the ortho-para transitions described in Ref. 1) involve spin-momentum coupling factors which threaten to violate unitarity unless there occur systematic cancellations of the type associated with the universal coupling constant of a gauge theory. (Spin-complexity corrections, whether or not they involve electroweak vector bosons, can be evaluated by Feynman-like rules, as described in Appendix B of Ref. 2.)

[6]G. F. Chew and C. Rosenzweig, Phys. Rep. 41C, No. 5 (1978).

[7]H. P. Stapp, Lawrence Berkeley Laboratory Report No. LBL-10774, 1980 (to be published).

[8]As shown in Ref. 1, if each core triangle on the quantum surface is to have one of its three edges touched by a Landau arc (thereby labeling quarks with "topological color") then hadron quantum disks contain a maximum of two core triangles.

[9]H. Harari, Phys. Rev. Lett. 22, 562 (1969); J. Rosner, Phys. Rev. Lett. 22, 689 (1969).

[10]P. Gauron, B. Nicolescu, and S. Ouvry, Orsay Report No. IPNO/TH 80-58, 1980 (to be published).

[11]There are believed to be no Regge branch points, only Regge poles, as discussed in Ref. 6.

[12]The quark-permutation degree of freedom has in Ref. 1 been called "topological color."

[13]G. F. Chew, J. Finkelstein, and M. Levinson, following Letter [Phys. Rev. Lett. 47, 767 (1981)].

[14]A plausible primary topological mechanism would be trivial vertices (see Ref. 1, Appendix A) attached to the internal lines of Landau tree graphs. This mechanism could shift masses without affecting coupling constants and would precede closed-loop corrections to zero entropy. At zero entropy it would be consistent to assign all elementary hadrons a single mass, m_0, while in closed-loop corrections mass differences would be included.

[15]A natural supposition is that continuous time is not a "primitive" notion, but an approximate representation of "entropy" growth in discrete topologies of high complexity.

Topological Theory of Elementary-Hadron Coupling Constants

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Topological particle theory is applied to the computation of elementary-hadron coupling constants. All are determined through topological supersymmetry by a single dimensionless "zero-entropy" constant g_0. The predicted coupling-constant ratios encompass and justify those of Mandelstam. The universality conjecture, $g_0=e$, is supported (to 6%) by the accurately measured value of the pion-nucleon coupling constant. An explanation emerges for the experimental failure to find baryonium.

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Within the topological theory of particles there occurs a degenerate collection of "elementary hadrons,"[1] all sharing a single mass m_0, which are supposed to stand in one-to-one correspondence with certain physical mesons (spin 0 and 1), baryons (spin $\frac{1}{2}$ and $\frac{3}{2}$), and baryonium (spin 0, 1, and 2).[2] A zeroth approximation to physical three-hadron coupling constants is achieved by superposing corresponding "zero-entropy" amplitudes—which all are interrelated by topological supersymmetry.[3,4] Precisely, if the dimensionless zero-entropy coupling constant is g_0, then every zero-entropy three-hadron M function is equal to $g_0 m_0$. Any *physical* hadronic coupling constant is approximated by counting how many different zero-entropy topologies associate to the physical three-vertex and, with appropriate Clebsch-Gordan coefficients, multiplying by g_0. This prescription turns out to encompass all the predictions of Mandelstam's model[5]—which was already an extension of $SU(6)_W$ symmetry—and adds a supersymmetry prediction for the ratios of baryon, meson, and baryonium couplings.

Although the value of g_0 is in principle calculable from the nonlinear zero-entropy bootstrap equations, this computation has not yet been accomplished. On the other hand a conjecture has been made that physical S-matrix unitarity will require $g_0=e$.[4] We report in this paper that not only do our predicted ratios of elementary-hadron dimensionless coupling constants agree with measured ratios but, to 6%, the *magnitudes* of physical strong-interaction couplings agree with the $g_0=e$ conjecture.

We begin by computing an elementary three-meson coupling constant. A physical meson's identity is fixed by the spin and flavor attached to quark and antiquark quantum triangles which topologically "build" the meson.[1] Given these spins and flavors, as well as the meson four-momenta[6] p_A, p_B, and p_C with $p_A+p_B+p_C=0$ and $p_A^2=p_B^2=p_C^2=m_0^2$, the first task is to enumerate the compatible zero-entropy topologies—associating the M-function value $g_0 m_0$ to any single topology, such as that depicted by the shorthand representation of Fig. 1. The solid arcs here carry quark spin and their direction distinguishes quark from antiquark. The direction rules for spin arcs are the same as those given originally for "quark lines" by Harari and Rosner,[7] and it is possible to attach to each spin arc a flavor-generation index.[8] The dashed lines in Fig. 1 are charge arcs as described by Chew *et al.*[9]; each quark has two possible values of electric charge according to whether its charge-arc direction agrees or disagrees with its spin-arc direction. Specifying a set of quark spins and flavors fixes the directions of both spin and charge arcs and also fixes the generation index, but within the topology of Fig. 1 are certain "hidden parameters" not determined by physical-meson properties.

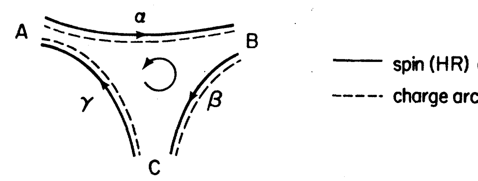

FIG. 1. The three-meson vertex.

One such hidden parameter is the patchwise orientation of the "classical" surface that houses the spin and charge arcs. This orientation—denoted by the central arrow in Fig. 1—relates to parity[1] and may either agree (ortho) or disagree (para) with the Harari-Rosner (HR) orientation. (The topology of Fig. 1 is para.) Physical amplitudes are always sums over pairs of topological amplitudes belonging to opposite patchwise orientations.

Another hidden parameter in Fig. 1 is the relative order within each quark of its charge and spin arcs. Either arc may be "on the outside." We assume (see Appendix B of Ref. 9 and Appendix D of Ref. 1) that these two possibilities correspond to using either $1+\gamma_5$ or $1-\gamma_5$ quark (four-component) M functions. Thus, for example, the particular para amplitude associated with Fig. 1 has the value

$$g_0 m_0[\overline{U}^{\alpha}(B)(1+\gamma_5)U^{\alpha}(A)][\overline{U}^{\gamma}(A)(1+\gamma_5)U^{\gamma}(C)][\overline{U}^{\beta}(C)(1-\gamma_5)U^{\beta}(B)], \tag{1}$$

where U is a conventional u or v spinor—depending on whether the attached particle is outgoing or ingoing. The U and $\overline{U}$ contain the boosts that convert M functions to S-matrix elements; the normalization is such that $\overline{U}(p)U(p)=1$. We are here expressing through four-component Dirac spinors the same two forms proposed by Mandelstam,[5] who described the two alternatives [corresponding to (0 and $\frac{1}{2}$) and ($\frac{1}{2}$ and 0) representations of the Lorentz group] through a two-valued "hidden" index added to each quark helicity. Stapp has discussed these alternatives with two-component M functions.[10] A natural notation employs M functions with dotted and undotted spinor indices so that zero-entropy quark-spin propagators take the form of Kronecher delta functions. Spin at zero entropy then behaves exactly like flavor, with each index taking four possible values even though physical spin is two valued. For computational convenience we nevertheless prefer Formula (1)—associating (for a specified classical-patch orientation) $1+\gamma_5$ with one order of charge and spin arcs and $1-\gamma_5$ with the other.

Stapp[10] has shown from S-matrix unitarity that physical amplitudes are sums of topological amplitudes where *each zero-entropy component occurs with coefficient +1.* This fact is the key to the results of the present Letter. Summing over the two charge-spin orders for each quark line leads to $(1+\gamma_5)+(1-\gamma_5)=2$, so that together with classical-patch-orientation doubling, the physical three-meson amplitude is approximated by

$$2^4 g_0 m_0[\overline{U}^{\alpha}(B)U^{\alpha}(A)][\overline{U}^{\gamma}(A)U^{\gamma}(C)][\overline{U}^{\beta}(C)U^{\beta}(B)]. \tag{2}$$

This formula is compatible with that of Mandelstam[5] when he employed quark wave functions symmetrized in the "hidden" index. Unitarity gives the reason for such symmetrization and also provides a definite overall normalization.[11]

The normalized spin and flavor superposition corresponding to a particular physical meson is straightforward. Attaching such a "wave function" to each meson location in Fig. 1 and summing over the two different cyclic orders (another hidden parameter) for the three mesons leads to a physical coupling constant. For example, we find for the $\rho^+\pi^-\pi^0$ amplitude F^0 the value

$$(F^0)_{\rho^+\pi^-\pi^0} = 12\sqrt{2}\, g_0 S_{\rho^+} \cdot (p_{\pi^-} - p_{\pi^0}), \tag{3}$$

where S_{ρ^+} is the ρ^+ polarization four-vector. The corresponding resonance width is

$$\Gamma_{\rho\to 2\pi} = \tfrac{4}{3}(12)^2 \left(\frac{g_0^2}{4\pi}\right) \frac{k_{cm}^3}{m_\rho^2}. \tag{3$'$}$$

All our three-meson coupling ratios are in agreement with Mandelstam[5] and with $SU(6)_W$ symmetry.[12]

The first stage in computing a meson-baryon-antibaryon coupling constant is similar to the foregoing. Corresponding to Fig. 2, after summing over classical-patch orientation and charge-spin order for each of the four-quark lines, we have

$$2^5 g_0 m_0 [\overline{U}^\alpha(A) U^\alpha(B)][\overline{U}^\beta(A) U^\beta(B)][\overline{U}^\gamma(C) U^\gamma(B)][\overline{U}^\delta(A) U^\delta(C)]. \tag{4}$$

Physical baryons, however, are not distinguished by the *order* of their three-quark lines—a hidden parameter which in Ref. 1 has been characterized as "topological color." For each physical baryon there are six possible quark permutations, each distinct at zero entropy. A convenient procedure is to attach to each baryon location (A and B) in Fig. 2 an ordinary three-quark spin-flavor baryon wave function, each quark carrying a (1, 2, and 3) color label as described in Ref. 1, but to assign the wave function a norm $\langle\psi/\psi\rangle$ equal to 6.[13] One thereby finds for the accurately measured $n\bar{p}\pi^+$ amplitude the value[14]

$$(F^0)_{np\pi^-} = 60 g_0 (\bar{u}_p \gamma_5 \gamma \cdot p_\pi u_n). \tag{5}$$

The well-known constant $g_{\pi NN}^2/4\pi$ (≈ 15) has a corresponding value of

$$\frac{g_{\pi NN}^2}{4\pi} = \frac{(60)^2}{2} \frac{g_0^2}{4\pi}. \tag{5$'$}$$

Again, all our baryon-antibaryon-meson couplings agree with Mandelstam.[5] We now additionally predict through supersymmetry (not via vector dominance) the ratios to three-meson couplings. These latter ratios agree with experiment. In particular, the measured $g_{\pi NN}^2/4\pi = 14.8$ predicts, through comparison of Formulas (3′) and (5′),

$$\Gamma_{\rho\to 2\pi} = \frac{8}{75} \frac{g_{\pi NN}^2}{4\pi} \frac{k_{cm}^3}{m_\rho^2}, \tag{6}$$

a ρ width of 124 MeV.[15]

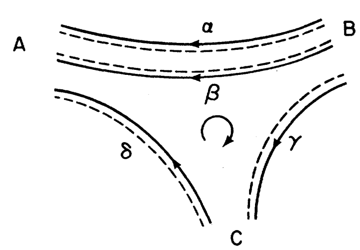

FIG. 2. The baryon-antibaryon-meson vertex.

A next question is how well the conjecture $g_0 = e$ is borne out. Formula (5′) with $g_0 = e$ gives a result only 12% smaller than the above experimental value of $g_{\pi NN}^2/4\pi$. The conjectured universality of electroweak and strong couplings is thus given encouraging support. It remains to confirm that coupling-constant corrections from higher-order terms of the topological expansion are no more than a few percent even though mass shifts are large.

Topological hidden parameters systematically make $B\overline{B}M$ coupling constants larger than MMM coupling constants by a factor of 4 (16 in the square of coupling constants).[2] One factor of 2 arises from the extra quark line and one factor of 2 from the always-allowed permutation of the two quark lines that do not touch the meson. The attachment of baryon wave functions obscures but does not remove the systematic factor of 4. Some readers may be puzzled by remembering that vector dominance of electromagnetic structure functions requires that *Dirac-type* coupling of vector mesons to baryons should have the *same* magnitude as the $\rho\pi\pi$ coupling in Formula (3) above. This fact is compatible with our theory; the additional enhancement is in *Pauli-type* coupling between baryons and vector mesons.

In the coupling of a baryonium to $B\overline{B}$ there is an *additional* factor of 4 coming from one more quark line and one more pair of permutable quark lines.[16] The experimental failure to find narrow baryonium thereby becomes understandable. Even if a baryonium lies below the $B\bar{B}$ threshold so that decay must lead to mesons via an Okubo-Zweig-Iizuka-rule-violating higher component of the topological expansion, the Okubo-Zweig-Iizuka suppression mechanism has to fight a factor of 16 in rate as well as the large phase space available to the final mesons. Although the Okubo-Zweig-Iizuka mechanism for baryonium decay has not yet been understood in detail, the extremely large baryonium coupling constants make it difficult for baryonium to have widths sufficiently narrow as to be detectable by a straightforward experiment.

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[a]Participating guest at Lawrence Berkeley Laboratory.

[1]G. F. Chew and V. Poénaru, Phys. Rev. Lett. 45, 229 (1980), and Lawrence Berkeley Laboratory Report No. LBL-11433 (to be published).

[2]The quantum numbers of elementary hadrons are those of ground ($l = 0$) states in constituent-quark models.

[3]P. Gauron, B. Nicolescu, and S. Ouvry, Orsay Report No. IPNO/TH 80-58, 1980 (unpublished).

[4]G. F. Chew, preceding Letter [Phys. Rev. Lett. 47, 764 (1981)].

[5]S. Mandelstam, Phys. Rev. 184, 1625 (1969), and D 1, 1745 (1970).

[6]Topological quarks do not carry momentum, as explained in Ref. 1.

[7]H. Harari, Phys. Rev. Lett. 22, 562 (1969); J. Rosner, Phys. Rev. Lett. 22, 689 (1969).

[8]The theory of Ref. 1 predicts four flavor generations, but this number is not needed for the computations described here.

[9]G. F. Chew, J. Finkelstein, Robert E. McMurray, Lawrence Berkeley Laboratory Report No. LBL-11435 (to be published).

[10]H. P. Stapp, Lawrence Berkeley Laboratory Report No. LBL-10774 (to be published).

[11]As explained in Ref. 10, unitarity also generates higher terms in the topological expansion that eliminate the wrong-parity "ghost poles" which occur in zero-entropy M functions for more than three particles.

[12]See, for example, B. Sakita and K. C. Wali, Phys. Rev. B 139, 1355 (1965).

[13]The usual baryon wave functions include a sum over the six quark permutations with a corresponding normalizing factor of $1/\sqrt{6}$. If each permutation is to be given a weight +1, this latter factor must be omitted.

[14]All our three-particle amplitudes have the dimension of mass.

[15]The Breit-Wigner ρ width listed in the Particle Data Group Tables is 158 MeV, but it is known for the $\Delta(1238)$ that the imaginary part of the mass of such broad P-wave resonances is significantly smaller than half the Breit-Wigner width.

[16]In calculating a physical baryonium coupling to $B\bar{B}$, ordinary two-quark two-antiquark spin-flavor wave functions are employed, the two quarks being distinguished by a color label and similarly for the two antiquarks. The norm of the wave function is four—corresponding to the four topologically distinct color permutations belonging to a single physical baryonium. See Sect. IX of Ref. 1.