Redetermination of the Newtonian Gravitational Constant G

Gabriel G. Luther
Center for Absolute Physical Quantities, National Bureau of Standards, Washington, D. C. 20234

and

William R. Towler
Department of Nuclear Engineering and Engineering Physics, University of Virginia, Charlottesville, Virginia 22901
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The universal Newtonian gravitational constant is being redetermined at the National Bureau of Standards with use of the method of Boyes in which the period of a torsion pendulum is altered by the presence of two 10.5-kg tungsten balls. The difference in the squares of the frequencies with and without the balls is proportional to G. The resulting value of G is $(6.6726\pm 0.0005)\times 10^{-11}$ $\mathrm{m^3\cdot sec^{-2}\cdot kg^{-1}}$.

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The value of G, defined by Newton's equation $F = GM_1M_2/R^2$ and listed in the table "Fundamental Constants" compiled by Taylor and Cohen in 1973 (CODATA)[1] was determined mainly from measurements made by Heyl and Chrzanowski in 1942.[2] Of the several attempts to improve this measurement made during the past 39 years,[3-7] none seems to have increased significantly the precision of this, the internationally accepted value of G, $(6.6720 \pm 0.0041)\times 10^{-11}$ $\mathrm{m^3\,sec^{-2}\,kg^{-1}}$.[1]

The theory of the present experiment is essentially the same as that of Boyes.[8] For a torsion balance with moment of inertia I and angular frequency ω_f, supported by a fiber with torsion constant K_f, the square of the angular frequency is given by

$$\omega_f^2 = K_f/I \text{ or } K_f = I\omega_f^2. \tag{1}$$

K_f is the second derivative of the potential energy stored by twisting the fiber through an angle θ. The torque is the first derivative of the potential energy with respect to θ; thus the torsion constant is the first derivative of the torque.

If the large masses are placed near this torsion pendulum, a small "fictitious" torsion constant K_g due to the gravitational attraction is added to the torsion constant of the fiber and the square of the frequency becomes

$$\omega_{f+g}^2 = (K_f + K_g)/I \text{ or } K_f + K_g = I\omega_{f+g}^2. \tag{2}$$

Subtracting Eq. (1) from Eq. (2) yields

$$K_g = I(\omega_{f+g}^2 - \omega_f^2) = I\Delta(\omega^2). \tag{3}$$

Just as K_f is the second derivative of the potential due to twisting the fiber, K_g is the second derivative of the gravitational potential,

$$K_g = d^2U/d\theta^2 \text{ evaluated at } \theta = 0$$

and U is the gravitational potential of the small-mass system in the gravitational field of the large masses calculated by integrating $U = GM_1 \times \rho dv/R$ over the volume of the small masses.

The detailed calculations of the potential U of a cylinder in the gravitational field of a spherical mass are given by Towler.[9] K_g is proportional to G, and is a function of the magnitude of the masses and the geometry of the apparatus; thus, G can be expressed as

$$G = \Delta(\omega^2)I/k_g \quad \text{where} \quad k_g = K_g/G.$$

Note that $\Delta(\omega^2)$ is determined by measuring the frequency of the torsion pendulum with and without the large masses in place and that I and k_g are calculated from the measurements of the appropriate dimensions, angles, masses, and densities of the apparatus. The above derivation is true of an undamped harmonic oscillator. However, the frequency of a harmonic oscillator is changed when damped. This frequency change is of second order in the damping constant for small values of the damping constant. Because the damping constant in our case is approximately 10^{-4}, the frequency change resulting from this damping is negligible.

The schematic diagram of the apparatus is shown in Fig. 1 and consists of the following:

(1) The two large spherical masses were fabricated from sintered tungsten by the Y-12 Plant of the Union Carbide Corporation at Oak Ridge, Tennessee. They are 10.165 072 and 10.165 108 cm in diameter and have masses 10.489 980 and 10.490 250 kg,[10] respectively.

(2) The small-mass system consists of two tungsten disks mounted in a dumbbell configuration. The discs are 2.5472 mm thick and 7.1660

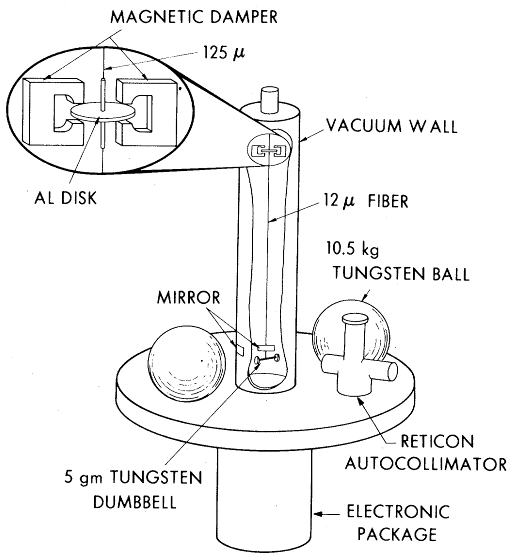

FIG. 1. Diagram of the apparatus with inset showing detail of the damper.

mm in diameter mounted on the ends of a 1.0347-mm centerless-ground tungsten rod 28.5472 mm long. The mass of the entire assembly is approximately 7 g.

(3) The small mass enclosed in a vacuum of 10^{-5} Torr or less is supported by a 10–12-μm quartz fiber 40 cm long. The fiber was plated with chromium and gold to make it conductive. The period without the large masses in place was approximately 6 min. The change in the period due to the large masses is a few percent.

(4) The damper, crucial to the success of this experiment, is shown in the inset in Fig. 1. It consists of a small circular aluminum disk suspended in a strong magnetic field approximately 10 cm from the top of the vacuum chamber by a 125-μm quartz fiber. The spindle through the center of the disk serves as the support for the much smaller 10–12-μm fiber which holds the bob. The larger top fiber is quite flexible with respect to motions of the bob in the pendulous mode which are introduced by seismic disturbances. These vibrations are damped in a few seconds. The torsion constant of the larger top fiber is much greater (10^5) than that of the smaller fiber, i.e., the top fiber is rigid, compared to the bottom fiber, with regard to torsional motions; therefore, the damping of the torsion mode is negligible.

(5) The detector consists of a 170-mm focal length autocollimator with a 25-mm aperture in which the light through a 15-μm slit is collimated, reflected twice from the mirror attached to the small-mass system, and refocused on a 1024-element diode array. The 1024 light-sensitive elements are 15 μm on centers, which allows the small system to be monitored over a range of 2×10^{-2} rad. The width of the refocused image of the slit covers 4 to 5 elements of the array. The angular position of the small-mass system is measured every 20 sec by determining the amount of light on each element of the array.

The apparatus is enclosed in an acoustically isolated, thermally controlled cube about 2.5 m on a side. This is mounted on a reinforced concrete slab of about 5000 kg. It is located in a large basement room of the Physics Building at the National Bureau of Standards, Gaithersburg, Maryland, where the temperature is held constant to within approximately 2 °C. The inner room temperature is controlled to within approximately 0.1 °C.

The large masses are positioned on three small pads located in wells machined into a monolithic aluminum plate. The separation of the centers of the large masses, 14.059 454 cm, was determined to within 0.3 μm. The consistency of the separation, after removing and replacing the large masses, was found to be less than 0.05 μm rms.

Data are taken alternately with the large masses in place and with them removed. Usually, each of these conditions would last from 6 to 12 h. Typically a series lasts from 50 to 75 h. The frequencies for each of the individual sets (6 to 12 h) were determined by use of the algorithm of Snyder.[11] These frequencies were squared. The midpoint time of the run was assigned to each run. Each set of runs was separated into two groups (masses on, masses off) and a linear least-squares fit was made of ω^2 versus time for each group with the constraint that the slopes of the fits are the same. The implication of this procedure is that any underlying drift (slope of the fits) affects the runs in the same way, i.e., is independent of whether the balls are on or off. As a check on the effects of the Earth's magnetic field, data were taken with the apparatus in different orientations.

The calculation of G was made from the average $\Delta(\omega^2)$ value listed in Table I.

The final assignment of the uncertainty in this determination of G was arrived at by the conventional method of taking the square root of the sum of the squares of the uncertainties listed in Table

TABLE I. $\Delta(\omega^2)$ for different orientations. Since the position is measured every 20 sec for convenience, ω has units of rad/(20 sec) and $\Delta(\omega^2)$ therefore has units of $rad^2/(400\ sec^2)$.

Orientations	$\Delta(\omega^2)$
NE-SW	0.004 978 66
NE-SW	837
NW-SE	816
NW-SE	729
NE-SW	773
NE-SW	788
Average	0.004 978 01 ± 0.000 000 20 (40 ppm)

TABLE II. Error budget.

Source of uncertainty	Uncertainty in ppm
Position of the large masses	10
Mass of the large masses	1
Length of the small mass	22
Thickness of the small mass	36
Density of the small mass	6
Moment of inertia of mirror	23
$\Delta(\omega^2)$	40
Total	64

II, and therefore represents a 1σ uncertainty.

The resulting value of G is $(6.6726 \pm 0.0005) \times 10^{-11}\ m^3 \cdot sec^{-2} \cdot kg^{-1}$.

The four largest components of the uncertainty budget are the uncertainty in the overall length and the thickness of the disks of the small-mass system, the uncertainty of the moment of inertia of the mirrors, and the uncertainty in the measurement of the frequency of the small-mass system's oscillation. We are now constructing a new small-mass system that will be more amenable to precise metrology than the present one, and the mirror will be made much less massive. The largest contribution to the overall error will remain the uncertainty in the measurement of the frequency of the oscillation of the small-mass system. This uncertainty will be the most difficult to reduce. In order to minimize this error, fibers which produce different periods will be tried. Experience has shown that the present small-mass system, and consequently the new one, may be suspended on fibers thin enough to produce periods of up to 20 min. The frequency change due to the large masses in that case will be about 20%.

It is difficult to imagine doing a measurement of this type without the personnel and facilities of an institution such as the National Bureau of Standards. The following list is by no means a complete list of all the assistance given to us.

R. D. Deslattes, who initiated this project, suggested the design of the autocollimator and participated in fruitful discussions. Bruce Borchardt measured the separation of the large masses. Bob Crosson transcribed data. Richard Davis and Randall Schoonover performed density measurements. Jim Filliben and Stephen Leigh provided time-series analysis and general computational assistance. P. Thomas Olsen checked the magnetic susceptibility of the large masses and the material of the small masses. J. J. Snyder supplied the algorithm for computing the frequencies. Clyde Turner performed metrology on the small-mass system.

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