Measuring Three-Gluon Coupling in Semi-Inclusive Neutrino and Antineutrino Scattering

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Asymmetry in the hadron momentum distribution with respect to the lepton scattering plane for deep-inelastic neutrino and antineutrino scattering is calculated in the one-loop order in quantum chromodynamics. Observation of an asymmetry of a few percent with the expected sign should provide clean evidence for the existence of the three-gluon coupling.

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The qualitative behavior of strong interactions at short distance has so far been successfully explained in quantum-chromodynamics (QCD) perturbation theory. Observation of three-jet structure in e^+e^- annihilation[1] and of rising transverse momentum spread of hadron jets in leptoproduction[2] strongly suggests the existence of a small effective quark-gluon coupling operating at short distance. These experiments (as well as classical structure function analyses), however, are not sensitive to the non-Abelian gluon self-coupling, whose existence is supposed to be essential for asymptotic freedom and color confinement.

Among various proposed tests to measure the gluon self-coupling, tests based on observation of T-odd quantities in hard processes[3,4] have at least two clear advantages: (1) They are insensitive to the poorly known gluon distribution in a nucleon and its fragmentation functions. (2) T-odd quantities are found to be quite sensitive to the gluon self-coupling, where vanishing of its magnitude causes vanishing of such quantities in one case[3] and makes their sign change in the other case.[4] In spite of these advantages, observation of T-odd effects in e^+e^- annihilation processes requires either longitudinally polarized beams or weak-electromagnetic interference. This makes the tests proposed so far very difficult.

In this Letter, we propose a new test to measure the three-gluon coupling through T-odd effects in semi-inclusive leptoproduction. We need longitudinally polarized lepton beams, and neutrinos and antineutrinos provide an ideal source of such beams. The T-odd quantity to be observed is just an asymmetry in the hadron momentum distribution with respect to the lepton scattering plane. This plane can be precisely determined even in (anti)neutrino-induced reactions.

We find that QCD perturbation theory predicts an asymmetry of the order of a few percent whose sign is the same for both neutrino and antineutrino reactions, while the Abelian gluon model with a comparable fixed point coupling predicts an asymmetry which changes its sign between these two reactions. Observation of such asymmetry with the expected sign should be a clear evidence for the existence of a three-gluon coupling whose magnitude is comparable to the "observed" quark-gluon coupling.

Consider the semi-inclusive process

$$\nu(k)[\bar{\nu}(k)] + N(P) \to \mu(k')[\bar{\mu}(k')] + h(P') + \text{anything}, \tag{1}$$

where N is a target nucleon and h an observed hadron. Integrating the azimuthal angle of the final lepton and the transverse momentum of the final hadron, we can write the differential cross section in terms of five variables,

$$Q^2 = -q^2, \quad x = Q^2/2q\cdot P, \quad y = q\cdot P/k\cdot P, \quad z = P'\cdot P/q\cdot P, \quad \varphi,$$

where $q = k - k'$ and φ is the azimuthal angle of $\vec{P}_T'$ relative to $\vec{k}_{T'}$ the transverse momenta perpendicular to $\vec{q}$ (Fig. 1).

The differential cross section reads

$$\frac{d\sigma(\nu/\bar{\nu})}{dx\,dy\,dz\,d\varphi} = \frac{G_F{}^2 Q^2}{4\pi^2 y}[A + B\cos\varphi + C\cos 2\varphi + D\sin\varphi + E\sin 2\varphi], \tag{2}$$

where G_F is the Fermi constant and the coefficients A to E can be expressed in terms of nine invariant functions, $F_i(x,z,Q^2)$,

$$A=-y^2F_1+[1+(1-y)^2]F_2\pm[1-(1-y)^2]F_3,\quad B=(1-y)^{1/2}[(2-y)F_4\pm yF_5],$$
$$C=(1-y)F_6,\quad D=(1-y)^{1/2}[\pm yF_7+(2-y)F_8],\quad E=(1-y)F_9. \tag{3}$$

Here we take the upper sign for neutrino scattering and the lower sign for antineutrino scattering.

It has been shown through factorization of mass singularities[5] that the semi-inclusive process can be expressed as a convolution of the effective parton density in a nucleon, $D_{j/N}(\xi,Q^2)$, the hard scattering part of the parton cross section, $\hat{F}_i^{k/j}$, and the effective fragmentation function, $D_{h/k}(\eta,Q^2)$, in each order of perturbation theory;

$$F_i(x,z,Q^2)=\sum_{jk}\int_x^1\frac{d\hat{x}}{\hat{x}}\int_z^1\frac{d\hat{z}}{\hat{z}}D_{h/k}\left(\frac{z}{\hat{z}},Q^2\right)\hat{F}_i^{k/j}(\hat{x},\hat{z},Q^2)D_{j/N}\left(\frac{x}{\hat{x}},Q^2\right). \tag{4}$$

The sum over j and k runs over all partons (quarks, antiquarks, and gluons). The parton scaling variables are defined by $\hat{x}=Q^2/2q\cdot p_j$ and $\hat{z}=p_k\cdot p_j/q\cdot p_j$, where p_i denotes the four-momentum of a parton i.

In zeroth order in $\alpha_s/\pi=g^2/4\pi^2$, the expansion parameter in QCD, the parton cross section reads

$$\hat{F}_2^{q/q}=\hat{F}_2^{\bar{q}/\bar{q}}=\hat{F}_3^{q/q}=-\hat{F}_3^{\bar{q}/\bar{q}}=\delta(1-\hat{x})\delta(1-\hat{z}), \tag{5}$$

and all other functions vanish. In the first order in α_s/π there are contributions to A, B, and C,[6] while D and E remain zero. This is because the coefficients D and E are proportional to the T-odd quantity $\langle\vec{k}\times\vec{k}'\cdot\vec{P}'\rangle$: T-odd effects vanish in the tree approximation if the interaction preserves time-reversal invariance.[3,7] They are proportional to the absorptive part of the parton scattering amplitudes. The Feynman diagrams which contribute to the coefficients D and E in the one-loop order in QCD are shown in Fig. 2.

We find in this order

$$\hat{F}_i^{k/j}=\pi(\alpha_s/\pi)^2f_i^{k/j}(\hat{x},\hat{z})\quad(i=7,8,9), \tag{6}$$

where

$$f_7^{q/q}=\frac{1}{2}\left[\frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})}\right]^{1/2}\{-[(1-\hat{x})(1-\hat{z})+2\hat{z}(1-\hat{z})L(1-\hat{z})]C_F^2$$
$$-[\hat{x}\hat{z}-(1-\hat{x})(1-\hat{z})-\hat{z}(1-\hat{z})L(1-\hat{z})]C_FC_A\},$$
$$f_8^{q/q}=\frac{1}{2}\left[\frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})}\right]^{1/2}\{[(1-\hat{x})(1-\hat{z})+2(1-2\hat{x})\hat{z}(1-\hat{z})L(1-\hat{z})]C_F^2 \tag{7}$$
$$-[\hat{x}\hat{z}+(1-\hat{x})(1-\hat{z})+(1-2\hat{x})\hat{z}(1-\hat{z})L(1-\hat{z})]C_FC_A\},$$
$$f_9^{q/q}=\tfrac{1}{2}\hat{x}\hat{z}\{-[1-2(1-2\hat{z})L(1-\hat{z})]C_F^2+[2-(1-2\hat{z})L(1-\hat{z})]C_FC_A\},$$

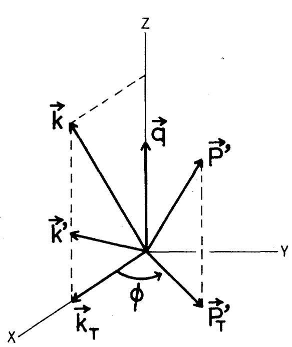

FIG. 1. Three-momenta for the process (1) in the target rest frame.

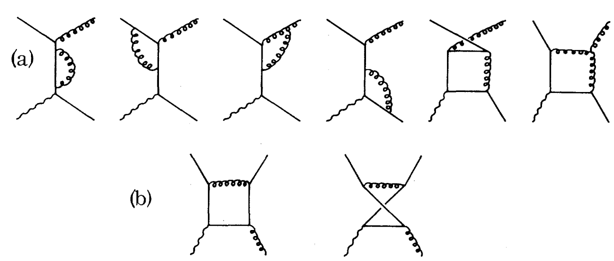

FIG. 2. Diagrams contributing to the absorptive part of the parton scattering amplitude, (a) current + quark → quark + gluon and (b) current + gluon → quark + antiquark, in the one-loop order. Wavy lines denote currents and curly lines are gluons.

and $f_7^{\bar{q}/\bar{q}} = f_7^{q/q}$, $f_i^{\bar{q}/\bar{q}} = -f_i^{q/q}$ ($i = 8, 9$);

$$f_7^{q/G} = \left[\frac{\hat{x}(1-\hat{x})}{\hat{z}(1-\hat{z})}\right]^{1/2} \{(1-\hat{x})[\hat{z}L(\hat{z}) - (1-\hat{z})L(1-\hat{z})] + \hat{x}(1-2\hat{z})\} T_F(C_F - \tfrac{1}{2}C_A),$$

$$f_8^{q/G} = \left[\frac{\hat{x}(1-\hat{x})}{\hat{z}(1-\hat{z})}\right]^{1/2} \{(1-\hat{x})[\hat{z}L(\hat{z}) + (1-\hat{z})L(1-\hat{z})] - \hat{x}\} T_F(C_F - \tfrac{1}{2}C_A), \tag{8}$$

$$f_9^{q/G} = \hat{x}(1-\hat{x})[L(\hat{z}) - L(1-\hat{z})] T_F(C_F - \tfrac{1}{2}C_A).$$

Here $C_F = \frac{4}{3}$, $C_A = 3$, and $T_F = \frac{1}{2}$ in QCD and $L(t) = [1 + \ln(1-t)/t]/t$. The functions $f_i^{G/q}$, $f_i^{G/\bar{q}}$, and $f_i^{\bar{q}/G}$ are obtained from $f_i^{q/q}$, $f_i^{\bar{q}/\bar{q}}$, and $f_i^{q/G}$, respectively, by the exchange $\hat{z} \leftrightarrow 1-\hat{z}$ and $\varphi \leftrightarrow \varphi + \pi$. In these formulas we neglect jet smearing effects and set the azimuthal angle φ of the hadron equal to that of its parent parton.

Substituting these parton cross sections (5)–(8) and an appropriate set of parton distribution and decay functions in Eqs. (2)–(4), we obtain the leading order predictions for the average quantities $\langle \sin\varphi \rangle$ and $\langle \sin 2\varphi \rangle$. In the following we neglect small scaling-violation effects and fix the expansion parameter at $\alpha_s/\pi = 0.1$. We use the quark distributions parametrized by Barger and Phillips[8] and the quark decay functions by Sehgal.[9] The gluon distribution in a nucleon is assumed to be

$$D_{G/N}(\xi) = 3(1-\xi)^5/\xi$$

and for the gluon decay function we use

$$D_{h/G}(\eta) = \tfrac{1}{2}[D_{h/q}(\eta) + D_{h/\bar{q}}(\eta)].$$

Our results are very insensitive to these unknown functions.

Shown in Figs. 3(a) and 3(b) are the predictions for $\langle \sin\varphi \rangle$ and $\langle \sin 2\varphi \rangle$ in neutrino and antineutrino scattering off an isoscalar target at $E_{\text{beam}} = 100$ GeV. Final states with both Q^2 and $(q+P)^2$ greater than 5 GeV^2 have been included. Solid lines denote predictions of QCD while dashed lines denote those of the Abelian gluon model ($C_F = T_F = 1$ and $C_A = 0$) with $\alpha_s/\pi = 0.1$ and the same parametrizations for parton distributions and decay functions.

Predictions for $\langle \sin\varphi \rangle$ are at the level of a few percent while those for $\langle \sin 2\varphi \rangle$ are one order of magnitude smaller. The most important feature of our results is that in QCD $\langle \sin\varphi \rangle$ should have the same sign for neutrino and antineutrino reactions while the Abelian gluon model predicts alteration of its sign between these two reactions. This happens because the dominant contribution to $\langle \sin\varphi \rangle$ comes from the quark scattering process [Fig. 2(a)] in QCD while it comes from the gluon annihilation process [Fig. 2(b)] in the Abelian gluon model. The most precisely measurable T-odd quantity in (anti)neutrino scattering should be

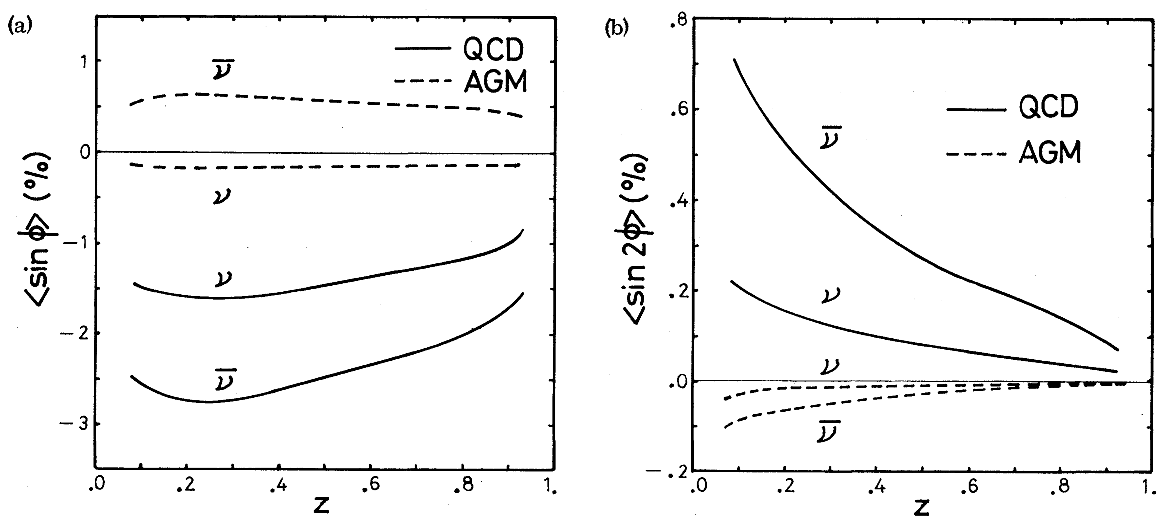

FIG. 3. Predictions for (a) $\langle \sin\varphi \rangle$ and (b) $\langle \sin 2\varphi \rangle$ in semi-inclusive neutrino and antineutrino scattering off an isoscalar target at $E_{\text{beam}} = 100$ GeV. Solid lines denote predictions of QCD and dashed lines are those of the Abelian gluon model.

the left ($0<\varphi<\pi$)-right ($\pi<\varphi<2\pi$) asymmetry in the hadron momentum distribution with respect to the lepton scattering plane,

$$A(z)=\frac{(dn/dz)(\text{left})-(dn/dz)(\text{right})}{(dn/dz)(\text{left})+(dn/dz)(\text{right})},$$

which is simply $(4/\pi)\langle\sin\varphi\rangle$. Observation of an asymmetry of a few percent with the expected sign should be clean evidence for QCD. Although small and difficult to measure, determination of the sign of $\langle\sin 2\varphi\rangle$ could provide further evidence.

The left-right asymmetry for longitudinally polarized electron scattering can be obtained from our results in a straightforward manner. Details will appear elsewhere.

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Fractional Quantum Numbers on Solitons

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A method is proposed to calculate quantum numbers on solitons in quantum field theory. The method is checked on previously known examples and, in a special model, by other methods. It is found, for example, that the fermion number on kinks in one dimension or on magnetic monopoles in three dimensions is, in general, a transcendental function of the coupling constant of the theories.

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Peculiar quantum numbers have been found to be associated with solitons in several contexts: (i) The soliton provides, of course, a different background than the usual vacuum around which to quantize other fields. The difference between these "vacuum polarizations" may induce unusual quantum numbers localized on the soliton.[1-3] (ii) Solitons may require unusual boundary conditions on the fields interacting with them, in particular leading to conversion of internal quantum numbers into rotational quantum numbers.[4-6] (iii) In the case of dyons, there is classically a family of solitons with arbitrary electric charge. The determination of which of these are in the physical spectrum requires quantum-mechanical considerations and brings in the θ parameter of non-Abelian gauge theories.[7,8]

At present all these phenomena seem distinct although there are suggestive relationships. In this note, we shall concentrate on (i), proposing a general method of analysis and working out a few examples.

