Free Decay of Superfluid Turbulence

F. P. Milliken and K. W. Schwarz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

and

C. W. Smith
Department of Physics and Astronomy, University of Maine, Orono, Maine 04469
(Received 12 February 1982)

The free decay of a turbulent field in superfluid ^{4}He is studied by using pulsed-ion techniques. The decay process is found to exhibit strikingly simple behavior. Analysis of the results indicated that current theoretical models need further refinement.

PACS numbers: 67.40.Vs

The turbulent state of superfluid ^{4}He consists of a tangle of quantized vortices, moving according to the laws of ideal fluid mechanics, with the additional feature that the normal-fluid excitations can exert a drag force on the vortex core. Measured properties of superfluid turbulence in steady-state channel flow include the line-length density L,[1] the associated mutual friction force density F_{sn} exerted by the superfluid on the normal fluid,[1-5] and the overall drift velocity $\langle v_l \rangle$ of the vortex tangle with respect to the superfluid.[6] Only the results for F_{sn} can be thought of as well established, the previously accepted measurements of L by Vinen having recently been challenged,[7] while the single experiment determining $\langle v_l \rangle$ is subject to uncertainties in interpretation. The phenomenological model by Vinen[1] and the more recent microscopic treatment by Schwarz[8] seem to be rather successful in explaining these features. It is recognized, however, that superfluid turbulence (like its classical counterpart) is a complicated phenomenon, and that present measurements and interpretations are but the beginnings of a serious study. The purpose of this paper is to present new measurements which have direct implications for the further theoretical development of the subject.

In our experiment, a spatially inhomogeneous steady-state turbulent field is generated in a relatively open geometry by a facing pair of ultrasonic transducers (Fig. 1). The transducers are then turned off and the decay of the L field is observed by using an interrupted-flight ion pulse technique[9] which makes use of the fact that quantized vortices trap ions at a known rate. A narrow ion pulse is injected to arrive at the point of interest x_0 a time t_0 after the transducers have been turned off. The pulse is stopped there by turning off the main drift field, thus allowing the free ions in the pulse to be trapped at a rate characteristic of the vortex line density at x_0, t_0. By varying the sampling time, this rate and hence $L(x_0, t_0)$ can be determined explicitly. The top curve of Fig. 2 shows a typical steady-state L profile, obtained by moving the sampling point across the high-excitation region between the two transducers. One sees that the profile is not exactly symmetrical and that it has internal structure. Such features depend on exactly how the transducers are aligned. We believe that the turbulence is generated and maintained by acoustic streaming, so that the actual shape of these profiles is determined by some very complicated physics. With our technique, however, one can follow the free decay of the entire profile with

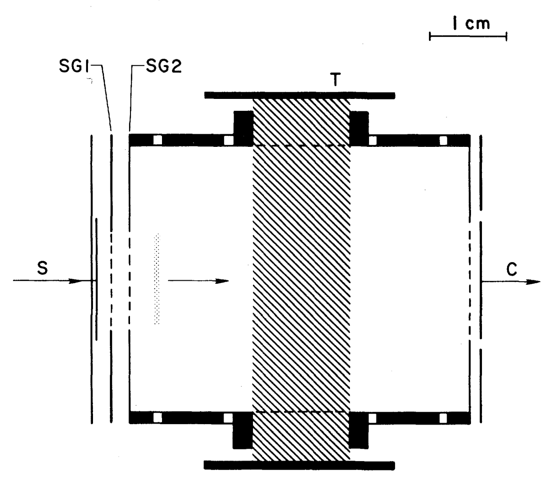

FIG. 1. Geometry of the experiment. Turbulence is created by the pair of ultrasonic transducers T which illuminate the hatched region. Ions are produced by the source S and manipulated by SG1 and SG2 to produce ion pulses of the size and shape indicated. The drifting ion pulse can be stopped at any position in the cell to sample L.

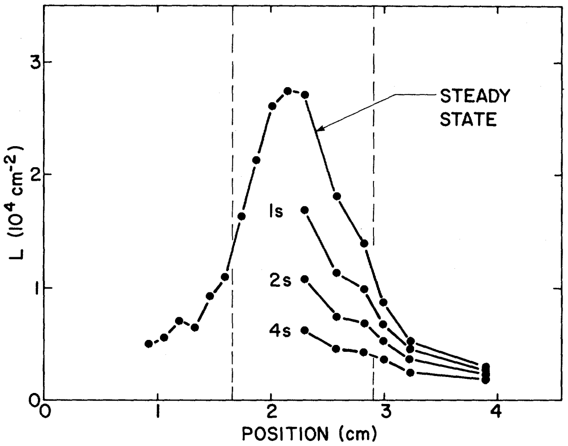

FIG. 2. Global characterization of the decay process. The top curve is the steady-state profile obtained by sweeping the sample point across the cell from SG2 to the field grid. Lower curves are at 1, 2, and 4 s after the transducers have been turned off. Note the persistence of structure as the profile decays. Lines have been drawn to guide the eye.

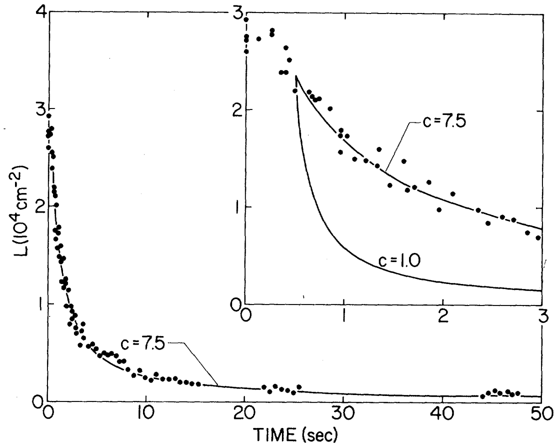

FIG. 3. Local characterization of the decay process, with measurements taken at the center of the cell. The temperature is 1.45 K. The curves are obtained from Eq. (5).

time. The result is an uncommonly well-characterized absolute measurement, in which the boundaries are far away, and where the effect of spatial diffusion of the vortex tangle can be investigated explicitly.

Decaying profiles $L(x,t)$ such as the one shown in Fig. 2 can be obtained starting from various steady-state turbulence levels. We find to within the resolution of our measurements that, after a short initial transient period, the decay rate dL/dt at any particular point in the fluid depends only on the instantaneous value of L at that point. The duration and shape of the initial transient are consistent with the idea that it represents the time required for the initial driving velocity $\vec{v}_n - \vec{v}_s$ to decay to an insignificant value. The absence of history-dependent effects at later times indicates that the tangle quickly reequilibrates its local internal structure as it decays. Moreover, one sees that even for this highly inhomogeneous distribution the transport of vorticity from high-density to low-density regions is very small, negligible compared to the bulk line decay mechanism which presumably depends only on the local structure of the vortex tangle.

Our results show that, save for the initial transient, the decay of the entire complicated profile from arbitrary starting levels can be described by a single curve, at least to first order. Figure 3 shows this curve, obtained by doing extensive measurements at the center of the cell. Each point is obtained from a separate determination of the free-ion trapping rate,[9] using a sampling time of 0.10 s. The data were obtained over a period of months, and in our estimation the scatter in Fig. 3 is truly representative of the repeatability of the measurements.

We now discuss the implications of these results by developing a straightforward model based on the ideas of Ref. 8. The configuration of the vortex lines which make up the tangle can be specified by a parametric curve $\vec{s}(\xi)$, where ξ is the arc length. The instantaneous velocity of the vortex singularity with respect to the local superfluid rest frame is then given approximately by

$$\dot{\vec{s}} \cong \beta\vec{s}\,' \times \vec{s}\,'' + \alpha\vec{s}\,' \times (\vec{v}_n - \vec{v}_s - \beta\vec{s}\,' \times \vec{s}\,''). \qquad (1)$$

Here $\vec{s}\,'$ is the vector tangent and $\vec{s}\,''$ the vector curvature of the line at the point in question; $\vec{v}_n$ and $\vec{v}_s$ are the local average normal and superfluid velocity fields; α is a friction constant related to the conventional Hall-Vinen[10] coefficient B by $\alpha = \rho_n B/2\rho$; and $\beta = (\kappa/4\pi)\ln(c_1/s''a_0)$, where κ is the quantum of circulation, $a_0 \sim 10^{-8}$ cm is the vortex-core cutoff parameter, and c_1 is a constant of order 1.

The approximations leading to Eq. (1) neglect nonlocal effects, the most important of which arise from line-line crossings. As in Ref. 8, we make the assumption that such crossings result in topological reconnections which introduce sharp kinks into the vortex lines. It is easy to show that if $v_{\rm rel}$ is the typical velocity of a given line segment in the vortex tangle relative to the other lines, then the rate at which such kinks appear per unit length of line is just $v_{\rm rel}L$. Each such kink changes $\vec{s}'$ by a vector of order unity and hence may be thought of as a δ function in $\vec{s}''$. If the characteristic local curvature $\langle s''\rangle$ is defined as the average of $|\vec{s}''|$ along the line, one concludes that line-line reconnections make this curvature increase at the rate $v_{\rm rel}L$. In terms of the characteristic local radius of curvature $R=\langle s''\rangle^{-1}$, this gives $\dot{R}\cong -R^2 v_{\rm rel}L$.

If $\vec{v}_n-\vec{v}_s$ in Eq. (1) is zero, the frictional term acts solely to destroy line. Since the decay is more rapid in regions of high curvature, the kinks are damped out, with an associated line loss. To estimate these effects, we model the locally distorted line by a sine wave with amplitude $\sim R$ and wave vector $\sim R^{-1}$. Straightforward analysis of this geometry gives $\dot{R}\cong\alpha\beta/R$, so that

$$\dot{R}\cong\alpha\beta/R-R^2Lv_{\rm rel}. \qquad (2)$$

This equation gives a rough representation of how the competing effects of kink creation and frictional damping determine the characteristic curvature in the vortex tangle. The instantaneous line loss rate calculated from the sine-wave model is then

$$\dot{L}\cong-\alpha\beta L/4R^2. \qquad (3)$$

In Ref. 8, $v_{\rm rel}$ is assumed to be the average self-induced velocity β/R. However, there are several possible refinements of the microscopic theory, each of which would reduce the effective value of $v_{\rm rel}$. First, not every line-line crossing necessarily results in a topological reconnection. Secondly, the vortex velocities tend to be polarized in the direction of the initial driving fields, so that $v_{\rm rel}$ may be less than β/R. Thirdly, the δ functions in $\vec{s}''$ generated by topological reconnections are produced in oppositely directed pairs. This interesting conservation law, which in fact is equivalent to momentum conservation, will also have the effect of slowing down the rate at which the tangle self-randomizes. Finally, it can be shown[11] that inhomogeneous vortex turbulence cannot exist without the continuous production of macroscopic vorticity. This raises the more complicated possibility that large-scale structures within the vortex tangle dominate the decay behavior. In the present context, all of these effects can be allowed for by writing $v_{\rm rel}=\beta/cR$, where $c\geq 1$ is an adjustable parameter.

Numerical integration of Eqs. (2) and (3) shows that, as L decays, R remains in quasiequilibrium. That is, $R(t)$ is always very close to $[\alpha c/L(t)]^{1/2}$. Making this substitution into Eq. (3) gives

$$\dot{L}\cong-(\beta/4c)L^2, \qquad (4)$$

which has the trivial solution

$$L(t)\cong L_0/(1+\beta L_0 t/4c). \qquad (5)$$

The curves in Fig. 3 were obtained from this equation using $c=1$ and $c=7.5$. In order to avoid the initial transient behavior, the starting point was taken to be the measured L_0 at 0.5 s. It is clear that Eq. (5) provides an excellent fit to the observed behavior. Decay curves were also obtained at the center of the cell starting from two lower power levels, and at five other points on the profile starting from the maximum power level. In addition, measurements were done at the cell center for $T=1.33$ and 1.60 K, a range over which the friction constant α varies by a factor of 2. All of these decay curves are described equally well by Eq. (5) with $c\simeq 7.5$. Thus, our simplified model appears to be quite successful. The fact that $c\gg 1$, however, leads us to conclude that the rate of internal randomization is considerably slower than assumed in current theory. As pointed out earlier, there are a number of mechanisms which could lead to such an effect.

Equation (4) is of the form originally proposed by Vinen,[1] although it must be emphasized that the physical reasoning which led us to Eq. (4) has no obvious connection with his original arguments. Vinen also measured the decay of turbulence in a channel. Because these early measurements take a complicated average over an entire channel in a situation where end effects and wall effects are expected to be important, their significance has been unclear. We find that, although Vinen's relatively crude decay curve is only very approximately described by Eq. (4), his overall decay coefficient is in reasonable agreement with our results. Since the decay rate is a strong function of L, this provides rather good evidence that Vinen's original measurements of L are approximately correct, a point which has recently been disputed.[7]

In summary, our experiment indicates that the free decay of superfluid turbulence is insensitive to vorticity transport effects and depends primarily on the local instantaneous value of L. Of the many experiments done on superfluid turbulence, only the recent work on turbulence in pure superflow[5] appears to be similarly unambiguous and well characterized. Our analysis indicates that the functional form of the decay is in excellent agreement with current theoretical ideas. However, the internal dynamics of the tangle appear to be considerably more sluggish than has been assumed. Various mechanisms to account for this have been suggested, but the problem of how to embody them in a realistic model remains to be investigated.

This research has been supported in part by National Science Foundation Grant No. DMR-8005358.

[1]W. F. Vinen, Proc. Roy. Soc. London, Ser. A 240, 114, 128 (1957), and 242, 493 (1957), and 243, 400 (1958).

[2]D. F. Brewer and D. O. Edwards, Philos. Mag. 7, 721 (1962).

[3]W. de Haas and H. van Beelen, Physica (Utrecht) 83B, 129 (1976).

[4]J. T. Tough, in *Progress in Low Temperature Physics*, edited by D. F. Brewer (North-Holland, Amsterdam, 1982), Vol. 8, Chap. 3.

[5]R. A. Ashotn, L. B. Opatowsky, and J. T. Tough, Phys. Rev. Lett. 46, 658 (1981); L. B. Opatowsky and J. T. Tough, Phys. Rev. B 24, 5420 (1981).

[6]R. A. Ashton and J. A. Northby, Phys. Rev. Lett. 35, 1714 (1975).

[7]C. F. Barenghi, K. Park, and R. J. Donnelly, Phys. Lett. 84A, 435 (1981).

[8]K. W. Schwarz, Phys. Rev. Lett. 38, 551 (1977), and Phys. Rev. B 18, 245 (1978).

[9]K. W. Schwarz and C. W. Smith, Phys. Lett. 82A, 241 (1981).

[10]H. E. Hall and W. F. Vinen, Proc. Roy. Soc. London, Ser. A 238, 215 (1956); P. Lucas, J. Phys. C 3, 1180 (1970).

[11]K. W. Schwarz, to be published.