Precise Measurement of the Hyperfine-Structure Interval and Zeeman Effect in the Muonic Helium Atom

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(Received 8 July 1981; revised manuscript received 16 February 1982)

Measurements of the hyperfine Zeeman transitions ($\Delta M_\mu = \pm 1$, $\Delta M_J = 0$) in the ground state of the muonic helium atom ($^4\mathrm{He}^{++}\mu^- e^-$) at magnetic fields of 11.5 and 13.6 kG have yielded values for the hfs interval, $\Delta\nu = 4465.004(29)$ MHz (6.5 ppm), and for the negative-muon magnetic moment, $\mu_{\mu^-}/\mu_p = 3.183\,28(15)$ (47 ppm). The theoretical value for $\Delta\nu$, including relativistic, radiative, and recoil contributions, agrees with our measured value but is much less precise because of inadequate knowledge of the Schrödinger wave function.

PACS numbers: 36.10.Dr

The muonic helium atom ($^4\mathrm{He}^{++}\mu^- e^-$) is the simplest example of a three-body atomic system with bound particles of widely different masses. It is a useful system for studying the electromagnetic interaction of the electron and negative muon, since the interaction between the spin magnetic moments of the muon and the electron can be determined through precise measurement of the ground-state hyperfine structure interval $\Delta\nu$. This provides a sensitive test of the Schrödinger wave function of the atom since the theoretical value of $\Delta\nu$ depends critically on the μ-e correlation. In addition, a measurement of the Zeeman effect in $^4\mathrm{He}^{++}\mu^- e^-$ can provide a precise direct determination of the ratio μ_{μ^-}/μ_p of the magnetic moments of the negative muon and the proton, which can be compared with the more accurately known μ^+ magnetic moment as a test of *CPT* invariance.

The muonic helium atom was discovered in a Larmor precession experiment at the Space Radiation Effects Laboratory,[1,2] and the first measurement of $\Delta\nu$ was made by observation of a microwave magnetic resonance transition at zero magnetic field at the Schweizerisches Institut für Nuklearforschung (SIN).[3] In this Letter we report a higher-precision measurement of $\Delta\nu$ and a determination of μ_{μ^-} through observation of Zeeman transitions at strong magnetic field at the Clinton P. Anderson Meson Physics Facility (LAMPF). We also measure the dependence of $\Delta\nu$ on the buffer-gas density, so that extrapolation to zero density can be made empirically.

The energy levels for the ground state of $^4\mathrm{He}^{++}$-$\mu^- e^-$ in a static magnetic field, $\vec{H}$, are obtained from the Hamiltonian[2]

$$\mathcal{H} = (-h\Delta\nu)\vec{I}_\mu\cdot\vec{J} + g_J\mu_B{}^e\vec{J}\cdot\vec{H} + g_\mu{}'\mu_B{}^\mu\vec{I}_\mu\cdot\vec{H} \tag{1}$$

in which $\Delta\nu$ is the hfs interval, $\vec{J}$ ($\vec{I}_\mu$) is the electronic (muonic) angular momentum operator, and $\mu_B{}^e$ ($\mu_B{}^\mu$) is the electron (muon) Bohr magneton. The g factors, to relative order α^2, are given approximately by

$$g_J \simeq g_e(1-\tfrac{1}{3}\alpha^2) = 2(1.001\,14);\qquad g_\mu{}' \simeq g_\mu(1-\tfrac{5}{3}\alpha^2) = 2(1.001\,08), \tag{1a}$$

in which g_e and g_μ are the free-particle g values[4,5] and the terms in α^2 are relativistic and diamagnetic-shielding contributions[6] which we have calculated for the $^4\mathrm{He}^{++}\mu^- e^-$ atom by using methods[7] similar to those developed for the ^{4}He atom.[8]

The strong-field transitions observed in our microwave magnetic resonance experiment are the muon spin-flip transitions, $\Delta M_\mu = \pm 1$, $\Delta M_J = 0$, where M_μ (M_J) is the muon (electron) magnetic quantum number. Their frequencies are given by

$$\nu_{12} = -\mu_B{}^{\mu} g_{\mu}'(H/h) + \tfrac{1}{2}\Delta\nu[1 - x + (1+x^2)^{1/2}], \quad (2)$$
$$\nu_{34} = +\mu_B{}^{\mu} g_{\mu}'(H/h) + \tfrac{1}{2}\Delta\nu[1 + x - (1+x^2)^{1/2}],$$

in which $x = (g_J\mu_B{}^e - g_\mu{}'\mu_B{}^\mu)H/(h\Delta\nu)$. An interesting and useful feature of Eq. (2) is that for $H \simeq 11.5$ kG the frequencies ν_{12} and ν_{34} are equal, and hence both transitions can be driven simultaneously.

The apparatus is similar to that used in a recent muonium experiment.[9] A beam of polarized μ^- from the muon channel[10] at LAMPF is stopped in a 35-cm-long, 30-cm-diam cylindrical pressure vessel containing He gas with a 1.5% admixture of Xe at 5 and 15 atm pressure and at ~25 °C. (The Xe serves as an electron donor to form neutral $^4\mathrm{He}^{++}\mu^-e^-$ atoms.[1]) The muon beam has a central momentum of 35 MeV/c and an intensity of 4×10^4 μ^-/s following a 6-cm-diam collimator. The beam passes through a 0.13-mm BeCu window, and most of the muons stop in the active region of the target, which is the gas in a 19-cm-long, 16-cm-diam TM_{110} microwave cavity. Inactive parts of the apparatus where muons might stop were made of high-Z materials in which nuclear capture is much more probable than muon decay. Static magnetic fields, H, of 11.5 and 13.6 kG, homogeneous to about 5 ppm over the active region, are provided by a precision solenoid[11] and are monitored and measured by a proton NMR system with an H_2O sample.

Incoming muons, μ_I, are detected by a 0.5-mm-thick plastic scintillator and decay electrons are detected by two scintillator telescopes in the forward and backward beam directions with respect to the target. The electron telescopes are vetoed for 0.5 μs after each μ_I to reject counts from muons stopped in the high-Z materials of the apparatus. The electrons from the muon decays are emitted preferentially in the direction opposite to the muon spin. Hence the spin-flip transitions ν_{12} and ν_{34} can be detected by observing the change in the rates in the forward (e_F) and backward (e_B) electron telescopes as the microwave field is switched on and off at 9 Hz. The signal S for each telescope is $S = [(e/\mu_I)_{\rm on} - (e/\mu_I)_{\rm off}]/(e/\mu_I)_{\rm off}$. Resonance curves are observed by sweeping either the microwave frequency ν to obtain $S(\nu)$ or the magnetic field H to obtain $S(H)$.

Data were taken at two gas pressures, 5 and 15 atm, and in the following modes: (i) sweeping H

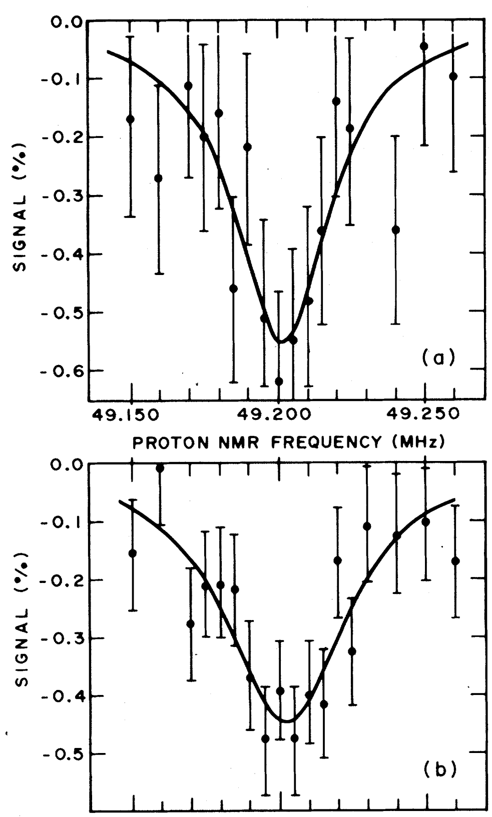

FIG. 1. Typical resonance curves for the ν_{12} transition obtained with the forward telescope at (a) 15 atm and (b) 5 atm. The data for these curves were obtained in (a) 24 h and (b) 100 h. For each curve obtained with the forward telescope there is a corresponding curve for the backward telescope.

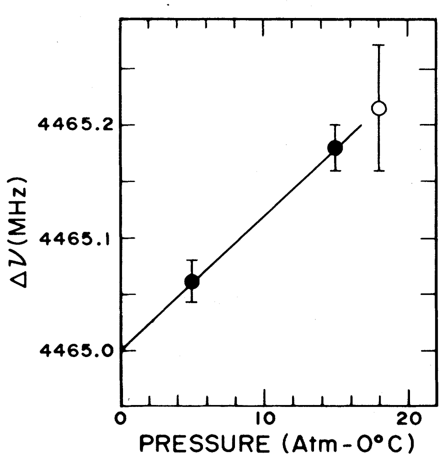

FIG. 2. $\Delta\nu$ as a function of He+Xe(1.5%) gas pressure. Closed circles show the results of this experiment; the open circle is the result of Ref. 3. The straight line shows the linear extrapolation used to extract $\Delta\nu(0)$.

through the value for which $\nu_{12}=\nu_{34}=\Delta\nu/2$ with the microwave frequency fixed at $\Delta\nu/2$ so that both transitions are driven simultaneously; (ii) sweeping ν with H fixed so that $\nu_{12}=\nu_{34}$, thereby driving ν_{12} and ν_{34} simultaneously; (iii) sweeping H with ν fixed so that only the ν_{12} transition is driven; and (iv) sweeping H with ν fixed so that only ν_{34} is driven. Most of our data were taken in modes (iii) and (iv) for which the ν_{12} and ν_{34} lines are resolved. The overlapping ν_{12} and ν_{34} lines of modes (i) and (ii) provide twice the signal heights of those of modes (iii) and (iv), but the line shapes are more difficult to analyze.

A Lorentzian line shape, which is an adequate approximation to the exact theoretical line shape, is fitted to a measured resonance curve to determine the line center, the linewidth, and the signal height. Typical resonance curves are shown in Fig. 1. The small signal height of about 0.5% is explained by the large depolarization of μ^- in He.[1,2] Fits with satisfactory χ^2 were obtained for the eighteen observed resonance curves.

A value of $\Delta\nu(P)$ is extracted from the data at each measured pressure, using Eq. (2) with the assumption that the μ^- moment is equal to the more accurately determined μ^+ moment.[9,12] The measured values of $\Delta\nu$ as a function of pressure are shown in Fig. 2, together with the zero-field measurement of Ref. 3. The hyperfine interval for the free atom $\Delta\nu(0)$ is found by fitting a linear pressure shift, $\Delta\nu(P)=\Delta\nu(0)(1+aP)$, to measured values. From our data we obtain

$$\begin{aligned}\Delta\nu(0)&=4465.004(29)\ \mathrm{MHz}\ (6.5\ \mathrm{ppm}),\\ a\Delta\nu(0)&=11.4(2.7)\ \mathrm{kHz/atom}\ (0\,^\circ\mathrm{C}),\end{aligned}\qquad(3)$$

in which the one-standard-deviation errors, predominantly statistical, are indicated.

We note that the earlier zero-field measurement[3] involved only one pressure point at 20 atm. The extrapolation to zero pressure used the more easily measured pressure shift of muonium in He-Xe gas and hence relied on the assumption that pressure shifts for μ^+e^- and for $\mathrm{He}^{++}\mu^-e^-$ are identical. This approach is justified to some extent by the absence of any observed isotope dependence of the pressure shift[13] for H, D, or T in Ar. In the present paper the pressure shift, a, is measured and hence no such assumption involving complicated collisional processes is required.

By treating both $\Delta\nu$ and μ_{μ^-} as free parameters in Eq. (2) we can obtain from our data $\mu_{\mu^-}/\mu_p=3.183\,28(15)$ (47 ppm), in agreement with the more accurately known value[9,12] for μ_{μ^+}/μ_p as required by CPT invariance for particle and antiparticle. Other measurements of magnetic moments of particles and antiparticles show that the moments are equal within uncertainties of 0.13 ppm for e^- and e^+,[14] and 7500 ppm for p and $\bar{p}$.[15]

Table I lists the significant sources of error in our measurement of $\Delta\nu(0)$ and μ_{μ^-}/μ_p. These are clearly dominated by the statistical counting errors. The systematic g_J and quadratic pressure-shift errors are based on the measured magnitudes of these effects for muonium in krypton.[9] On the basis of theoretical[16] and experimental[17] information on the g_J pressure shift for Rb in He and on the quadratic pressure shifts in the hfs intervals for alkali atoms in noble gases,[18] we believe that the errors listed represent conservative upper bounds on these systematic effects for $^4\mathrm{He}^{++}\mu^-e^-$ in He.

The theoretical expression for $\Delta\nu$ is

$$\begin{aligned}\Delta\nu&=\Delta\nu_F[1+\delta^{\mathrm{rel}}+\delta^{\mathrm{rad}}+\delta^{\mathrm{rec}}]\\ &=\Delta\nu_F(1+2227\ \mathrm{ppm}),\end{aligned}\qquad(4)$$

TABLE I. Experimental uncertainties.

Source of error	Fractional error in $\Delta\nu(0)$ (ppm)	Fractional error in μ_{μ}^-/μ_p (ppm)
Counting statistics	6.4	41
Quadratic pressure shift	0.5	0
g_J pressure shift	0.0	18
Field inhomogeneity	1.3	9
Quadrature sum	6.5	47

TABLE II. Higher-order contributions to $\Delta\nu$.

Term	Expression	ppm
Relativistic (δ^{rel})[a]	$0.17\alpha^2$	8.9
Radiative (δ^{rad})[a,b]		
a_e, a_μ	$\alpha/\pi+0.76(\alpha/\pi)^2$	2326.9
Vacuum polarization[c]	$7.66(\alpha/\pi)^2$	41.3
Vertex	$-25.2(\alpha/\pi)^2$	−136.0
Recoil (δ^{rec})[b]	$-44(\alpha/\pi)m_e/(m_\mu+m_\alpha)$	−13.7
Total contributions		2227

[a]Ref. 19.
[b]Ref. 22.
[c]The value of the vacuum polarization radiative contribution of +41.3 ppm is taken from Ref. 19; the corresponding contribution from Ref. 22 is +29.0 ppm.

TABLE III. Comparison of theoretical results with this experiment.

Method	$\Delta\nu$ (MHz)
Variational calculation[a]	4465.1(1.0)
Perturbation theory[b]	4462.6(3.0)
Born-Oppenheimer theory[c]; global operator technique	4460
Present experiment	4465.004(29)

[a]Ref. 19.
[b]Ref. 20.
[c]Ref. 21.

in which $\Delta\nu_F$ is the lowest-order (Fermi) term including the effective reduced mass factor. Calculation of $\Delta\nu_F$, which requires an accurate Schrödinger wave function in the region of μ-e overlap and which has been done by different methods,[19-21] contains the dominant theoretical error. Table II gives the higher-order contributions[19,22] which amount to 2227 ppm. Table III gives our experimental value of $\Delta\nu$, and the various calculated values including the higher-order contributions.

Our experiment is in good agreement with the value, $\Delta\nu = 4464.95(6)$ MHz (13 ppm), obtained from measurements at SIN[3] and is consistent with, but about 2 orders of magnitude more precise than, the theoretical results. An improved solution of the three-body Schrödinger equation for this unusual atom is clearly needed.

This work was supported in part by the U. S. Department of Energy (Contract No. DE-AC02-76ERO3074) and by NATO Grant No. 1589. In addition we acknowledge the support of the Schweizer National Fonds and the Max Kade Foundation. We gratefully acknowledge important contributions to this experiment by B. Dichter, A. Disco, R. Fong-Tom, and L. Trudell of Yale University and also the support and technical assistance of R. Damjanovich, B. Rector, and the entire operating staff at LAMPF. We also thank K.-N. Huang, S. Lakdawala, and P. Mohr for helpful discussions.

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