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Is There an Aharonov-Bohm Effect for Neutrons?

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A neutron interferometer experiment testing the existence of an Aharonov-Bohm effect has been performed and no measurable phase shift has been found upon reversal of the enclosed magnetic flux. A positive result would have provided evidence for a breakdown in the standard minimal-coupling scheme for the electromagnetic interaction of a neutron. The sensitivity of the experiment sets the ratio of the Aharonov-Bohm effect for a neutron to that of a particle of charge e to be less than 5×10^{-12}.

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For a charged particle, there are intrinsically nonlocal effects produced by an electromagnetic field. The Aharonov-Bohm (AB) effect[1-3] is a striking demonstration of this phenomenon. Consider an electron beam which is split into two coherent subbeams and is allowed to interfere upon subsequent recombination (see Fig. 1). Then, if a magnetic flux is passes somewhere through the area between the separated beams (for instance by an infinite solenoid perpendicular to their plane, or a toroidal coil wrapped around one of them) there will be a phase shift induced in the interference pattern, even though neither beam ever passes through the magnetic field. This effect has been demonstrated experimentally and discussed many times for electrons.[4]

The effect is caused by the topological properties of the coupling between a charged particle and the electromagnetic field. This coupling itself is a representation of the gauge invariance of the theory, which is a consequence of the fact

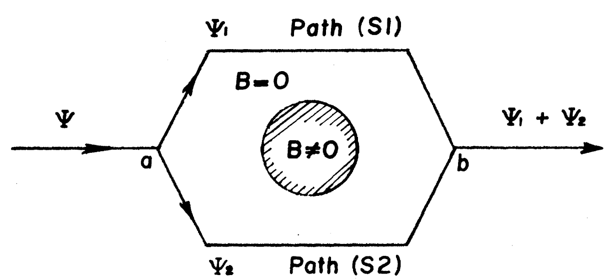

FIG. 1. The Aharonov-Bohm effect. An electron beam is split coherently at a and recombined at b. A magnetic flux through the region between the two subbeams, ψ_1 and ψ_2, will induce a phase shift in the interference pattern even if neither beam is ever in the magnetic field.

that the absolute phase of the wave function is unobservable. Gauge invariance is an expression of this symmetry; charge conservation and Maxwell's equations are consequences. This "minimal-coupling" scheme was generalized by Yang and Mills[5] to isotopic spin, and by Utiyama[6] to arbitrary Lie groups. Today it has become the standard model for local gauge groups for both the weak and strong interactions.[7]

According to the minimal-coupling scheme, whenever the momentum $\vec{p}$ of a charged particle appears in the Lagrangian, it is to be replaced by

$$\vec{p} \rightarrow \vec{p} - (e/c)\vec{A}(x), \tag{1}$$

where $\vec{A}$ is the vector potential. Then when the vector potential undergoes the gauge transformation

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla\lambda \tag{2}$$

the wave function undergoes a local phase change

$$\psi \rightarrow \psi' = \psi \exp[(ie/\hbar c)\lambda(x)], \tag{3}$$

so that

$$\left(\vec{p} - \frac{e}{c}\vec{A}'\right)\psi' = \exp\left[\frac{ie}{\hbar c}\lambda(x)\right]\left(\vec{p} - \frac{e}{c}\vec{A}\right)\psi. \tag{4}$$

Thus, the gauge transformation reduces to a phase factor, proportional to the charge, and any overall neutral (real) Lagrangian will remain invariant. The magnetic field itself is invariant under the transformation in Eq. (2). One can also represent the effect of the vector potential by introducing the path-dependent integral[8]

$$\psi_S = \exp[(ie/\hbar c)\textstyle\int_{(S)}\vec{A}\cdot d\vec{l}\,]\,\psi_0, \tag{5}$$

where ψ_0 is the wave function in the absence of the potential.

In the case where one has a free-particle beam and the magnetic field introduces only a weak perturbation into the wave function, Eq. (5) can be evaluated by taking ψ_0 to be the unperturbed wave function and integrating over the classical path (S) of the unperturbed beam.[3] When the two beams ψ_1 and ψ_2 are coherently recombined, as shown in Fig. 1, the wave function of the recombined beam will be

$$\psi = \psi_{S1} + \psi_{S2} = [\,\psi_{01} + \psi_{02}\exp(ie/\hbar c)\oint \vec{A}\cdot d\vec{l}\,]\ \exp[(ie/\hbar c)\textstyle\int_{(S1)}\vec{A}\cdot d\vec{l}\,], \tag{6}$$

where

$$\oint \vec{A}\cdot d\vec{l} = \int \vec{B}\cdot d\vec{S} = \Phi, \tag{7}$$

the flux through the area bordered by the two beams. Thus, this flux produces a real, observable phase shift between the beams, even though neither beam ever feels any force. Yet this nonlocal effect is a necessary consequence of quantum mechanics.[3]

For a neutral particle, like the neutron, the minimal-coupling scheme implies that there is no direct coupling to the electromagnetic potential [$e=0$ in Eq. (1)]. In the nonrelativistic limit, all electromagnetic effects combine via a Pauli term to produce a magnetic moment which couples directly to the magnetic field, not to the potential. Therefore, the neutron is unaffected by a gauge transformation. So one immediate consequence of the standard minimal-coupling scheme is that a neutral particle will not exhibit an AB effect.

One may ask, however, whether such a conclusion is experimentally borne out. After all, the neutron has a spin, and therefore a complex wave function and nontrivial charge-conjugation properties. Also, the Pauli interaction is only heuristic, and the neutron wave equation, even at very low energies, has not been fully explored. So it is not obvious that the neutron should be excluded from all possible gauge transformations.

We can envision couplings other than the standard one, which generalize the usual gauge invariance, and give rise to an AB effect. Observing such an effect would prove that the standard coupling scheme cannot completely describe the electromagnetic interaction, even in the extreme nonrelativistic limit. Conversely, the lack of an observable AB effect would set an upper limit on such possible couplings.

For a charged particle, the Hamiltonian $(\vec{p} - e\vec{A}/c)^2/2m + e\varphi$ leads to a potential (to first order in A),

$$V = e\varphi - (e/mc)\vec{A}\cdot\vec{p}, \tag{8}$$

which is the nonrelativistic limit of the coupling $(-j_\mu A_\mu)$. For the neutron, one can ask whether there is a coupling

$$V_1 = \gamma_1 e\varphi f_1\left[\left(\frac{p}{mc}\right)^2\right] - \gamma_2\left(\frac{e}{mc}\right)\vec{A}\cdot\vec{p}\,f_2\left[\left(\frac{p}{mc}\right)^2\right], \tag{9}$$

where the form factors obey $f_1(0) = f_2(0) = 1$. Then the first term would be due to a static charge on the neutron, while the second term would be due to a "dynamic" charge, which would also exhibit a generalized $\vec{v}\times\vec{B}$ force in a magnetic field and an AB effect. (A possible third term proportional to $\vec{\sigma}\cdot\vec{A}\times\vec{p}$ would violate time reversal invariance.)

It is already known from experiments in strong static electric fields[9] that $\gamma_1 < 10^{-19}$; but no experiments sensitive to the existence of γ_2 have been performed. It might appear that $\gamma_1 \neq \gamma_2$ would violate Lorentz invariance. However, one can always devise an invariant coupling that will reduce nonrelativistically to Eq. (9). (One can also introduce a generalized gauge invariant prescription that will include this coupling.) For particles with considerable kinetic energy, it would be difficult to distinguish the effects produced by the dynamical part of the potential in a magnetic field from those given by Foldy.[10] But the Foldy interaction does not exhibit an AB effect, so one can separate them even at very low energies.

To detect γ_2, we have searched for the existence of an AB effect on low-energy neutrons (1.564 Å) in an interferometer system. In analogy with Eq. (6), the two coherently recombined neutron wave functions will show a phase difference due to the dynamical potential of Eq. (9), of

$$\varphi = \gamma_2 e\Phi/\hbar c, \tag{10}$$

where we assume $f_2 = 1$, since $(p/mc)^2 \sim 10^{-11}$.

The experiment was performed with the same two-crystal neutron interferometer used earlier[11] in a search for nonlinear terms in the Schrödinger equation and details concerning its operation are reported there. In this system, two spatially separated coherent and parallel beams pass through the interferometer before recombination. One of the beams was passed through the center of a magnetized rectangular loop of single crystal Fe as shown in Fig. 2(a). Under ideal magnetization conditions, there is complete magnetic-field closure within the crystal and circulating flux passes through the four domains shown in Fig. 2(b). Translation of the crystal loop from beam (2) to beam (1) provides a reversal of the magnetic flux passing between the coherent neutron beams. Neutron phase effects associated with this flux reversal can be studied by observing intensity changes when the interferometer phase is positioned on the steep side of a fringe. This phase adjustment can be provided conveniently by changing the angular position of an aluminum phase plate in one of the neutron beams.

The magnetic domain pattern of Fig. 2(b) was induced by passing a small current through a primary coil wrapped around one of the crystal

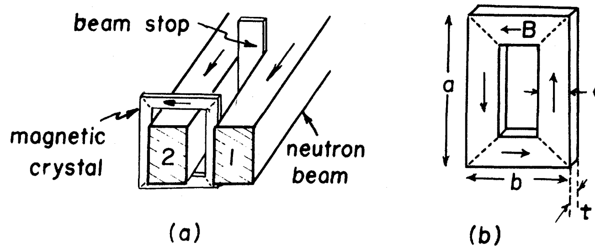

FIG. 2. Arrangement of magnetic loop crystal relative to interferometer neutron beams. (a) Circulating flux passing upward between coherent beams. Sidewise translation of the loop crystal to beam (1) reverses this flux direction. (b) Domain pattern and dimensions of Fe crystal (silicon stabilized). Magnetization directions are all along [100] axes. Dimensions (mm) are $a = 15.00$, $b = 10.04$, $c = 2.80$, and $t = 0.28$.

legs and the saturation magnetization was checked with a secondary coil. Upon collapse of the activating field it was found that 90% of the saturation field was retained in the crystal. This is significant because the later neutron measurements were performed without the presence of an activating field and correction for this incomplete magnetization was given in the analysis. Of more consequence, this incomplete magnetization implies the possible presence of external fringe fields at positions along the neutron trajectory in either of beams (1) and (2). In turn, this could introduce artificial interferometer phase effects because of neutron spin rotation[12] in the Larmor-acting fringe field. This was studied by measurements of the interference fringe contrast of the interferometer when the magnetic crystal surrounded, or was adjacent to, the cohering beams. Within experimental uncertainty of 1%, no change of contrast was found and we conclude that fringe field effects were of no concern. At the conclusion of the neutron experiments, the circulating magnetization in the crystal was again measured with search coils and found to be the same as the original measurement within experimental uncertainty.

In the search for an AB effect, the magnetic crystal was shuttled transversely from one beam to another and intensity changes were looked for. This was performed many times and no measurable intensity change beyond experimental uncertainty was found. This observation combined with the measured intensity modulation of the fringe pattern sets an upper limit to the neutron phase shift of $(0.56 \pm 0.67)^\circ$ upon enclosed flux reversal.

Using a conservative value of 1.3° for the upper limit phase shift and a circulating flux of 152 Gcm^2, we evaluate from Eq. (10) the upper limit for γ_2 to be

$$\gamma_2 < 4.9 \times 10^{-12}.$$

Thus within experimental uncertainty, no measurable AB *effect has been found for neutrons.* This conclusion suggests more than the mere absence of a dynamic charge on the neutron. More generally, the experiment was capable of detecting the existence of any nonstandard coupling to the electromagnetic field resulting in an AB effect, even though the particle involved was uncharged.

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Instanton Contributions in Two-Dimensional Nonlinear O(3) σ Model

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The correlation length in the two-dimensional nonlinear O(3) σ model is calculated with contributions of instantons included. It is given by $\xi = 0.0125a \exp[2\pi/f(a)]f(a)/2\pi$ where $f(a)$ is the coupling constant defined in lattice regularization scheme and a is the lattice spacing. This number remarkably coincides with the result of Monte Carlo simulations by Shenker and Tobochnik.

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The O(3) nonlinear σ model in two dimensions bears many similarities with a non-Abelian gauge model in four dimensions: Both possess asymptotic freedom, n-instanton solutions, and no intrinsic scale parameters. An O(3)-invariant regularization of the former gives the O(3) Heisenberg spin model, while a gauge-invariant regularization of the latter gives a non-Abelian lattice gauge model. In the O(3) Heisenberg model no phase transition would occur at any finite temperature. This corresponds to the fact that in the lattice gauge theory a confining phase would survive even when $g \ll 1$. It is believed that these kinds of low-temperature (weak coupling) behavior are due to nonperturbative effects. Numerical calculations in the lattice-regularization scheme include automatically all such effects. Indeed, recent Monte Carlo simulations per-

