to hold for large values of λ. We have checked explicitly the agreement of the free energy of the standard and reduced models in lower orders of perturbation theory in arbitrary dimensions [up to $(1/\lambda)^5$]. It is extremely important to check if the equivalence of the models persists at small values of λ.

We are grateful to Professor E. Brézin for his critical remark on the original version of our manuscript.

Note added.—After the submission of this paper we received a preprint by G. Bhanot, U. Heller, and H. Neuberger where some evidence for a spontaneous breakdown of U(1) symmetry at small λ is presented. We understand that M. Peskin and K. Wilson have obtained similar results.

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Strangeness Production in the Quark-Gluon Plasma

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Rates are calculated for the processes $gg\rightarrow s\bar{s}$ and $u\bar{u}, d\bar{d}\rightarrow s\bar{s}$ in highly excited quark-gluon plasma. For temperature $T \geq 160$ MeV the strangeness abundance saturates during the lifetime ($\sim 10^{-23}$ sec) of the plasma created in high-energy nuclear collisions. The chemical equilibration time for gluons and light quarks is found to be less than 10^{-24} sec.

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Given the present knowledge about the interactions between constituents (quarks and gluons), it appears almost unavoidable that, at sufficiently high energy density caused by compression and/or excitation, the individual hadrons dissolve in a new phase consisting of almost-free quarks and gluons.[1] This quark-gluon plasma is a highly excited state of hadronic matter that occupies a volume large as compared with all characteristic length scales. Within this volume individual color charges exist and propagate in the same manner as they do inside elementary particles as described, e.g., within the Massachusetts Institute of Technology (MIT) bag model.[2]

It is generally agreed that the best way to create a quark-gluon plasma in the laboratory is with collisions of heavy nuclei at sufficiently high energy. We investigate the abundance of strangeness as function of the lifetime and excitation of the plasma state. This investigation was motivated by the observation that significant changes in relative and absolute abundance of strange particles, such as $\bar{\Lambda}$,[3] could serve as a probe for quark-gluon plasma formation. Another interesting signature may be the possible creation of exotic multistrange hadrons.[4] After identifying the strangeness-producing mechanisms we compute the relevant rates as functions of the energy density ("temperature") of the plasma state and compare them with those for light u and d quarks.

In lowest order in perturbative QCD $s\bar{s}$-quark pairs can be created by annihilation of light quark-antiquark pairs [Fig. 1(a)] and in collisions of two gluons [Fig. 1(b)]. The averaged total cross sections for these processes were calculated by

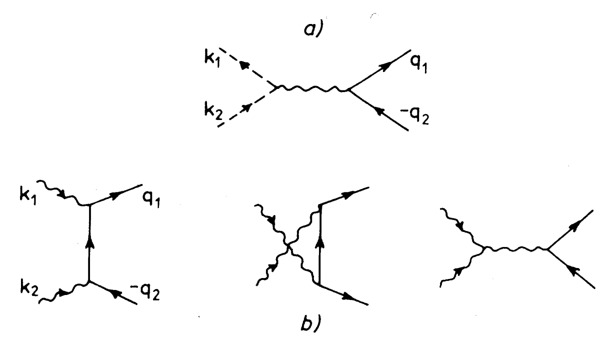

FIG. 1. Lowest-order QCD diagrams for $s\bar{s}$ production: (a) $q\bar{q}\rightarrow s\bar{s}$, (b) $gg\rightarrow s\bar{s}$.

Combridge.[5] For fixed invariant squared mass $s=(k_1+k_2)^2$, where k_i are the four-momenta of the incoming particles $\{w(s)=(1-4M^2/s)^{1/2}\}$,

$$\bar{\sigma}_{q\bar{q}\to s\bar{s}}=\frac{8\pi\alpha_s^2}{27s}\left(1+\frac{2M^2}{s}\right)w(s), \tag{1a}$$

$$\bar{\sigma}_{gg\to s\bar{s}}=\frac{2\pi\alpha_s^2}{3s}\left[\left(1+\frac{4M^2}{s}+\frac{M^2}{s^2}\right)\tanh^{-1}w(s)-\left(\frac{7}{8}+\frac{31}{8}\frac{M^2}{s}\right)w(s)\right]. \tag{1b}$$

For the mass of the strange quark we assume (a) the value fitted within the MIT bag model, $M=280$ MeV, and (b) the value found in the study of quark currents, $M=150$ MeV.[6] When discussing light-quark production we use $M=15$ MeV. The effective QCD coupling constant $\alpha_s=g^2/4\pi$ is an average over spacelike and timelike domains of momentum transfers in reactions shown in Fig. 1. We use (a) $\alpha_s=2.2$, the value consistent with $M=280$ MeV in the MIT bag model, and (b) the value $\alpha_s=0.6$, expected at the involved momentum transfer.

Given the averaged cross sections it is easy to calculate the rate of events per unit time,[7] summed over all final and initial states:

$$\frac{dN}{dt}=\int d^3x\int\frac{d^3k_1}{(2\pi)^3|k_1|}\sum_i\rho_i(k_1,x)\int\frac{d^3k_2}{(2\pi)^3|k_2|}\sum_i\rho_i(k_2,x)\int_{4M^2}^{\infty}ds\,\delta(s-(k_1+k_2)^2)k_1{}^{\mu}k_{2\mu}\bar{\sigma}(s). \tag{2}$$

The sum over initial states involves the discrete quantum numbers i (color, spin, etc.) over which Eq. (1) was averaged. The factor $k_1\cdot k_2/|k_1||k_2|$ is the relative velocity for massless particles, and we have introduced a dummy integration over s in order to facilitate the calculations. We now replace the phase-space densities $\rho_i(k,x)$ by momentum distributions $f_g(k)$, $f_q(k)$, and $f_{\bar{q}}(k)$ of gluons, quarks, and antiquarks that can still have a parametric x dependence, i.e., through $T=T(x)$. The (invariant) rate per unit time and volume for the elementary processes shown in Fig. 1 is then

$$A=\frac{dN}{dt\,d^3x}=\tfrac{1}{2}\int_{4M^2}^{\infty}s\,ds\,\delta(s-(k_1+k_2)^2)$$
$$\times\int\frac{d^3k_1}{(2\pi)^3|k_1|}\int\frac{d^3k_2}{(2\pi)^3|k_2|}\{(2\times8)^2f_g(k_1)f_g(k_2)\bar{\sigma}_{gg\to s\bar{s}}(s)+2\times(2\times3)^2f_q(k_1)f_{\bar{q}}(k_2)\bar{\sigma}_{q\bar{q}\to s\bar{s}}(s)\}, \tag{3}$$

where the numerical factors count the spin, color, and isospin degrees of freedom.

The anticipated lifetime of the plasma created in nuclear collisions is of the order 6 fm/c =2 $\times10^{-23}$ sec. After this time the high internal pressure will most likely have caused the state to expand to below the energy density required for the global restoration of the perturbative QCD vacuum state.[8] From various considerations[9] the transition between the hadronic and the quark-gluon phase is expected at an energy density of approximately 1 GeV/fm^3. Under these conditions, it is possible to estimate that each perturbative quantum (light quark, gluon) in the plasma state will rescatter several times during the lifetime of the plasma. Hence the momentum distribution functions $f(p)$ can be approximated by the statistical Bose (Fermi) distribution functions

$$f_g(p)\approx(e^{\beta\cdot p}-1)^{-1} \tag{4a}$$

(gluons)

$$f_{q/\bar{q}}(p)\approx(e^{\beta\cdot p}\lambda^{\pm}+1)^{-1} \tag{4b}$$

(quarks/antiquarks), where $\beta\cdot p=\beta_0|\vec{p}|-\vec{\beta}\cdot\vec{p}$ for massless particles, $(\beta\cdot\beta)^{-1/2}=T$ is the temperaturelike parameter, and $\lambda^{\pm}$ is the baryon number (antibaryon number) fugacity. In the rest frame of the plasma $\beta\cdot p=|\vec{p}|/T$. The distributions (4) can only be taken seriously for $|\vec{p}|$ not much larger than T; to populate the high-energy tail of the distributions too many collisions are required, for which there may not be enough time during the lifetime of the plasma. While in each individual nuclear collision the momentum distribution may vary, the ensemble of many collisions may lead to better statistical distributions. The interesting quantities, in particular the energy density, can now be expressed in terms of the parameters T and $\lambda^{\pm}$. In the high-temperature limit (i.e., $\lambda^{\pm}\sim1$) one finds, neglecting also perturbative QCD interactions,

$$\epsilon\gtrsim(2\times8+2\times2\times2\times3\times\tfrac{7}{8})\frac{\pi^2}{30}T^4$$
$$\approx\left(\frac{T}{160\text{ MeV}}\right)^4\frac{\text{GeV}}{\text{fm}^3}. \tag{5}$$

The large number of degrees of freedom leads to the relatively large value of the energy density at given T. Under conditions attainable in collisions of heavy nuclei, the parameter T, i.e., the "temperature," is of the order of 150–200 MeV at the phase boundary.

Finally let us discuss the values of the fugacities $\lambda^{\pm}$ in Eq. (4b). As quarks are brought into the reaction by the colliding nuclei, baryon number conservation permits us to relate the baryon density ν to the fugacities by integrating Eq. (4b) over all momenta:

$$\nu(T,\lambda^+,\lambda^-)=\tfrac{1}{3}\times 12\int\frac{d^3p}{(2\pi)^3}\left[(e^{|p|/T}\lambda^+ +1)^{-1}-(e^{|p|/T}\lambda^- +1)^{-1}\right]. \tag{6}$$

The factor $\frac{1}{3}$ takes into account the fractional baryon number of quarks. As we will show, the $gg\to q\bar{q}$ reaction time is much shorter than that for $q\bar{q}\to s\bar{s}$ production since the light quark masses are only of the order of 15 MeV. Consequently we may assume chemical equilibrium between q and $\bar{q}$: $\lambda^+=1/\lambda^- =\exp(-\mu_q/T)$. As long as gluons dominate the plasma state, i.e., for $T\gtrsim 200$ MeV, the precise conditions at the phase transition, such as abundance of q and $\bar{q}$, will not matter for the $s\bar{s}$ abundances at times comparable to the lifetime of the plasma. We will use the value $\mu_q=300$ MeV in order to estimate, when necessary, the baryon density at a given temperature. We can now return to the evaluation of the rate integrals, Eq. (3).

In the glue part of the rate A, Eq. (3), the k_1,k_2 integral can be carried out exactly by expanding the Bose function, Eq. (4a), in a power series in $\exp(-k/T)$:

$$A_g=\frac{8}{\pi^4}T\int_{4M^2}^{\infty}ds\, s^{3/2}\bar{\sigma}_{qq\to s\bar{s}}(s)\sum_{n,n'=1}(nn')^{-1/2}K_1\!\left(\frac{(nn's)^{1/2}}{T}\right). \tag{7}$$

In the quark contribution an expansion of the Fermi function is not possible and the integrals must be evaluated numerically. It is found that the gluon contribution, Eq. (7), dominates the rate A. For $T/M \gtrsim 1$ we find

$$A\approx A_g=\frac{7}{3\pi^2}\alpha_s^{\,2}MT^3e^{-2M/T}\left(1+\frac{51}{14}\frac{T}{M}+\dots\right). \tag{8}$$

The abundance of $s\bar{s}$ pairs cannot grow forever; at some point the $s\bar{s}$-annihilation reaction will deplete the strange-quark population. This loss term is proportional to the square of the density n_s of strange and antistrange quarks. With $n_s(\infty)$ being the saturation density at large times, the following differential equation determines n_s as a function of time[10]:

$$dn_s/dt\approx A\{1-[n_s(t)/n_s(\infty)]^2\}, \tag{9a}$$

$$n_s(t)=n_s(\infty)\tanh(t/\tau),\quad \tau=n_s(\infty)/A. \tag{9b}$$

$n_s(t)$ is a monotonically rising, saturating function, controlled by the characteristic time constant τ. In a thermally equilibrated plasma the asymptotic strangeness density, $n_s(\infty)$, is that of a relativistic Fermi gas [Eq. (6) with $\lambda=1$], provided the volume V is large.[11] We find that the relaxation time

$$\tau=\tau_g=\frac{9}{7}\left(\frac{\pi}{2}\right)^{1/2}\alpha_s^{\,-2}M^{1/2}T^{-3/2}e^{M/T}\left(1+\frac{99}{56}\frac{T}{M}+\dots\right)^{-1} \tag{10}$$

is falling rapidly with increasing temperature.

We now discuss the numerical results for the rates, time constants, and the expected strangeness abundance. In Fig. 2(a) we compare the rates for strangeness production by the processes depicted in Fig. 1 for the two different choices of parameters discussed below Eq. (1). The rate for $q\bar{q}\to s\bar{s}$ alone (shown separately) contributes less than 10% to the total rate. In Fig. 2(b) we show the corresponding characteristic relaxation times toward chemical equilibrium, τ, defined in Eq. (9). While our results for strangeness production by light quarks agree in order of magnitude with those of Biró and Zimányi[12] (considering their choice of parameters), it is here obvious that gluonic strangeness production, which was not discussed by these authors, is the dominant process. If we compare the time constant τ with the estimated lifetime of the plasma state we find that the strangeness abundance will be chemically saturated for temperatures of 160 MeV and above, i.e., for an energy density above 1 GeV/fm^3. We note that τ is quite sensitive to the choice of the

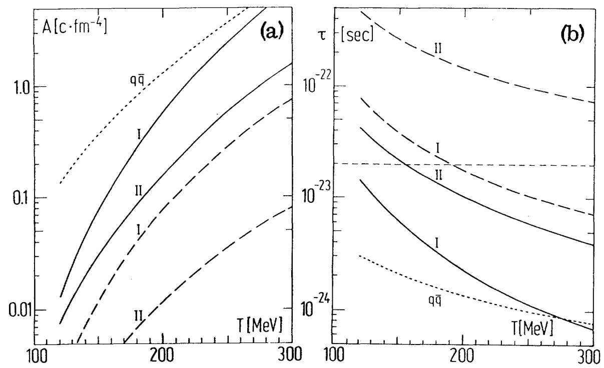

FIG. 2. (a) Rates A. (b) Time constants τ as functions of temperature T. Full lines, $q\bar{q} \to s\bar{s}$ *and* $gg \to ss$; dashed lines, $q\bar{q} \to s\bar{s}$; dotted lines, $gg \to q\bar{q}$ ($M = 15$ MeV). Curves marked I are for $\alpha_s = 2.2$ and $M = 280$ MeV; those marked II are for $\alpha_s = 0.6$ and $M = 150$ MeV.

strange-quark mass parameter and the coupling constant α_s which must, however, be chosen consistently. A measure of the uncertainty associated with the choice of parameters is illustrated by the difference between our results for the two parameter sets taken here.

Also included in Figs. 2(a) and 2(b) are our results for gluon conversion into light quark-antiquark pairs. The shortness of τ for this process indicates that gluons and light quarks reach chemical equilibrium during the beginning stage of the plasma state, even if the quark/antiquark (i.e., baryon/meson) ratio was quite different in the prior hadronic compression phase.

The evolution of the density of strange quarks, Eq. (9), relative to the baryon-number content of the plasma state, is shown in Fig. 3 for various temperatures. The saturation of the abundance is clearly visible for $T \geq 160$ MeV. To obtain the measurable[13] abundance of strange quarks, the corresponding values reached after the typical lifetime of the plasma state, 2×10^{-23} sec, can be read off in Fig. 3 as a function of temperature. The strangeness abundance shows a pronounced threshold behavior at $T \sim 120$–160 MeV.

We thus conclude that strangeness abundance saturates in sufficiently excited quark-gluon plasma ($T > 160$ MeV, $E > 1$ GeV/fm^3), allowing us to utilize enhanced abundances of rare, strange hadrons ($\bar{\Lambda}$, $\bar{\Omega}$, etc.) as indicators for the formation of the plasma state in nuclear collisions.

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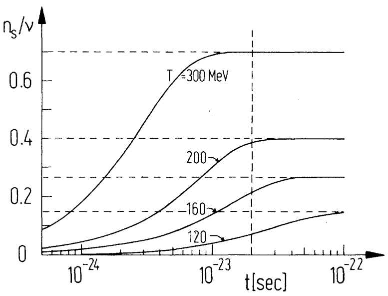

FIG. 3. Time evolution of the relative strange-quark to baryon-number abundance in the plasma for various temperatures ($M = 150$ MeV, $\alpha_s = 0.6$).

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