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Axions, Domain Walls, and the Early Universe

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Axion models have a spontaneously broken $Z(N)$ symmetry. The resulting discretely degenerate vacua and domain-wall solitons are incompatible with the standard cosmology. It is possible, however, to introduce a small $Z(N)$ breaking interaction into axion models without upsetting the Peccei-Quinn mechanism. In that case the domain walls disappear a certain time after their formation in the early universe. Their presence for a limited time period might lead to galaxy formation.

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When a gauge interaction explicitly breaks a global symmetry, it often happens that a discrete subgroup of the global symmetry remains unbroken. Such is the case in axion models[1] which, as I am about to show, have a spontaneously broken $Z(N)$ symmetry. The result applies to all models which have a Peccei-Quinn symmetry $U_{PQ}(1)$ which is broken only by the QCD gluon anomaly. N is the number of quark flavors that rotate under $U_{PQ}(1)$. To be specific however, I analyze the Dine-Fischler-Srednicki model[2] in which the axion can be made "invisible." The Yukawa couplings and scalar self-interactions of that model,

$$-\sum_{i,j=1}^{N/2} K_i^{uj}(u_{Li}'^{\dagger} d_{Li}'^{\dagger})\begin{pmatrix}\varphi_1^{0}\\ \varphi_1^{-}\end{pmatrix}u_{Rj}' + \text{H.c.} - \sum_{i,j=1}^{N/2} K_i^{dj}(u_{Li}'^{\dagger} d_{Li}'^{\dagger})\begin{pmatrix}-\varphi_2^{-*}\\ \varphi_2^{0*}\end{pmatrix}d_{Rj}' + \text{H.c.} - V(\varphi_1, \varphi_2, \Phi), \quad (1)$$

have the $U_{PQ}(1)$ symmetry:

$$q_i \to e^{i\alpha\gamma_5} q_i, \quad \varphi_1 \to e^{-2i\alpha}\varphi_1,$$
$$\varphi_2 \to e^{+2i\alpha}\varphi_2, \quad \Phi \to e^{-2i\alpha}\Phi. \quad (2)$$

Φ is a singlet under the standard gauge group, which is coupled to φ_1 and φ_2 through V. $U_{PQ}(1)$ is explicitly broken by and only by the QCD gluon anomaly. The corresponding anomalous Ward identity requires the change

$$\theta_{QCD} \to \theta_{QCD} - 2N\alpha \quad (3)$$

when the transformation (2) is applied.

However, a $Z(N)$ subgroup of $U_{PQ}(1)$ remains unbroken. Indeed, consider the subgroup $Z_L(N) \otimes Z_R(N) \otimes U_V(1)$ of the global symmetry group $SU_L(N) \otimes SU_R(N) \otimes U_V(1)$ of QCD[3]:

$$q_{Li} \to \exp[i(2\pi k_L/N + \beta)]\, q_{Li},$$
$$q_{Ri} \to \exp[i(2\pi k_R/N + \beta)]\, q_{Ri}, \quad (4)$$

where k_L and k_R are integers. These are also symmetries of the $SU_L(2) \otimes U_Y(1)$ gauge interactions and indeed of the full theory provided that we make the transformations

$$\varphi_1 \to \exp[i(2\pi/N)(k_L - k_R)]\varphi_1, \quad \varphi_2 \to \exp[-i(2\pi/N)(k_L - k_R)]\varphi_2, \quad \Phi \to \exp[i(2\pi/N)(k_L - k_R)]\Phi \tag{5}$$

along with (4). The generator of the subgroup $Z(N)$ of $U_{PQ}(1)$ corresponds to setting $k_L = 0$, $k_R = 1$, $\beta = -\pi/N$ for which we obtain (2) with $\alpha = \pi/N$.

Let us adopt the quark phase convention for which $\theta_{QCD} = 0$, and diagonalize the Yukawa couplings:

$$-\sum_{j=1}^{N/2} K_j{}^u e^{i\gamma}(u_{Lj}{}^\dagger d_{Lj}{}''^\dagger)\begin{pmatrix}\varphi_1{}^0\\ \varphi_1{}^-\end{pmatrix} u_{Rj} + \text{H.c.} - \sum_{j=1}^{N/2} K_j{}^d e^{i\gamma}(u_{Lj}{}''^\dagger d_{Lj}{}^\dagger)\begin{pmatrix}-\varphi_2{}^{-*}\\ \varphi_2{}^{0*}\end{pmatrix} d_{Rj} + \text{H.c.}, \tag{6}$$

where $K_j{}^u$ and $K_j{}^d$ are real and positive, and $q_L'' = \mathfrak{U}_{KM}\, q_L$.

The vacuum is characterized by[4]

$$\langle q_{Lj}{}^\dagger q_{Rj}\rangle_0 = \mu_j{}^3, \quad \langle \varphi_1{}^0\rangle_0 = v_1 \exp(i\alpha_1), \\ \langle \varphi_2{}^0\rangle_0 = v_2 \exp(i\alpha_2), \quad \langle \Phi\rangle_0 = v_\Phi \exp(i\alpha_\Phi). \tag{7}$$

The phases α_1 and α_2 are determined by minimizing the vacuum expectation value of the Yukawa interactions, which couples the phases of $\langle\varphi_1\rangle_0$ and $\langle\varphi_2\rangle_0$ to the vanishing[4] phase of the quark-antiquark condensates. One finds $\alpha_1 = -\gamma$ and $\alpha_2 = \gamma$, and hence the quark current masses $m_i{}^u = \exp[i(\alpha_1 + \gamma)]K_i{}^u v_1$ and $m_i{}^d = \exp[i(-\alpha_2 + \gamma)]K_i{}^d v_2$ are real when $\theta_{QCD} = 0$. That is the Peccei-Quinn mechanism. The axion field a is the pseudo-Goldstone boson produced by the spontaneous breaking of $U_{PQ}(1)$. The axion mass[1,2] $m_a \simeq f_\pi m_\pi / v$ and the axion couplings[2] to quarks, leptons, the photon, gluons, and weak vector bosons are all proportional to $1/v = (v_1{}^2 + v_2{}^2 + v_\Phi{}^2 + f_\pi{}^2)^{-1/2}$.

The $Z(N)$ symmetry of axion models implies that there are $N-1$ other vacua degenerate with vacuum (7). The N vacua can be characterized by

$$\langle q_{Li}{}^\dagger q_{Ri}\rangle_k = \mu_i{}^3 \exp(+2ik\pi/N), \\ \langle\varphi_1\rangle_k = v_1 \exp[i(\alpha_1 - 2k\pi/N)], \\ \langle\varphi_2\rangle_k = v_2 \exp[i(\alpha_2 + 2k\pi/N)], \\ \langle\Phi\rangle_k = v_\Phi \exp[i(\alpha_\Phi - 2k\pi/N)], \tag{8}$$

for $k = 1, 2, \ldots, N$. Although distinct, the N vacua all have exactly the same physical properties. Local experiments cannot determine which one is the local vacuum. However, neighboring regions which happen to be in different vacua must be separated by domain-wall solitons. The main properties of these domain walls can be obtained by considering the effective action

$$S_{\text{eff}} = v^2 \int d^4x \left[\tfrac{1}{2}\partial_\mu \alpha\, \partial^\mu \alpha + (m_a{}^2/N^2) f(N\alpha)\right], \tag{9}$$

where $\alpha \sim a/v$ denotes collectively all the phases that rotate under a $U_{PQ}(1)$ transformation, and f is a periodic function of period 2π, whose Taylor expansion begins with $f(x) = f(0) - \frac{1}{2}x^2 + \ldots$. One easily finds that the domain wall between two domains respectively in vacua $\alpha = 2k\pi/N$ and $\alpha = 2(k+1)\pi/N$ has thickness of order $m_a{}^{-1}$ and mass per unit surface

$$m \simeq m_a v^2 \simeq f_\pi m_\pi v. \tag{10}$$

A $\Delta\alpha = 4\pi/N$ domain wall (for $N > 2$) will tend to split into two $\Delta\alpha = 2\pi/N$ domain walls. On the other hand, neighboring domain walls of opposite $\Delta\alpha$ attract and are likely to annihilate each other. The corresponding forces have range of order $m_a{}^{-1}$. Because of their thickness, domain walls can only absorb or reflect radiation of momentum[5] $\lesssim m_a \simeq (1.4\times10^{-3}\text{ eV})[(10^{10}\text{ GeV})/v]$. For v large enough, the domain walls are transparent for most practical purposes. Because the stress density in domain walls is of the order of their energy density,[6] Newtonian gravity cannot be applied to their case. The gravitational fields produced by domain walls have been investigated by Vilenkin.[7] They produce accelerations of order

$$Gm \simeq \frac{1}{6\times10^5\text{ sec}}\,\frac{v}{10^{10}\text{ GeV}}. \tag{11}$$

As has been emphasized before,[6] the spontaneous breakdown of an exact discrete symmetry such as the $Z(N)$ symmetry of axion models exhibited here is incompatible with the standard cosmology. When the universe cools after the "big bang," different regions will as a rule settle into different vacua. In particular, regions which are outside each other's event horizon are uncorrelated. Requiring $\rho_{\text{domain wall}} \lesssim \rho_{\text{critical}}$ at the present time, one finds $m \lesssim 10^{-5}\text{ GeV}^3$, which is impossible in axion models, unless $N = 1$.[8]

The above argument holds only if the N vacua all have exactly the same physical properties. It might be, however, that the $U_{PQ}(1)$ symmetry is softly broken. That need not upset the Peccei-

Quinn mechanism. For example, adding a term of the form $e^{i\delta}\mu\Phi$ + H.c. to $V(\varphi_1, \varphi_2\Phi)$ would produce an effective value for $\theta_{\rm QCD}$ of order $\mu^3/m_a^2 v$, which is consistent with the upper limit of 10^{-9} from the neutron electric dipole moment[3] provided that

$$\mu^3 \leq 10^{-9} f_\pi^2 m_\pi^2/v. \quad (12)$$

The soft breaking of $U_{\rm PQ}(1)$ would produce shifts in energy density among the vacua of order $\langle\Delta\mathcal{H}\rangle_0 = \mu^3 v$. This results in pressure $p = \langle\Delta\mathcal{H}\rangle_0$ on the domain walls in the direction of the domain with highest vacuum energy density. The corresponding acceleration is c/τ with

$$\tau = \frac{m}{p} \simeq \frac{f_\pi m_\pi v}{\mu^3 v} = (5\times10^{-4}\ \text{sec})\frac{v}{10^{10}\ \text{GeV}}\frac{f_\pi^2 m_\pi^2 10^{-9}}{v\mu^3}. \quad (13)$$

$\lambda = c\tau$ is the size of domains for which the differences in volume energy are of the order of the surface energy. Once the domain bubbles have average size λ, the $Z(N)$-breaking effects become important and the true vacuum takes over.

Let us then consider the early universe. Domain walls appear when the temperature is of the order of

$$T_0 = (m_a^2 v^2)^{1/4} = (f_\pi m_\pi)^{1/2} = 1.4\times10^{12}\ \text{K}. \quad (14)$$

The universe is then about $t_0 \simeq 6\times10^{-5}$ sec old. For a time of order t_0 thereafter, thermal fluctuations can easily create and destroy domain walls. I assume that during that time, regions of size ct_0 settle into the same vacuum. After that the domain walls are "frozen," that is, they can only annihilate by meeting other domain walls. The initial mass density of domain walls is thus

$$\rho_{w0} \simeq \frac{f_\pi m_\pi v}{ct_0} \simeq 3\times10^{-8}(10^{12}\ \text{K})^4\frac{v}{10^{10}\ \text{GeV}}. \quad (15)$$

It seems plausible that the main aspects of the subsequent evolution of the domain walls are controlled by, first, the Hubble expansion of the bubbles along with the rest of the universe, and second, the expansion at close to the speed of light of those bubbles which happen to be very large. If one looks at sufficiently large regions of space, one will always find bubbles which are much larger than average. This is because adjacent causally disconnected regions of space can happen to be in the same vacuum, in which case they are part of the same bubble. Bubbles which are larger than a certain critical size will expand at approximately the speed of light because, by expanding, they "eat up" more domain wall at their periphery than the amount of domain wall needed to increase their surface. Domain-wall annihilation at the periphery of the large expanding bubbles produces radiation in the form of axions, gravitational waves, and so on. The ratio of the number density of large expanding bubbles to the number density of average bubbles is of course a decreasing function of N.

We thus arrive at a picture in which the largest bubbles always have size of order ct. The smaller bubbles are regularly "eaten up" by the large expanding bubbles. Therefore, unless N is *very* large, the size of the smallest bubbles at any one time is not much smaller than ct either. Let $l \sim ct$ be the average size of bubbles at time t. Then the amount M_w of matter in the form of domain walls in a large comoving volume V increases with time during the radiation-dominated era as

$$M_w \simeq \frac{V}{l^3} l^2 m = \frac{R^3}{l} m \sim \frac{t^{3/2}}{t} = \sqrt{t}. \quad (16)$$

To M_w one must add the mass M_a of radiation produced by domain-wall annihilation. We have

$$\frac{dM_{w+a}}{dt} = \frac{dM_w}{dt} + \frac{dM_a}{dt} = 2HM_w - HM_a. \quad (17)$$

$H = 1/2t$ in the radiation-dominated era. The first term in (17) represents the stretching of domain walls due to Hubble expansion; the second term represents the cooling of the radiation that has been produced by domain-wall annihilation. Combining (16) and (17), one finds that M_{w+a} also increases with time as $\sqrt{t}$. On the other hand, the mass M_r of ordinary radiation in the comoving volume V decreases with time as $1/\sqrt{t}$. Hence

$$\frac{\rho_{w+a}}{\rho_r} \simeq \frac{t}{2t_0}\left.\frac{\rho_{w+a}}{\rho_r}\right|_{2t_0}. \quad (18)$$

Comparing with (15), we find that ρ_{w+a} does not dominate the matter density of the universe till about a time of order

$$t \sim 10^8 t_0(10^{10}\ \text{GeV}/v) \sim (6\times10^3\ \text{sec})(10^{10}\ \text{GeV}/v). \quad (19)$$

By requiring $\tau < t$, we obtain the constraint

$$\frac{f_\pi m_\pi 10^{-9}}{\mu^3 v}\left(\frac{v}{10^{10}\ \text{GeV}}\right)^2 < 10^7. \quad (20)$$

From (12) and (20), one concludes that $v \leq 10^{13.5}$

GeV for the present upper limit on the neutron electric dipole moment.

The appearance of domain walls for a limited time period in the early universe may of course be useful to explain the formation of galaxies. The strong gravitational fields (11) due to domain walls can produce large-scale inhomogeneities that may survive until decoupling time and collapse into galaxies. If one demands that the final average size $c\tau$ of the domain bubbles grows by Hubble expansion into the present average intergalactic distance, one obtains $\tau \sim 3\times 10^6$ sec. To obtain the average distance between clusters of galaxies, one needs $\tau \sim 10^{11}$ to 10^{12} sec.

Finally, I would like to emphasize that spontaneously broken discrete symmetries appear in other contexts, for example, the one-hyperfamily extended hypercolor model[9] with Pati-Salam unification. The Pati-Salam unification is necessary to break explicitly a spontaneously broken $U_0(1)\otimes U_3(1)$ global symmetry and give mass to the two concomitant pseudo-Goldstone bosons P^0 and P^3. However, the Pati-Salam interactions leave unbroken[10] a $Z_0(4)\otimes Z_3(4)$ subgroup of $U_0(1)\otimes U_3(1)$. The resulting $(4\times 4=16)$-fold discrete vacua are incompatible with the standard cosmology, unless additional $Z_0(4)\otimes Z_3(4)$-breaking interactions are introduced into the model.

My conclusions are as follows: (1) All axion models in which the $U_{PQ}(1)$ symmetry is broken only by the QCD gluon anomaly are incompatible with standard cosmology, unless $N=1$.[8] (2) One can break the $Z(N)$ symmetry of axion models without upsetting the Peccei-Quinn mechanism. Provided (20) is satisfied, the domain walls disappear before dominating the matter density of the universe. (3) The appearance of domain walls for a limited time period in the early universe might be useful to explain galaxy formation. Other models, in particular scalarless theories, can have that property.

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