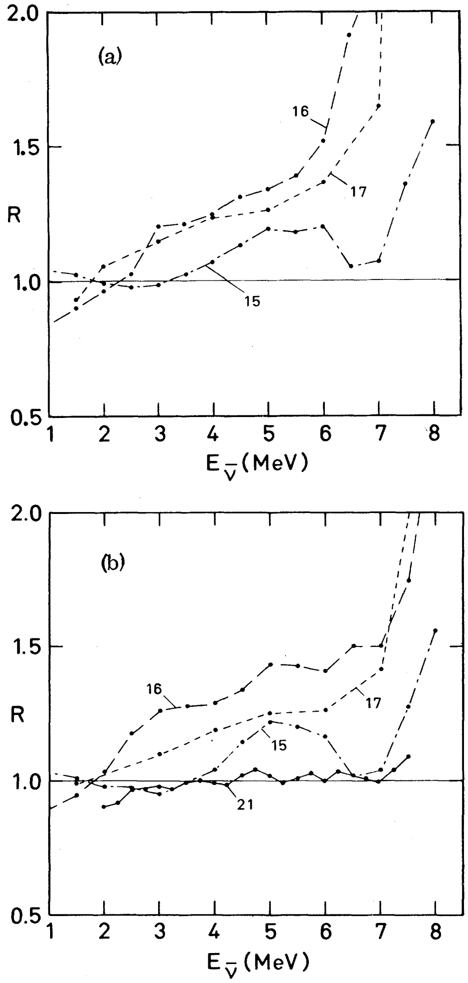

FIG. 1. (a) Comparison of the most recent calculations (Refs. 15–17) of the $\bar{\nu}$ spectra from thermal fission of ^{239}Pu with this work. R denotes the ratio between the number of antineutrinos per megaelectronvolt calculated in the given references and in this work. (b) The same as (a) for ^{235}U. The normalization is done here to the theoretical $\bar{\nu}$ spectrum of Ref. 22. The $\bar{\nu}$ spectrum of Ref. 21 is deduced from the measured e^- spectrum.

The agreement of the calculation for ^{235}U with the experiment is excellent, the deviation being $<5\%$ over the range of importance for the $\bar{\nu}$-oscillation experiments (the slight systematic positive deviation at lower energies originates from the assumption of secular equilibrium in the calculation, whereas the measurement[21] was limited to 1.5 days). Figure 1 shows that the deviations of the results of Refs. 15–17 from our calculation (and the experiment) of ^{235}U show up again with the same characteristics in the case of ^{239}Pu, indicating the systematical errors in the treatment of the β-decay properties.

In summary, calculations of the e^- and $\bar{\nu}$ spectra from the fission products of ^{239}Pu are presented with an uncertainty in the absolute and relative shapes of $<5\%$, i.e., with errors about an order of magnitude smaller than in previous calculations. Together with the recent calculation of the e^- and $\bar{\nu}$ spectra from fission of ^{235}U (Ref. 22) the results for ^{239}Pu yield a reliable theoretical basis for the interpretation of reactor antineutrino experiments. They might be helpful for understanding the partly different values for M^2 and θ deduced from the two existing reactor $\bar{\nu}$ experiments.[12,13,25] The Grenoble experiment[12] finds in addition to the solution around $M^2 = 1$ eV^2 reported in the Irvine experiment[13] a solution for $M^2 < 0.14$ eV^2 and full mixing. An analysis[26] of the measured e^+ spectrum of the Grenoble experiment with different calculated $\bar{\nu}$ spectra for $R = 0$ gives the following results. Use of the $\bar{\nu}$ spectrum calculated with microscopic S_β (or those from Refs. 21 and 14) leads to a solution for $M^2 < 0.14$ eV^2. The other "class" of calculated $\bar{\nu}$ spectra,[15-17] however, excludes this solution as does the Irvine experiment which was analyzed[13] by using the $\bar{\nu}$ spectrum of Ref. 16.

The results of this paper will be of importance as a basis for the interpretation of $\bar{\nu}$-oscillation experiments in progress at power reactors, where the problem of a variation of the fuel composition as a function of time occurs.

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Term-Dependent Hybridization of the $5f$ Wave Functions of Ba and Ba^{++}

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It is shown that, unlike in neutral Ba, the $4d \to 5f$ transitions cannot be neglected in the interpretation of the $4d$ spectrum of Ba^{++}. A term-dependent hybridization of the $5f$ wave functions occurs, the effects of which reverse between Ba and Ba^{++}, and oscillator strength reappears in the $4d \to nf$ ($n \geq 5$) transitions. A second kind of wave-function collapse is identified and its effects are described.

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The formation and properties of the rare-earth elements have been explained[1] in terms of the transfer of the $4f$ wave function from the outer to the inner well of a double-valley potential. The sudden contraction of the $4f$ wave function (wave-function collapse) and consequent spatial segregation of nf solutions between the inner and outer wells, which occurs at about $Z = 57$, leads to highly non-Rydberg $4d$-subshell spectra in the rare earths and preceding elements. In particular the spectra of these elements show very strong $4d \to 4f$ resonances while higher members of $4d \to nf$ series possess negligible oscillator strength. In neutral Ba prime interest attaches to the transition to the $4d^9 4f 6s^2\ {}^1P_1$ upper state which is the only $4d \to 4f$ transition allowed by *LS* selection rules for electric dipole excitation. The ${}^1P$ resonance is placed in the continuum following the $4d$ ionization threshold by SCF (self-consistent-field) configuration-average calculations and is better described[2] as $4d \to (4+\epsilon)f\ {}^1P_1$, this notation indicating that the $4f\,{}^1P$ and $\epsilon f\ {}^1P$ wave functions are hybridized.

In a recent experiment[3] photoexcitation of the $4d$ subshell in the species Ba, Ba^+, and Ba^{++} has been observed. Thus qualitative changes in the spectral distribution of oscillator strength with increasing stage of ionization can be followed experimentally. However, the interpretation of the Ba^{++} spectrum presents difficulties, where more discrete structure exists than might be expected. The present Letter makes a start on this complex problem. We point out that, in contrast to the situations which have previously been analyzed,[4] the $d \to f$ channel in Ba^{++} *is not entirely dominated* by the $4d \to (4+\epsilon)f\ {}^1P_1$ transition. Indeed, the $4d \to 5f$ transitions possess significant oscillator strength—superior to that of the