Superposition of Traveling Waves in the Circular Couette System

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Experiments have been performed on the circular Couette system that reveal the existence of two simultaneous traveling azimuthal waves in a highly nonlinear fluid flow. These waves can have the same or different wave numbers, but they are nearly dispersionless. Flow visualization, scattering intensity, and Doppler velocimetry techniques were used, and intensity measurements were made in both laboratory and rotating reference frames.

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In recent years nonlinear waves have been found to occur in a variety of systems. Two well-studied types of nonlinear waves are solitons and Stokes waves. We report here on experiments which reveal the existence of two wave trains that propagate simultaneously in a highly nonlinear flow. These waves pass through one another like solitons, but, in contrast to solitons, these waves are nonlocal and dispersionless. Although we know of no other observations of two simultaneous dispersionless wave trains in a nonlinear system, recent experiments on several hydrodynamic systems have revealed doubly periodic flow states that may correspond to the same type of waves described here.

We begin by reviewing briefly the sequence of transitions that occur in flow between concentric cylinders (the circular Couette system) with the outer cylinder at rest and the inner cylinder rotating. If the Reynolds number R (proportional to the rotation rate of the inner cylinder) is increased, the following is observed for systems with radius ratio 0.88 and a height-to-gap ratio 20, as in our experiments[1]: At small R the velocity field is time independent and azimuthal, if we neglect end effects. When R exceeds a critical value R_c, the azimuthal flow becomes unstable and a vortex structure resembling a stack of doughnuts forms; *Taylor vortex flow* has an axial periodicity but it is time independent and axisymmetric. At larger R, traveling azimuthal waves appear on the vortices. At a fixed R in this *wavy vortex flow* regime, several steady states are possible, each characterized by different numbers m of waves around the annulus[2]; velocity power spectra have only a single fundamental frequency F_1 and harmonics. The m state of the system depends on the Reynolds-number history and other factors, but each state, once established, is stable over a range in R. An interesting property of the waves is the absence of dispersion[2]: At a given R the wave speed is essentially independent of m.

At still larger R another transition occurs: The rotating waves then appear to be periodically modulated, and the velocity power spectra require two independent frequencies (and combinations) for their characterization. Gorman and Swinney[3] observed and classified twelve of these *modulated wavy vortex flow states*. Rand[4] and Gorman, Swinney, and Rand[5] showed that, as a consequence of the circular symmetry of the boundary conditions, only certain space-time symmetries in the flow patterns are allowed for doubly periodic flows; the observations agreed with Rand's predictions.[3-5] While one fundamental frequency can be identified with the original waves passing by the fixed detector, the identification of the second fundamental frequency has been unclear, even though a well-defined modulation frequency was measured visually and found in the power spectrum.[3]

We present here experimental evidence for the following hypothesis: The second fundamental frequency is associated with a *second* set of traveling azimuthal waves that rotate around the annulus at a different speed from the first. Modulated wavy vortex flow can then be simply described by two integers, m_1 and m_2, which correspond to the number of waves in the first and second sets, respectively. In this picture there are two special rotating coordinate frames, one comoving with the first set of traveling waves and one comoving with the second set. A power spectrum taken in either of these special frames would show only a single independent frequency.

We will now describe how two traveling waves can produce the observed modulation patterns, and then we will present results from a number of experiments, conducted on two Couette systems previously described,[5,6] that demonstrate the existence of the two traveling-wave trains.

Figure 1 illustrates how two sets of traveling waves can generate different modulation patterns. The right-hand column shows schematic diagrams for two of the modulated wavy vortex flow states, as observed in a reference frame comoving with the first set of waves. The left-hand column shows how two traveling waves could generate these patterns. In an appropriate rotating reference frame, one set of waves would be fixed and would appear to be modulated by the relative motion of the other set. For example, if a pattern were the result of simple superposition of the two wave trains, and both had the same wave number, then the "modulation" or flattening of all the waves around the annulus would occur at the same time, the cancellation being always in phase, as in the first example in Fig. 1. If, however, $m_2 = m_1 + 1$, the point of maximum cancellation would precess around the annulus, as in the second example in Fig. 1.

The observed space-time symmetries of the other modulated wavy vortex flow states can also be generated by the superposition of two azimuthal waves traveling with different speeds. However, this does not mean that the nonlinear equations of motion have separate solutions for each wave that can be superposed to obtain the solution for modulated wavy vortex flow.

Having determined that a two-wave picture provides a simple explanation of the modulated wavy vortex flow patterns, we conjectured that the hypothetical second wave train might have a phase velocity independent of m, just as Coles[2] found for the first wave train. If so, there would be a spectral component F_2 such that the wave speed $\omega_2/k_2 = \text{const} \times (F_2/m_2)$ would be the same for all m_2. Indeed, with m_2 inferred from the modulation patterns, we found that every spectrum contained a peak F_2 such that $F_2/m_2 \simeq 0.44$, as Fig. 2 illustrates.[7] (Frequencies in Fig. 2 and elsewhere in this paper are expressed relative to the cylinder frequency in the laboratory frame.) Figure 2 also shows the nearly dispersion-free character of the first wave train ($F_1/m_1 \simeq 0.33$, independent of m_1), and the very weak Reynolds-number dependence of both wave trains.

As a further test of the two-traveling-wave hypothesis we have conducted some experiments with two laser beam probes positioned at the same height but separated azimuthally. Now the observation of two independent frequencies in the power spectrum implies that there are two independent temporal phase coordinates for the system, and that the motion of the system through an abstract phase space lies on a torus. Any measurement of the flow at some point in space and time will not measure these phases directly but rather some unknown function of these phases, $I(\varphi_1, \varphi_2)$ where $\varphi_1 = 2\pi F_1 t - m_1\theta$ and $\varphi_2 = 2\pi F_2 t - m_2\theta$. Nevertheless, the traveling-wave picture can be tested by using a second identical detector separated from the first by an azimuthal angle δ and shifted in time by τ; this detector measures $I(\varphi_1', \varphi_2')$ where $\varphi_1' = \varphi_1 + 2\pi F_1\tau - m_1\delta$ and $\varphi_2' = \varphi_2$

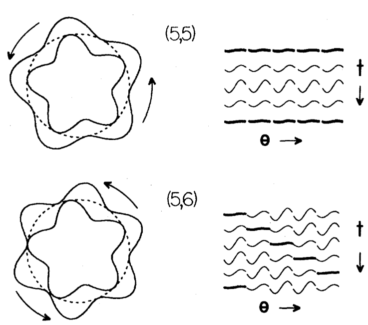

FIG. 1. Schematic diagrams illustrating for two states (m_1, m_2) how the modulation patterns can be generated by two traveling azimuthal waves. The column on the left shows schematically the first set of traveling waves inside the circle ($m_1 = 5$) and the second set ($m_2 = 5, 6$) outside the circle. The column on the right shows schematically the temporal evolution of the modulation patterns in a reference frame comoving with the first set of waves. Each line represents the entire flow pattern at an instant of time (all vortex pairs are in phase in the axial direction); succeeding lines show the temporal evolution of the flow pattern for one modulation period. The angle θ increases from zero to 2π rightward.

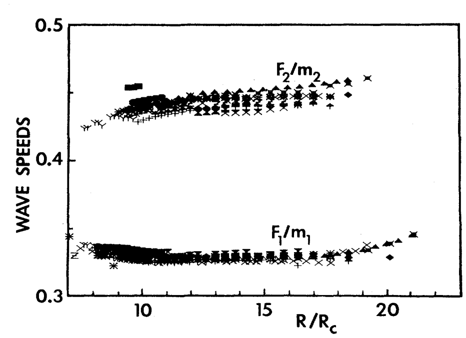

FIG. 2. The wave speeds F_1/m_1 and F_2/m_2 of the two sets of traveling azimuthal waves. Each of the thirteen symbols corresponds to a different state (N, m_1, m_2), where N is the number of vortices; m_1 and m_2 each range from 3 to 7. Wave speeds were obtained from power spectra of velocities determined by laser Doppler velocimetry.

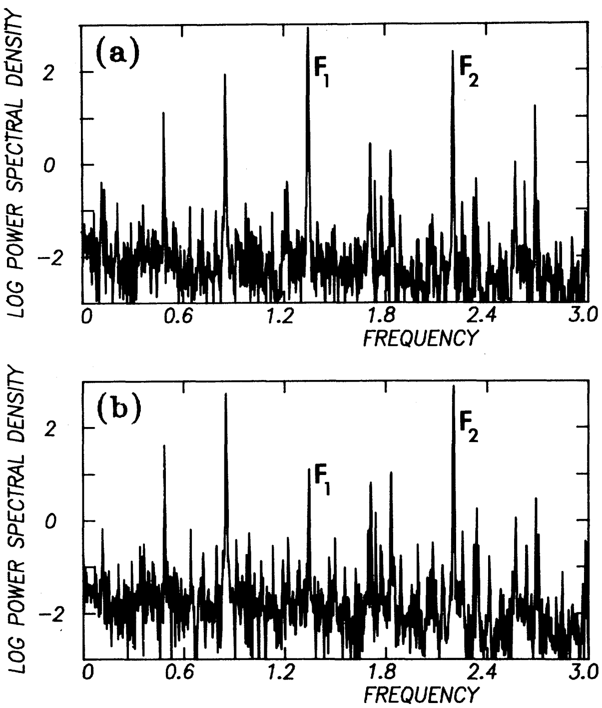

FIG. 3. (a) Power spectrum of the photocurrent of a single detector for a flow with $m_1 = 4$, $m_2 = 5$, and $R/R_c = 10.6$. (b) Power spectrum of $D(\delta, \tau; t)$ for $\delta = 90°$ and $\tau \simeq \delta/(2\pi F_1/m_1)$. The delay time is that required for the first wave train to propagate from the first to the second detector; hence the amplitude at F_1 is near a minimum.

$+2\pi F_2 \tau - m_2\delta$. The difference between the two phase functions,

$$D(\delta, \tau; t) = I(\varphi_1', \varphi_2') - I(\varphi_1, \varphi_2), \quad (1)$$

has a power spectral density at frequency $pF_1 + qF_2$ which is given by[8]

$$P(pF_1 + qF_2) \propto \{1 - \cos[2\pi(pF_1 + qF_2)\tau - (pm_1 + qm_2)\delta]\}, \quad (2)$$

where p and q are integers.

Tests of (2) have been made on a flow visualized by means of a suspension of Kalliroscope flakes in water.[6] The intensity of light scattered from the two probe beams was detected with photodiodes and recorded as a function of time in a computer. The spectrum of the resulting time series for a single detector is shown in Fig. 3(a).[3] Figure 3(b) shows a power spectrum of the difference function $D(\delta, \tau; t)$ for a time delay value near that at which the component at F_1 should be absent according to (2); this result is in good agreement with the prediction, and a corresponding result obtains for F_2. Moreover, the sinusoidal dependence of the spectral power on τ predicted by (2) is supported by the data, as Fig. 4 illustrates. Also, the phase of each sinusoid at $\tau = 0$ in Fig. 4 provides a check that the assignments of integers m_1 and m_2 were correct.

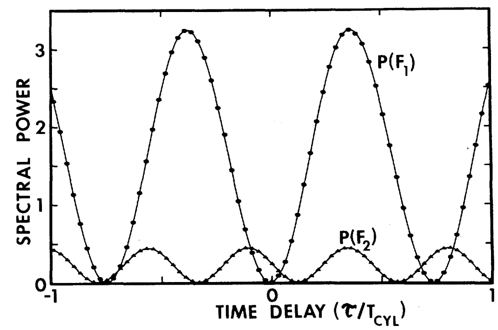

FIG. 4. The dependence of the spectral power of $D(\delta, \tau; t)$ on τ at frequencies F_1 ($p = 1$, $q = 0$) and F_2 ($p = 0$, $q = 1$); $m_1 = 4$, $m_2 = 5$, $\delta = 90°$. The curves are best-fit sine functions.

We have made measurements for states with $m_1 = 4$, 5, 6, and 7, and $m_2 = 4$, 5, 6, and 7. For each state the power in several different spectral components $pF_1 + qF_2$ was determined as a function of τ for several angles δ. These data are all accurately described by (2).

Finally, the existence of the two special comoving reference frames has been confirmed in scattering intensity measurements made with a single probe mounted on a rotating table that is coaxial with the Couette system. Figure 5 shows a power spectrum obtained with the table rotating at a frequency F_2/m_2. The flow is characterized by a single fundamental frequency F_1' (the shifted value of F_1) and its harmonics, in accord with the two-traveling-wave hypothesis; a corresponding result is obtained with the table rotating at $F_1/$

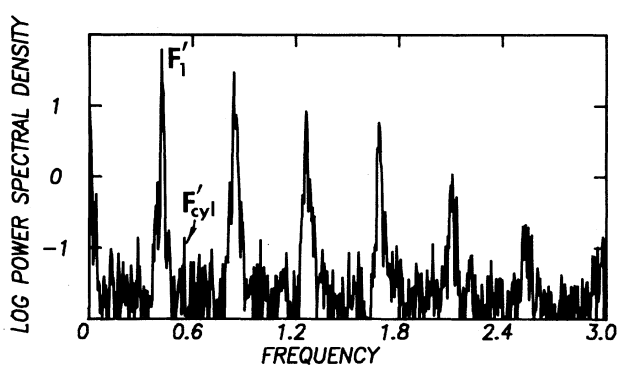

FIG. 5. Power spectrum obtained with a single detector mounted on a table rotating at frequency F_2/m_2, for a flow with $m_1 = 4$, $m_2 = 5$, and $R/R_c = 10.6$.

m_1. Our spectra also contain a weak component at the cylinder frequency, F_{cyl}', due to slight misalignment within the Couette system, and the spectral lines are broadened because of poor table speed control. Detailed measurements with improved system alignment and speed control will be made for all of the observed states (m_1, m_2).[9]

In conclusion, we have shown that doubly periodic flow in the Couette system corresponds to two traveling waves that propagate with different speeds. The symmetry arguments of Rand[4] and Gorman, Swinney, and Rand[5] suggest that there may be other cylindrically symmetric fluid systems with this feature. In fact, Pfeffer and Fowlis[10] have described an asymmetrical flow state observed in a differentially heated rotating annulus in terms of the superposition of two traveling waves.

Several questions are raised by these results. For example, why does simple superposition appear to generate the observed space-time symmetries? Also, what is the physical significance of two wave speeds? If the flow is locally perturbed, will the effect of the perturbation travel at either or both speeds? These and other questions remain for future investigations.

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[7]The relations between the frequencies denoted here F_1 and F_2 and the frequencies denoted f_1 and f_2 in Ref. 3 are $F_1=f_1$ and $F_2=f_1+f_2$.
[8]Equation (2) can be obtained by substituting

$$I(\varphi_1,\varphi_2)=\Sigma a_p b_q \exp[i(p\varphi_1+q\varphi_2)]$$

and

$$I(\varphi_1',\varphi_2')=\Sigma a_p b_q \exp[i(p\varphi_1'+q\varphi_2')]$$

into $D(\delta,\tau;t)$.
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Observations of Double Layers and Solitary Waves in the Auroral Plasma

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Small-amplitude double layers and solitary waves containing magnetic-field-aligned electric field components have been observed for the first time in the auroral plasma between altitudes of 6000 and 8000 km in association with electron and ion velocity distributions that indicate the presence of electric fields parallel to the magnetic field. The double layers may account for a large portion of the parallel potential drop that accelerates auroral particles.

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Double layers, small localized regions of a single electric field polarity, have been studied analytically,[1] but until now have been observed only in computer simulations[2] and in laboratory plasmas.[3] We report the first observation of small-amplitude double layers in a naturally occurring plasma. These double layers differ from the previously reported electrostatic shocks[4] in that their electric field is much smaller (typically no greater than 15 mV/m), their electrostatic polarization relative to the magnetic field is predominantly parallel rather than perpendicular, and the duration of an individual double layer is much shorter—typically 2–20 ms rather than 0.1–10 s for the electrostatic shocks. The dominant polarity of the electric field through an

