tempt to pump $|a\rangle$-state atoms to electron spin-flipped states which would then be ejected from the HSC by the field gradients, leaving a gas enriched in $|b\rangle$-state atoms. If T_1 is fast, however, this technique is also frustrated. Although this unique system, H↓, presents a number of exciting new phenomena and areas of research, it seems that one of the most fascinating, the weakly interacting Bose-Einstein condensed gas, will not be easily achieved.

We thank J. Berkhout for various useful contributions to the experiment, R. Sprik, A. Lagendijk, and V. V. Goldman for stimulating discussions, and O. H. Höpfner for mechanical constructions. The financial support of the foundation Fundamenteel Onderzoek der Materie is gratefully acknowledged.

[1]I. F. Silvera and J. T. M. Walraven, Phys. Rev. Lett. 44, 164 (1980).
[2]I. F. Silvera and J. T. M. Walraven, Phys. Rev. Lett. 45, 1268 (1980).
[3]M. Morrow, R. Jochemsen, A. J. Berlinsky, and W. N. Hardy, Phys. Rev. Lett. 46, 195 (1981).
[4]A. P. M. Matthey, J. T. M. Walraven, and I. F. Silvera, Phys. Rev. Lett. 46, 668 (1981).
[5]I. F. Silvera and V. V. Goldman, Phys. Rev. Lett. 45, 915 (1980); V. V. Goldman and I. F. Silvera, to be published.
[6]D. O. Edwards and I. B. Mantz, J. Phys. (Paris), Colloq. 41, C7-257 (1980).
[7]R. W. Cline, D. A. Smith, T. J. Greytak, and D. Kleppner, Phys. Rev. Lett. 45, 2117 (1980).
[8]J. W. Statt and A. J. Berlinsky, Phys. Rev. Lett. 45, 2105 (1980).
[9]E. D. Siggia and A. E. Ruckenstein, Phys. Rev. B 23, 3589 (1981).
[10]I. B. Mantz and D. O. Edwards, Phys. Rev. B 20, 4518 (1979); F. A. Guyer and M. D. Miller, Phys. Rev. Lett. 42, 1754 (1979).
[11]F. M. Ellis, R. B. Hallock, M. D. Miller, and R. A. Guyer, Phys. Rev. Lett. 46, 1461 (1981).
[12]This argument was suggested by D. O. Edwards.
[13]R. Jochemsen, M. Morrow, A. J. Berlinsky, and W. N. Hardy, to be published, and private communication. They measure $\epsilon_a/k \simeq 0.4$ K at low density in zero field by NMR.
[14]A. Lagendijk, to be published.
[15]Y. Kagan and G. Shlyapnikov, private communication.

Longitudinal Acoustic Impedance and the Squashing Mode in Superfluid ^{3}He-B

O. Avenel
Centre d'Etudes Nucléaires de Saclay, Service de Physique du Solide et de Résonance Magnétique, 91191 Gif-sur-Yvette, France

and

L. Piche, W. M. Saslow,[(a)] and E. Varoquaux
Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay, France

and

R. Combescot
Groupe de Physique des Solides, Ecole Normale Supérieure, 75231 Paris, France
(Received 16 March 1981)

We have observed an unexpectedly strong effect of the squashing mode in ^{3}He-B on the longitudinal acoustic impedance Z. Most peculiar is the behavior of the reactive part Z'' which displays both large positive and negative excursions. Our analysis illustrates the role of the boundary conditions on the superfluid order parameter and of the crossover with temperature of the squashing mode from a propagating wave with velocity c_{sq} (which is measured) to a nonpropagating surface wave (whose imaginary velocity accounts for the anomalies of Z'').

PACS numbers: 67.50.Fi

The squashing mode in superfluid ^{3}He-B is an eigenmode of the complex tensorial order parameter which couples strongly to density fluctuations.[1] In weak-coupling theory its eigenfrequency is proportional to the gap function: $\omega_{sq} = (\frac{12}{5})^{1/2}\Delta_{BCS}(T)$. This distinctive mode was identified first in the vicinity of the transition temperature T_c.[1] In more recent sound-propagation experiments at higher frequency and lower temperature,[2-4] only the wings of the resonance peak could be observed because of the high attenuation at resonance. This work is a *direct* study of the squashing mode through measurements of the longitudinal acoustic impedance Z of the superfluid. This type of experiment, which has been performed previously only on normal liquid ^{3}He,[5] essentially amounts to measuring complex phase velocities. The new feature described in this Letter is the influence of two coupled modes on the transfer of acoustic energy from the wall.

The experiment was performed with use of the same cell and in the same cryogenic apparatus as in Ref. 4. We electronically monitor the free ringing decay of an X-cut quartz crystal weakly excited by a short (4-μs) tone burst at its resonant frequency $\omega/2\pi$. The decay envelope can be represented by $\exp[-(\tau_R^{-1}+i\Delta\omega)t]$. The real and imaginary components of Z (ReZ and ImZ) are easily shown[6] to be proportional to τ_R^{-1} and $\Delta\omega/\omega$, respectively. We operate at constant frequency (45, 75, and 105 MHz) and pressure (0 to 15 bars) and sweep the temperature in very slow demagnetization and remagnetization cycles (typically 50 μK/h). Temperatures are converted into frequencies by use of the weak-coupling gap function $1.764 k_B T_c \Delta_{BCS}(T)/\Delta_{BCS}(0)$. The excitation power level is kept below the threshold of nonlinear phenomena. With this setup, changes in Z/ρ_0 as small as $0.01\,c$ can be resolved; c is the velocity of sound. As shown in Fig. 1, typical variations of Z/ρ_0 are a sizable fraction of c. Hence, the squashing mode has a pronounced effect on the longitudinal acoustic impedance. Two features are particularly noteworthy: (1) The line shapes are markedly asymmetric with long tails extending to the low-temperature side; and (2) ImZ/ρ_0 is as much affected by the squashing mode as is ReZ/ρ_0 and has large positive as well as large negative deviations. These features have never been encountered before to our knowledge and may be accounted for by the following phenomenological model.

The superfluid is treated as a continuous medium, in which small motions occur, giving rise to a mass transport velocity $\vec{v}$ and to small deviations ρ and Π of the mass and momentum flux densities about their equilibrium values ρ_0 and Π_0. Although in the collisionless regime, we still have local conservation of mass and momentum

$$\dot{\rho}+\rho_0\nabla\cdot\vec{v}=0, \qquad (1)$$

$$\dot{\vec{v}}+\nabla\Pi/\rho_0=0. \qquad (2)$$

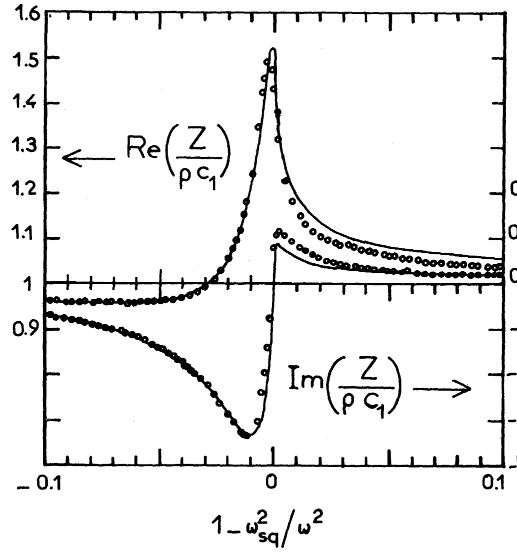

FIG. 1. Variations of the real and imaginary parts of the longitudinal acoustic impedance Z normalized to the low-temperature, low-frequency limit $\rho_0 c_1$, vs the reduced frequency $1-\omega_{sq}^2/\omega^2$. The dots represent only a part of the experimental data points obtained at 13.5 bars and 104.3 MHz in ^{3}He-B. Reduced frequencies are computed from the measured reduced temperatures T/T_c using the BCS gap function as explained in the text. The plain line comes from the theoretical expression for Z with the following values of the model parameters: $\bar{\Lambda}=1.52\times10^{-2}$, $c_{sq}/c_1=0.16$, $\tau_{sq}=\tau_0=2.5\,\mu$s, $\xi=780$ Å. The resonance at $\omega=\omega_{sq}$ is found at $T/T_c=0.680$.

We also need an equation of motion for the superfluid order-parameter eigen component which is relevant to our problem. From a microscopic calculation based on the kinetic equation,[7] we know that the appropriate combination is $\delta=d_{zz}''-(d_{xx}''+d_{yy}'')/2$. [Sound propagates along z (parallel to $\vec{k}$) and $d_{i\alpha}''=\mathrm{Im}\,d_{i\alpha}(\omega,k)$; $d_{i\alpha}$ is the usual order-parameter matrix.] Reference 7 indicates that the behavior of δ close to the resonance may be described by the following differential equation:

$$\ddot{\delta}+\frac{\dot{\delta}}{\tau_{sq}}+\omega_{sq}^2\delta-c_{sq}^2\nabla^2\delta=\gamma\nabla^2\rho. \qquad (3)$$

Equation (3) reproduces the dispersion law for the squashing mode (i.e., $\omega^2=\omega_{sq}^2+c_{sq}^2k^2$), contains a phenomenological collision time τ_{sq}, and includes the coupling to sound, to first order in k^2, through the parameter γ. This coupling also enters the equation of motion for ρ because the generalized pressure Π is a function of both ρ and δ:

$$\nabla^2\Pi=\partial\Pi/\partial\rho\,\nabla^2\rho+\partial\Pi/\partial\delta\,\nabla^2\delta-\dot{\rho}/\tau_0, \qquad (4)$$

where the last term has been introduced to describe zero-sound damping. Putting $c^2 = \partial\Pi/\delta\rho$ at constant δ, we readily find from Eqs. (1), (2), and (4)

$$\ddot{\rho} + \dot{\rho}/\tau_0 - c^2\nabla^2\rho = \partial\Pi/\partial\delta\,\nabla^2\delta\,. \qquad (5)$$

The model described by Eqs. (3) and (5) is quite similar to that used by Calder *et al.*[8] to analyze the transmission of sound. Its solution with fixed ω consists of an arbitrary superposition of four plane waves with phase velocities $\pm c_+$ and $\pm c_-$, where

$$\frac{c_\pm^2}{\tilde{c}^2} = \frac{1}{2}\left\{1 + \frac{1}{w} \pm \left[\left(1 - \frac{1}{w}\right)^2 + \frac{4\Lambda}{w}\right]^{1/2}\right\},$$

$$w = (c_{\rm sq}^2/\tilde{c}^2)^{-1}(1 - \omega_{\rm sq}^2/\omega^2 + i/\omega\tau_{\rm sq}),$$

$$\Lambda = \gamma c^{-2} c_{\rm sq}^{-2}\partial\Pi/\partial\delta, \quad \tilde{c}^2 = c^2/(1 + i/\omega\tau_0).$$

Besides a pole at $w = 0$ (i.e., for the $k = 0$ limit of the squashing mode) $c_\pm^2$ possesses two branch points at $w_0 = 1 - 2\Lambda \pm 2[\Lambda(\Lambda - 1)]^{1/2}$ (where the zero-sound mode crosses the pair-vibration mode). It may develop an imaginary part even in the absence of damping when $\Lambda(\Lambda - 1) > 0$. In this case, the system is unstable with respect to fluctuations at large enough wave vector. On experimental grounds, such an instability is ruled out and we shall require that the condition $0 < \gamma\,\partial\Pi/\partial\delta < c^2 c_{\rm sq}^2$ be satisfied. This stability condition also ensures that c_+ and c_- give rise to waves which decay when propagating away. As we are dealing with a semi-infinite homogeneous medium and since the fluid is at rest at infinity ($z = \infty$ boundary conditions), we retain only the two outgoing waves:

$$\rho(t,z) = \rho_+ \exp[-i\omega(t - z/c_+)] + \rho_- \exp[-i\omega(t - z/c_-)]$$

and similar forms hold for δ, v, and Π.

The acoustic impedance of the liquid follows from the definition $Z = \Pi_0/v_Q$ (v_Q, interface velocity; Π_0, generalized pressure at $z = 0$). From Eq. (2), we have that $\Pi_0/\rho_0 = c_+v_+ + c_-v_-$. We note that, in a usual liquid where sound is the only acoustic mode, $Z/\rho_0 = c$, where c is the complex velocity. In this case, Z'' is directly proportional to the absorption and does not change sign. In ^{3}He-B, the acoustic impedance arises from the interplay of two modes, which explains the remarkable behavior of Z''. We need two boundary conditions at the interface to eliminate the amplitudes v_+ and v_-. From continuity at $z = 0$, $v_+ = v_- = v_Q$. We must also specify the behavior of the pair-vibration amplitude δ close to a wall. On general grounds, we expect that δ goes from its bulk value to zero at a distance ζ from the wall which is of the order of the superfluid coherence length. Assuming that this simple result holds, the second $z = 0$ boundary condition reads $\partial\delta/\partial z = -\delta/\zeta$. The parameter ζ is akin to the slip-length concept justified in detail by Højgaard Jensen *et al.*[9] in the case of shear waves. Applying this complete set of boundary conditions, we find that

$$Z = Z_0(\rho_0 c_+ c_- + i\omega\zeta Z_\infty)(\rho_0 c_+ c_- + i\omega\zeta Z_0)^{-1},$$

$$Z_0 = \tilde{c}^2(c_+ + c_-)(\tilde{c}^2 + c_+c_-)^{-1}\rho_0,$$

$$Z_\infty = (c_+^2 + c_-c_+ + c_-^2 - c_+^2c_-^2/\tilde{c}^2)(c_+ + c_-)^{-1}\rho_0.$$

Z involves six physical quantities: the squashing frequency $\omega_{\rm sq}$, the dimensionless coupling factor $\bar{\Lambda} = \Lambda c_{\rm sq}^2/c^2$, the velocity of the squashing mode $c_{\rm sq}$, two quasiparticle collision times $\tau_{\rm sq}$ and τ_0, and the accommodation length ζ. Of these six parameters, $\omega_{\rm sq}$ is measured directly[10] and $\bar{\Lambda}$ known approximately.[8] At very low temperatures, dissipation is very small, so τ_0 and $\tau_{\rm sq}$ become very large and do not give a relevant contribution to Z. An approximate starting value can be given to $c_{\rm sq}$ by theory.[10] Thus only ζ is totally undetermined. A numerical fitting procedure yields precise values for these remaining four quantities. The full curve in Fig. 1 shows the outcome of such a fit ($\tau_0 = \tau_{\rm sq}$).

We present in Fig. 2 the values of $\bar{\Lambda}$ obtained at various frequencies in the vicinity of $T/T_c \sim 0.6$. If $F_2^s = 0$, the theoretical expression for $\bar{\Lambda}$ is given by[1]

$$\bar{\Lambda} = \tfrac{4}{5}\lambda(\omega, T)(c_0 - c_1)/c_1, \qquad (6)$$

where c_0 and c_1 are the zero- and first-sound velocities. $\lambda(\omega, T)$ is an integral defined in Ref. 1 which embodies the frequency and temperature dependence of the coupling between modes. Then we set $\Delta = \Delta(T_{\rm sq})$ and obtain the full curve in Fig. 2. The fitting procedure is sensitive mainly to the combination $\Lambda - 1$ and, especially at low pressures, the accuracy on $\bar{\Lambda}$ is somewhat impaired.

As the velocity of the squashing mode $c_{\rm sq}$ is determined to a fair level of accuracy, we were led to evaluate again its theoretical expression including the Fermi-liquid corrections.[10] At $T \ll T_c$, we find

$$c_{\rm sq}^2 = v_{\rm F}^2[10/21 + 1/21J + (8J/225)F_1^s], \qquad (7)$$

where $J = (5/\sqrt{6})\arctan(\sqrt{6}/2)$ and $v_{\rm F}$ is the Fermi velocity. Result (7) is seen in Fig. 2 to account well for the experimental values and, together

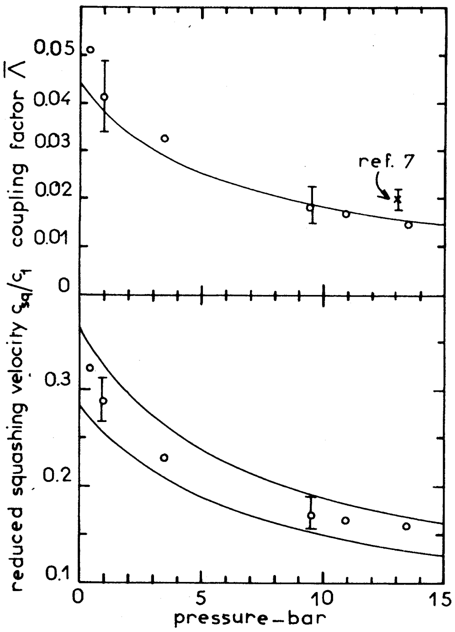

FIG. 2. Dimensionless coupling factor $\overline{\Lambda}$ (on top) and reduced squashing velocity c_{sq}/c_1 (at the bottom) vs pressure. These parameters are obtained from fits to the experimental data as shown in Fig. 1 at temperatures in the vicinity of $T/T_c = 0.6$. The plain lines correspond to expressions (6) and (7): Values of F_1^s from Ref. 11 (upper curve) give squashing velocities in slightly better overall agreement than those from Ref. 12 (lower curve).

with (6), satisfies the stability condition. The error bars in Fig. 2 reflect uncertainties in the fitting procedure. The accommodation length ζ is found at all pressures to be of the order of $(1.8 \pm 0.2)\xi_0$, ξ_0 being the coherence length $\hbar v_F/\pi k_B T_c$. We find it a remarkable fact that acoustic properties can be so sensitive to such an abstract entity as the boundary condition on the order parameter. Also remarkable is the importance of the evanescent branch of the squashing mode ($\omega < \omega_{sq}$).

Finally, we note that the simple model described here accounts fairly well for the complex behavior of the longitudinal acoustic impedance in ^{3}He-B and that the model parameters extracted from the data agree satisfactorily with theoretical expectations. However, we want to point out that this model is equivalent, in microscopic theory, to a series expansion in $(kv_F/\Delta)^2$ truncated after the first order. As the expansion parameter is not small, higher-order terms will be significant. Their contribution should not change drastically the phenomenological model and may even improve the already fair agreement displayed in Fig. 1. But it may also alter the correspondence, illustrated in Fig. 2, between the model parameters $\overline{\Lambda}$ and c_{sq} and their microscopic counterparts, expressed by Eqs. (6) and (7).

This work is supported in part by Direction des Recherches Etudes et Techniques Contract No. 79/567. Laboratoire de Physique des Solides and Groupe de Physique des Solides are Laboratoires Associées au Centre National de la Recherche Scientifique.

[(a)]On leave from Texas A&M University, College Station, Tex. 77843.

[1]For a review, see P. Wölfle, in *Progress in Low Temperature Physics*, edited by D. F. Brewer (North-Holland, Amsterdam, 1978), Vol. VIIa.

[2]R. W. Giannetta, A. Ahonen, E. Polturak, J. Saunders, E. K. Zeise, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 45, 262 (1980).

[3]D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder, J. B. Ketterson, and W. P. Halperin, Phys. Rev. Lett. 45, 266 (1980).

[4]O. Avenel, E. Varoquaux, and H. Ebisawa, Phys. Rev. Lett. 45, 1952 (1980).

[5]J. Wilks, *Liquid and Solid Helium* (Clarendon, Oxford, 1967), p. 524.

[6]P. Roach and J. B. Ketterson, in *Quantum Fluids*, edited by S. B. Trickey, E. Adams, and J. Dufty (Plenum, New York, 1977), p. 223.

[7]V. E. Koch and P. Wölfle, Phys. Rev. Lett. 46, 486 (1981), and private communication.

[8]I. D. Calder, D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, J. B. Ketterson, and W. P. Halperin, Phys. Rev. Lett. 45, 1866 (1980). In their model, these authors have set c_{sq} equal to zero, in which case our stability condition can never be satisfied.

[9]H. Højgaard Jensen, H. Smith, P. Wölfle, K. Nagai, and T. Maack Bisgaard, J. Low Temp. Phys. 41, 473 (1980).

[10]The dependence of ω_{sq} on pressure and on magnetic field deserves a full discussion, which will be given elsewhere. Our result for c_{sq}, in the absence of the F_1^s term, corresponds to that given by P. N. Brusov and V. N. Popov, Zh. Eksp. Teor. Fiz. 78, 2419 (1980) [Sov. Phys. JETP 51, 1217 (1980)].

[11]T. A. Avesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, Phys. Rev. Lett. 44, 1076 (1980).

[12]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).