Thermal Instability and Magnetic Field Generated by Large Heat Flow in a Plasma, Especially under Laser-Fusion Conditions

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Representing a large heat flow in a plasma as a flow of hot relatively collisionless electrons balanced by an opposing current of cold collisional electrons, it can be shown that an electrothermal instability can be induced in the cold resistive plasma when the heat flow exceeds about 3% of the free-streaming limit. This is nonconvective if the temperature of the cold plasma is less than $0.2n_e^{1/4}Z^{1/2}$°K, with n_e in m^{-3}. The instability will lead to filamentary magentic structures with a typical growth time of a few picoseconds and wavelength about 10 μm.

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In this Letter a new mechanism for an unstability is proposed in laser-produced plasmas. It can generate a filamentary density and temperature structure with associated magnetic fields in the plasma between the critical and ablation surfaces in which a nonlinear heat flux occurs, and could be dangerous for any inertial confinement scheme.

A large heat flow can be approximately represented by an inward flow of relatively collisionless hot electrons balanced by a return current of cold electrons driven by an electric field. This is because the mean free path of electrons with a velocity v is proportional to v^4/Zn_0, n_0 being the number density of electrons and Z the ionic charge number. Representing the hot and cold equilibrium currents by $\vec{J}_{h0}$ and $\vec{J}_{e0}$, respectively, the heat flux q can be written as

$$q=\tfrac{5}{2}(k_B/e)\vec{J}_{e0}(T_{h0}-T_{e0}), \tag{1}$$

where T_{h0} and T_{e0} ($<T_{h0}$) are the temperatures of the hot and cold electrons, respectively, and k_B is Boltzmann's constant. The electric field necessary to drive the resistive cold return current is given by Ohm's law, $\vec{J}_{e0}=\sigma_0\vec{E}_0$ whilst quasineutrality ensures that $\vec{J}_{h0}+\vec{J}_{e0}=0$.

If the cold-electron temperature is spatially perturbed with a wave vector $\vec{k}$ orthogonal to the heat flow, the $T_e^{3/2}$ dependence of the electrical conductivity σ will cause the cold-electron current to be increased where the cold-electron temperature is higher and vice versa. In turn, the Ohmic dissipation by the cold current will be increased in the temperature peaks, so tending to increase the electron temperature. This is the basis of the electrothermal instability discussed earlier by the author[1] in another context. Locally the net current will no longer be zero, and a spatially oscillating magnetic field will be set up. Through Faraday's law, electric field changes will occur so as to reduce the current perturbations, but because the hot and cold electrons obey different equations of motion a local net current will persist, and the main effect of Faraday's law will be found to reduce the growth rate at longer wavelength. Short-wavelength modes will be damped out by the term in electron thermal conductivity κ.

A perturbation analysis requires a steady state, and for an instability based on a dissipative energy process some artificiality in the form of the energy sink is required. Here, the Ohmic dissipation of the return cold current is assumed to be balanced by equipartition to the ions (which are kept at a fixed temperature) and by bremsstrahlung radiation loss, as follows (with coefficient $\beta_b=1.42\times10^{-40}$ mks):

$$J_{e0}^2=3n_0^2e^2k_B(T_{e0}-T_i)/m_i+\sigma_0\beta_b n_0^2T_{e0}^{1/2}. \tag{2}$$

In reality the ions and electrons advect the energy in the ablation process. However it can be shown *a posteriori* that replacing the equipartion term by $v\cdot\nabla\frac{3}{2}n_ek_B(T_e+T_i/Z)$ for sonic flow yields a scale length of the same order as the optimum wavelength of the instability, making the model reasonably plausible. Taking the equilibrium heat flow to be in the z direction, perturbed quantities are assumed to vary as $\exp(\alpha t+ikx)$. As in Ref. 1 it is found that for the fastest growing mode α/k is of the order of the sound speed $(k_BT_0/m_i)^{1/2}$. Therefore ion motion is also included. The perturbed electron-energy equation is

$$\tfrac{3}{2}\sigma_0\alpha k_B(n_0T_{e1}+n_1T_{e0})+\tfrac{5}{2}\sigma_0n_0k_BT_{e0}ikv_{x1}+\sigma_0\kappa_0k^2T_{e1}$$
$$=2J_{e0}J_{e1}-3n_0^2e^2k_BT_{e1}/m_i-6n_0n_1e^2k_B(T_{e0}-T_i)/m_i-2\sigma_0\beta_bn_0(n_1T_{e0}+n_0T_{e1})T_{e0}^{-1/2}. \tag{3}$$

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The cold-plasma equations of motion and continuity are

$$n_0 m_i \alpha v_{x1}/Z = -ikn_0 k_B T_{e1} - ikn_1 k_B(T_{e0} + T_i/Z) - J_{e0}B_{y1}, \tag{4}$$

$$\alpha n_1 + n_1 ikv_{x1} = 0. \tag{5}$$

The perturbed Faraday's law and Ohm's law are $ikE_{z1} = B_{y1}$ and $J_{e1} = \sigma_1 E_{z0} + \sigma_0 E_{z1}$ where the perturbed conductivity σ_1 is given by $\frac{3}{2}T_{e1}\sigma_0/T_{e0}$.

The use of the full $J_{e0}B_{y1}$ force on the cold-plasma ions is a good approximation provided that the number density of hot electrons n_h is much smaller than that of cold n_e. Then the ion density n_i is close to n_e/Z and the forces $Zen_i E_{x1}$, $-en_e E_{x1}$, $J_{e0}B_{y1}$, and $J_{h0}B_{y1}$ are of the same order, whilst $-en_h E_{x1}$ is much smaller. It follows that the equations of motion and continuity for the hot electrons are

$$\alpha n_{h0} m_e v_{hx1} = J_{h0}B_{y1} - k_B T_{h0} ikn_{h1}, \tag{6}$$

$$\alpha m_e v_{hz1} = -eE_{z1}, \tag{7}$$

$$\alpha n_{h1} + ikn_{h0} v_{hx1} = 0. \tag{8}$$

The hot electrons couple to the cold plasma through Ampere's law

$$ikB_{y1} = \mu_0(J_{e1} + J_{h1}) \tag{9}$$

with $J_{h1} = -n_{h1}ev_{hz0} - n_{h0}ev_{hz1}$ and $J_{h0} = -n_{h0}ev_{hz0}$.

In Eq. (6) the hot-electron temperature is assumed to be unperturbed because of the large hot-electron thermal conductivity. Combining it with Eq. (8) gives

$$n_{h1} = iJ_{h0}B_{y1}/(\alpha^2 m_e/k + k_B T_{h0} k). \tag{10}$$

The first term in the denominator can be ignored since for the fastest growing mode it is of order $(m_e/m_i)(T_{e0}/T_{h0})$ times the other term. Without further approximation the equations can be reduced to an algebraic dispersion equation, quartic in α and cubic in k^2:

$$\left[(x+y+1+2f)(x+ay+h) - 3\left(\frac{t-1}{t}+f\right)(ay+h)\right]\left[\frac{x^2}{Z} + dy\,\frac{(Zt+1)}{Zt}\right] + \left[\frac{2}{3}x - 2\left(\frac{t-1}{t}\right) - 2f\right]\left[dy(x+ay+h) - \frac{9}{4}dy\left(\frac{t-1}{t}+f\right)\right] = 0, \tag{11}$$

where the dimensionless parameters are

$$x = \frac{\sigma_0 m_i \alpha}{2n_0 e^2}, \quad y = \frac{\sigma_0 \kappa_0 m_i k^2}{3n_0^2 k_B e^2}, \quad a = \frac{3n_0 k_B}{2\mu_0 \sigma_0 \kappa_0}, \quad t = \frac{T_{e0}}{T_i}, \quad d = \frac{3k_B^2 T_{e0}\sigma_0}{4\kappa_0 e^2} = 0.7054, \quad f = \frac{\sigma_0 \beta_b m_i}{3e^2 k_B T_{e0}^{1/2}},$$

and

$$h = \frac{ay}{k^2}\left[-\frac{\mu_0 J_{h0}^2}{n_{h0}k_B T_{h0}} + \frac{n_{h0}e^2\mu_0}{m_e}\right] = -\frac{3}{2}\frac{n_0}{n_{h0}}\frac{T_{e0}-T_i}{T_{h0}} - \frac{3f}{2}\frac{n_0}{n_{h0}}\frac{T_{e0}}{T_{h0}} + \frac{1}{2}\frac{n_{h0}}{n_0}\frac{m_i}{m_e}. \tag{12}$$

The influence of the hot collisionless electrons is through the parameter h which, if $J_{h0} = n_h e(k_B T_{h0}/m_e)^{1/2}$ is satisfied, becomes zero. Dispersion curves for $f = 0$, $h = 0$, and $Z = 1$ are to be found in Ref. 1. Figure 1 illustrates the positive x root (demonstrating unstable growth) versus y ($\propto k^2$) for $a = 5.375$, $d = 0.7054$, $t = 3$, $Z = 10$, and for various values of h. The left section of roots for $h = 0$ and -1 are complex, the real part only being drawn. The important property to note is that the fastest growing mode is real and has x_{max} and $y(x_{max})$ of order 1. It is likely that the nonconvective instability is the most dangerous. This branch is generally real when it crosses the y axis, and the conditions for its onset can be found by putting $x = 0$ in Eq. (11) and examining the resulting quadratic in y for the necessary conditions that y is real and positive. The particular case of $h = 0$ simply leads to conditions similar to those found in Ref. 1, namely

$$t \geq \frac{(5Z-2-f) + [(5Z-2-f)^2 + 12Z(3f+4)]^{1/2}}{2Z(3f+4)} \to 1.32(f=0,\ Z=1) \text{ and } 1.25(f=0,\ Z\to\infty) \tag{13}$$

and

$$\left(\frac{a}{18}\right)^{1/2} \geq \frac{[Zt(Zt+1)]^{1/2}(ft+t-1)}{Zt^2(4+3f) - t(5Z-2-f) - 3} \to \frac{f+1}{3f+4} \text{ as } t\to\infty. \tag{14}$$

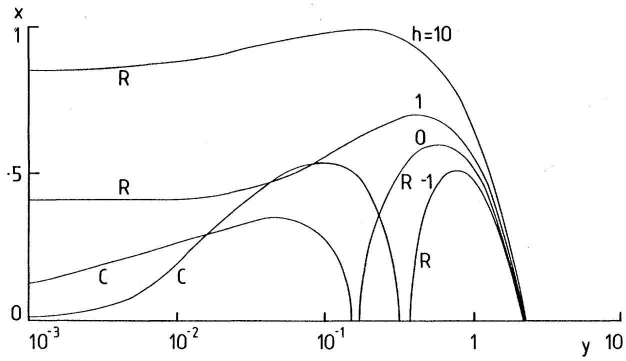

FIG. 1. The unstable roots of the dispersion equation are shown in a plot of positive real x vs y for $Z = 10$, $a = 5.375$, $d = 0.7054$, $f = 0$, $t = 3$, and for $h = 10$, 1, 0, and -1. C signifies that the root is complex, R that it is real.

Recalling the definition of a, the condition (14) is, for $f \ll 1$, $t \to \infty$,

$$T_{e0} \leq 0.2 n_0^{1/4} Z^{1/2}, \qquad (15)$$

where T_e is in degrees Kelvin and n_0 in inverse meters cubed. Returning to Eqs. (1) and (2) the condition (13) on t can be written as a condition on the heat flux, i.e., for $f \ll 1$, it is unstable if

$$q > q_{\text{crit}}$$

$$= n_0 m_e \left(\frac{kT_{e0}}{m_e}\right)^{3/2} 2.13 \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_{h0}}{T_{e0}} - 1\right). \qquad (16)$$

Therefore it can be seen, for example, that for solid density ($n_0 \simeq 10^{29}$ m^{-3}) with $Z = 10$ the plasma is unstable to the thermal instability if $T_{e0} < 1.1 \times 10^7$ °K and, for $T_{h0} = 4T_{e0}$, if the heat flow exceeds 3% of the free-streaming limit. ($T_{h0} = 4T_{e0}$ is taken to represent the fact that heat flow is dominantly carried by electrons with 2 to 3 times the thermal velocity.) This instability might be the cause of the well-known flux limiter in laser-plasma experiments which is also about 3% of the free-streaming limit.

Under the assumption that if these conditions are satisfied the growth rate of the fastest growing mode corresponds to $x \simeq 1$ and $y \simeq 1$, the order of magnitude can be found from Eq. (12) where the growth rate α is

$$\alpha = 2.8 \times 10^{-8} \frac{n_0 Z x}{A T_{e0}^{3/2}} \simeq \frac{2 m_e}{m_i} \nu_{ei} \qquad (17)$$

and the wavelength λ is, in terms of the mean free path λ_{mfp}

$$\lambda = \frac{2\pi}{k} = 2.41 \times 10^{10} \frac{T_e^2 A^{1/2}}{n_0 Z y^{1/2}}$$

$$\simeq 3.8 \left(\frac{m_i}{m_e}\right)^{1/2} \lambda_{\text{mfp}}. \qquad (18)$$

Taking the same example of $n_0 = 10^{29}$ m^{-3}, $T_{e0} = 10^7$ °K, $Z = 10$, and $A = 20$, the growth rate is 4.4×10^{10} s^{-1} with a wavelength of 11 μm. The scaling with Z is interesting in that higher Z gives faster growth and shorter wavelengths, as does lower T_{e0}. It would be less model dependent to take the growth rate as the Ohmic heating rate by the cold return current rather than the equipartition rate. The term f enhances the instability because the radiation loss, being proportional to n^2, is greater in the denser colder plasma, and in principle can be the dominant cause of the thermal instability.

The more general case of $h \neq 0$ is more complex for the marginal stability condition, $x = 0$. The condition for y to be real at $x = 0$ is

$$\left[\frac{h}{a} - 1 + f + \frac{3(t-1)}{t} + \frac{2Zt}{Zt+1}\left(\frac{t-1}{t} + f\right)\right]^2$$

$$\geq \frac{18}{a}\left(\frac{t-1}{t} + f\right)^2. \qquad (19)$$

This condition is illustrated in Fig. 2 in the h-a plane for $t \to \infty$ and for $t = 1.4$, the regions enclosed by the parabolas being forbidden regions of complex y. Outside a parabola defined by t, Z, and f the two roots y_1 and y_2 are real, but when condition (13) is satisfied the area, bounded by the curve and the lines $a = 0$ and $h = h_{\max}$ (of the curve), contains two negative real roots and is therefore also forbidden. In the remaining regions of h-a parameter space at least one real positive root x can be found for some real positive y.

In the present work the source of magnetic field is not the usual $\nabla n \times \nabla T$ term[2-5] but arises from the modeling of a large heat flow by counterstreaming collisionless hot electrons and collisional cold electrons driven by an electric field. If the electrons were to be modeled as a single fluid, the magnetic field would originate through the perturbed electron stress-tensor component P_{xz}. It is therefore not unlike the Weibel[6] instability, which is, however, collisionless. An adaptation of the Weibel instability to counterstreaming of collisional hot and cold electrons[7] leads to very long wavelengths and small growth rates. In the present theory a hybrid model is

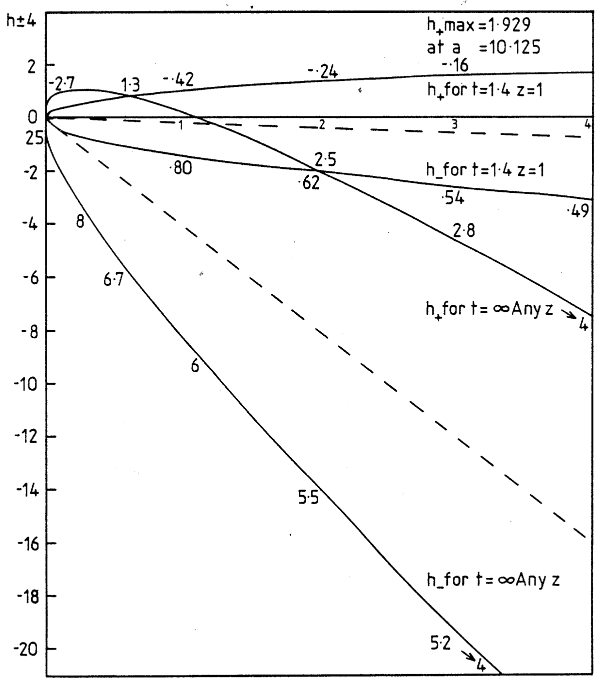

FIG. 2. Condition (19) is shown in h-a space for $t=\infty$, any Z, and for $t=1.4$, $Z=1$, with $f=0$. Values of y are shown as numbers on the h_- and h_+ curves illustrating the region of negative (and hence forbidden) y.

employed.

The saturation of the instability could occur through convection of the ablating plasma or when the magnetic pressure equals the plasma pressure. If the filamentary current structures are frozen into the ablating plasma, then, with use of suitable arguments for the ablation velocity and scale length, any perturbation will grow by at least $e^3 \simeq 20$. If instead the magnetic field grows until its pressure equals the plasma pressure, a density of 10^{22} cm^{-3} at temperature 10^6 °K will give 5.9 MG. Shell breakup might occur, and in the maximum magnetic-field regions the Hall parameter $\omega\tau$ could be much larger than one, resulting in a much reduced transverse thermal conduction. The shell breakup or even density fluctuations will have a deleterious effect on the attainment of a symmetric high compression.

In the experimental work of Willi and Rumsby[8] large magnetic fields have been indicated by Faraday rotation measurements associated with fine filamentary structure in the ablating plasma. Whilst the origin of these filaments could be due to laser self-focusing, many of the filaments or jets do not follow the laser-ray path and indeed in plane targets can occur outside the focal-spot area. This would suggest that the origin of the filaments might also be due to heat flow, especially as variation of Z of the target leads to a variation in wavelength similar to that of Eq. (18). The spatial scale length of the laser hot spots is about 50 μm in contrast to the scale length of the jets which is up to 10 μm for high Z. Grek *et al.*[9] reported the occurrence of filamentary structures or jets originating in the overdense plasma at up to sixteen times the critical density. In this region heat-flow–driven instabilities could be the origin of the filamentary structure.

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