First-Order Phase Transition in Four-Dimensional SO(3) Lattice Gauge Theory

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We present Monte Carlo evidence of a first-order phase transition in four-dimensional SO(3) lattice gauge theory. This result stands in sharp contrast to the known single-phase structure of the corresponding SU(2) theory, and suggests that the Z_2 center degrees of freedom may be important to the thermodynamics of the SU(2) gauge system.

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One of the major advantages of the lattice regularization of continuum gauge theory is that confinement can be understood as a property of the strong-coupling (or "high-temperature"), disordered phase of the gauge system. However, according to asymptotic freedom, $g^2 \to 0$ in the continuum limit; so the problem is to show that the "high-temperature" property of confinement persists in the "low-temperature" region down to $g^2=0$. The extrapolation of strong-coupling properties into the weak-coupling regime is possible if there is no phase transition separating the strong- and weak-coupling regimes, and this is the motivation for studying the thermodynamics of lattice gauge systems. The numerical results of Creutz[1] and Lautrup and Nauenberg[2] are strong evidence of the absence of any phase transition in four-dimensional SU(2) lattice theory.

It is natural to think that the absence of phase transitions in the SU(2) theory can be generalized to lattice theories with any continuous non-Abelian gauge group. However, the essential mechanism which frustrates a phase transition between the strong- and weak-coupling regimes in SU(2) is not really understood at present, while phase transitions have in fact been observed in certain other four-dimensional lattice gauge systems. Mean-field theory, for example, predicts the occurrence of first-order transitions in gauge systems,[3] and these are known to occur for four-dimensional lattice theories with some finite gauge groups.[4] Compact U(1) gauge theory seems to have a second-order transition in four dimensions.[5] Even in the SU(2) case, the specific heat has a sharp peak in the crossover region[2]; so it seems that there is "almost" a phase transition in the crossover region. One would like to somehow isolate the degrees of freedom in the SU(2) theory which prevent this transition from actually occuring. Now it is widely believed that topological configurations associated with the gauge-group center are responsible for confinement in the SU(2) weak-coupling regime[6]; so it might also be true that the Z_2 center degrees of freedom are crucial to the SU(2) thermodynamics.[7] It is this conjecture which has motivated us to study, by Monte Carlo techniques, the thermodynamics of a theory with a trivial group center, namely, four-dimensional SO(3) lattice gauge theory.

SO(3) lattice theory can be expressed in terms of integrals over the SU(2) group variables, however, the action is expressed as a trace in the adjoint, rather than the fundamental, representation of SU(2):

$$\begin{aligned} Z &= \int \prod_l dU_l\, e^{\beta \sum_p S_p}, \\ S_p &= \chi_A(g_p)/\chi_A(1) \\ &= \tfrac{1}{3}[\chi_F{}^2(g_p) - 1] \\ &= \tfrac{1}{3}[\mathrm{tr}(UUU^\dagger U^\dagger)^2 - 1], \end{aligned} \tag{1}$$

where the link variables U_l are SU(2) matrices, and the χ_A, χ_F are traces over the SU(2) group variables in the adjoint and fundamental representations, respectively. It should be noted that in SO(3) lattice gauge theory, in fact in any lattice gauge theory with a trivial center, quark color always can be screened by glue, i.e., a quark can bind with gluons to form a color singlet. Hence the Wilson loop must always follow a perimeter law even at strong couplings, and the strong-coupling (or "disordered") phase of such theories corresponds to color screening, rather than quark confinement *per se*.[8] The same color screening effect would occur in an SU(N) gauge theory with quarks in the adjoint representation. But the more interesting distinction between SU(2) and SO(3), apart from their available quark representations, is the fact that SO(3) cannot distinguish between link variables U and $-U$; this means, for example, that a "thin fluxon" configuration in SU(2) is indistinguishable from vacuum in the SO(3) theory. Conceivably, this loss of Z_2 degrees of freedom in the SO(3) theory could affect the thermodynamics so severely that a phase transition between strong and weak

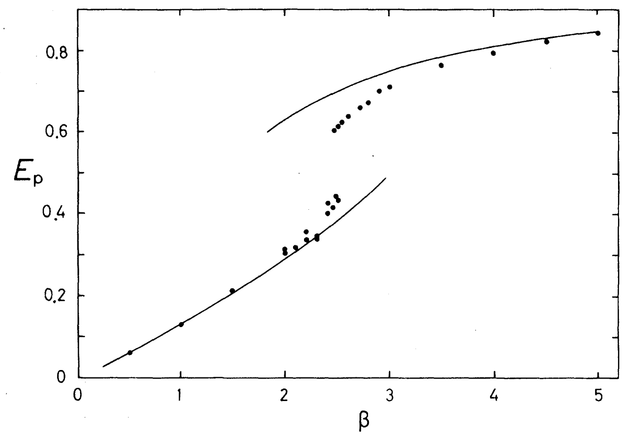

FIG. 1. The plaquette energy E_p as a function of β. The solid lower curve is a plot of the high-temperature expansion

$$E_p = \frac{1}{9}\beta + \frac{1}{54}\beta^2 - \frac{1}{1459}\beta^4 + \frac{199}{944784}\beta^5$$

and the upper solid curve is a plot of the low-temperature expansion $E_p = 1 - \frac{3}{4}\beta^{-1}$.

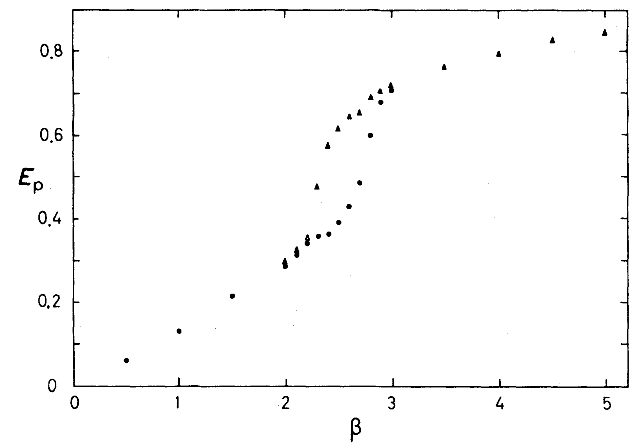

FIG. 2. SO(3) hysteresis curve in the plaquette energy E_p, taken from step-mode Monte Carlo runs with 100 iterations for each datum point. Points represented by triangles were taken from hot starts; circles represent cold starts.

couplings would occur. Using Monte Carlo methods on a 3^4 lattice, we have in fact found such a transition.

In Fig. 1 we show our results for the mean plaquette energy $E_p = \langle S_p \rangle$ as a function of $\beta = 3/2g^2$. This Monte Carlo data can be compared to the high- and low-temperature expansions for E_p, which are also plotted in Fig. 1. There is a very strong signal of a phase transition at $\beta = 2.48$, where E_p makes a sudden jump from $E_p = 0.43$ to $E_p = 0.60$ in a region of $\Delta\beta \simeq 0.01$. The only question is whether this is a first- or second-order transition. We conclude that it is a first-order transition because, to the limits of our computer data, there is a discontinuity between the upper and lower branches of the E_p curve. We were unable to resolve any intermediate points in the transition region. After 3000 iterations at the transition temperature $\beta = 2.48$, a Monte Carlo run with a cold start gives a data point on the upper branch of the curve, while a run at the same temperature with a hot start gives a data point on the lower branch. We have also run the Monte Carlo program in step mode with only 100 iterations per step and obtained the hysteresis curve of Fig. 2, which again shows a very pronounced two-phase structure. Practical constraints on computer time did not allow us to determine the normalized specific heat

$$\rho = \sigma^{-2}(\partial E_p/\partial\beta) = \sum_{p'} (\langle S_p S_{p'} \rangle - \langle S_p \rangle \langle S_{p'} \rangle)/(\langle S_p^2 \rangle - \langle S_p \rangle^2) \quad (2)$$

with much reliability; we can only report that in the data available to us, in 3^4 as well as 4^4 lattices, the specific heat does not show any clear sign of forming a sharp peak, which would be characteristic of a second-order transition, and so this data seems consistent with a first-order transition. We have also seen, in the neighborhood of the transition point at $\beta = 2.48$, that the lattice system will remain in the metastable phase for hundreds of iterations, and then suddenly jump to the stable phase during the Monte Carlo run, which again is evidence of a well-separated two-phase structure. It is interesting to note that the first-order transition in SO(3) and the peak in the specific heat occurs at roughly the same value of β ($\beta = 2.48$ and 2.2, respectively). On the other hand, as a function of g^2, the SO(3) transition and SU(2) peak well separated. In Fig. 3 we have plotted the SU(2) and SO(3) plaquette action densities as a function of g^2, and on this scale the SO(3) transition seems very precocious.

We conclude that since the SO(3) gauge theory in four dimensions has a phase transition while the SU(2) theory does not, the center degrees of freedom could well be responsible for the difference in the thermodynamics of these very

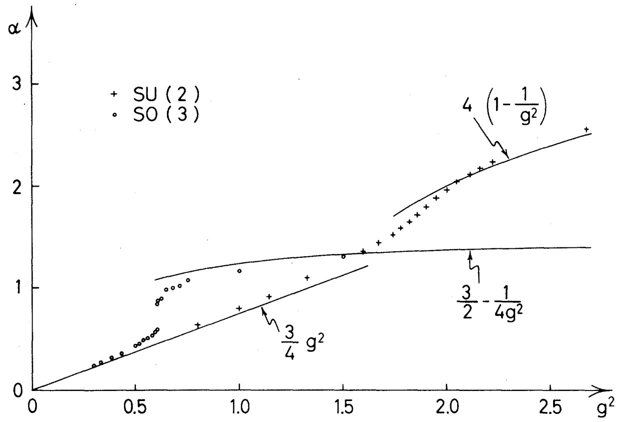

FIG. 3. The plaquette action densities plotted as a function of g^2. The action densities are $\alpha = 4 - 2\ \mathrm{tr}g$ for SU(2), and $\alpha = \frac{5}{2} - \frac{1}{2}\ \mathrm{tr}g$ for SU(3). In the weak-coupling limits the two groups must agree with each other for the case of the action density.

similar systems. It is also of interest that we now have an example of a continuous gauge group in four dimensions which, like some finite groups, undergoes a first-order phase transition as predicted by mean-field theory. In fact, a naive mean-field calculation by the Weiss self-consistent method gives a value $\beta_c = 2.59$ for the transition point, which is not far from our observed value $\beta_c = 2.48$. Further applications of mean-field methods to lattice gauge theories will be given in a subsequent paper.[9]

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