Example of a Negative Effective Poisson's Ratio

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An effective Poisson's ratio is introduced for anisotropic materials as the negative ratio of transverse and longitudinal strains averaged over all transverse directions. It is shown that for certain orientations of the applied force this effective Poisson's ratio assumes negative values for α-quartz. This implies that such a bar increases its cross section under length extension.

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Poisson's ratio μ for isotropic bodies is generally assumed to be positive and smaller than 0.5. While the upper limit may be derived from thermodynamics, the lower limit of zero is merely a matter of experience. In some renowned text books (e.g., Landau and Lifshitz[1] and Feynman, Leighton, and Sands[2]) it is remarked that μ might theoretically be negative down to a value of -1, which would imply transverse contraction associated with longitudinal compression. It is added however, that no material with negative μ is actually known. This last observation, however, is not an absolute one. In fact, by including anisotropic bodies, at least one example may be given for a longitudinal extension being accompanied by an increase in cross section.

The concept of Poisson's ratio has been generalized for anisotropic media.[3,4] Denoting the infinitesimal strain components as referred to the Cartesian crystal axes x_1, x_2, and x_3 in abbreviated notation by S_i, generalized Poisson's ratios are given by

$$\mu_{ij} = -S_i/S_j \quad (i,j=1,2,3;\ i\neq j), \tag{1}$$

where index j indicates the direction of the applied force. Making use of a linear stress-strain relation μ_{ij} may be expressed in terms of the compliances s_{ij}:

$$\mu_{ij} = -s_{ij}/s_{jj}. \tag{2}$$

It is known that for a host of crystals certain μ_{ij} are negative. From this fact alone, however, no judgment on overall expansion or contraction may be made. The general impression seems to be that "anomalous" expansion in one direction is more than compensated for by "normal" behavior in the direction perpendicular to it. For most known materials this is so. In the case of α-quartz, however, there is an exception.

In order to demonstrate this, we consider a general orientation of the crystal bar. We use a Cartesian coordinate system x_1', x_2', and x_3' with axes parallel to the edges of the bar and with x_3' taken as the length direction, rotated with respect to x_1, x_2, and x_3 (Fig. 1). All quantities referred to the primed system are also written with primes.

A generalized Poisson's ratio as given in (1) for two specific transverse directions may, of course, be defined for an arbitrary lateral direction. That means that it can be given as a function $\mu_3'(\varphi)$ of an azimuthal angle φ in the $x_1'x_2'$ plane, describing a rotation about x_3'. We now define the "effective Poisson's ratio" $\bar{\mu}_3'$ as the average value of $\mu_3'(\varphi)$ over all transverse directions:

$$\bar{\mu}_3' = (2\pi)^{-1}\int_0^{2\pi}\mu_3'(\varphi)\,d\varphi. \tag{3}$$

With use of the general transformation equations given by Cady[3] it may be shown that

$$\bar{\mu}_3' = -(s_{13}'+s_{23}')/2s_{33}'. \tag{4}$$

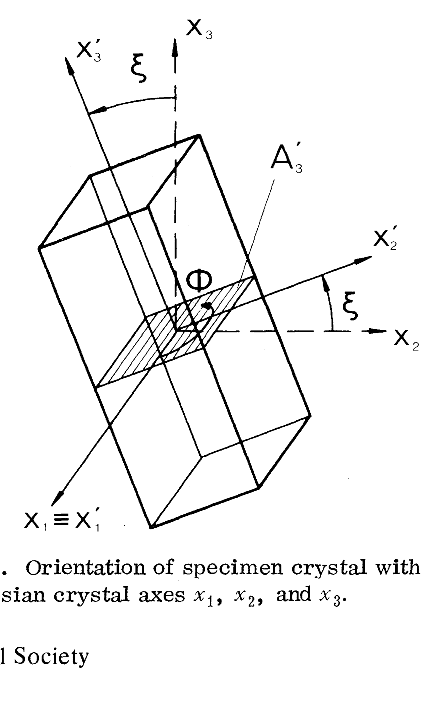

FIG. 1. Orientation of specimen crystal with respect to Cartesian crystal axes x_1, x_2, and x_3.

Since for α-quartz (and other crystals of classes 32, $3m$ and $\bar{3}m$) $\bar{\mu}_3'$ turns out to be symmetric about the x_2x_3 plane, all extrema must occur if the direction of the applied force lies in this plane. It is therefore sufficient to consider bars singly rotated about x_1 through an angle ξ as indicated in Fig. 1. The primed compliances are given by[3]

$$\begin{aligned} s_{13}' &= s_{12}\sin^2\xi + s_{13}\cos^2\xi - s_{14}\sin\xi\cos\xi, \\ s_{23}' &= s_{13}(\cos^4\xi+\sin^4\xi) + (s_{11}+s_{33}-s_{44})\cos^2\xi\sin^2\xi + s_{14}\sin\xi\cos\xi\cos2\xi, \\ s_{33}' &= s_{11}\sin^4\xi + s_{33}\cos^4\xi + (2s_{13}+s_{44})\cos^2\xi\sin^2\xi + 2s_{14}\cos\xi\sin^3\xi. \end{aligned} \tag{5}$$

With use of numerical data from McSkimin, Andreatch, and Thurston[5] for the s_{ij} (adjusting the sign of s_{14} to conform to the new Institute of Electrical and Electronics Engineers standard on piezoelectricity[6]) $\bar{\mu}_3'$ was calculated as a function of ξ. The results are shown in Fig. 2. It is seen clearly that in the angular range $36.1° < \xi < 71.2°$ the effective Poisson's ratio is negative. The largest negative value is taken on for $\xi = 53.9°$. With the previously accepted convention[7] of Institute of Radio Engineers standards on piezoelectric crystals (1949) these values of ξ are to be replaced by their supplementary angles. For most of the angular range given above both μ_{13}' and μ_{23}' are negative such that a bar stretched in this direction will expand in all lateral directions simultaneously. The effect is a large one and may not be explained by an unfortunate combination of measurement errors. In fact, differences between compliances at constant electric field and at constant dielectric displacement or between isothermal and adiabatic values as well as the use of data from different authors do not change the results for $\bar{\mu}_3'$ on a scale demonstrable in Fig. 2.

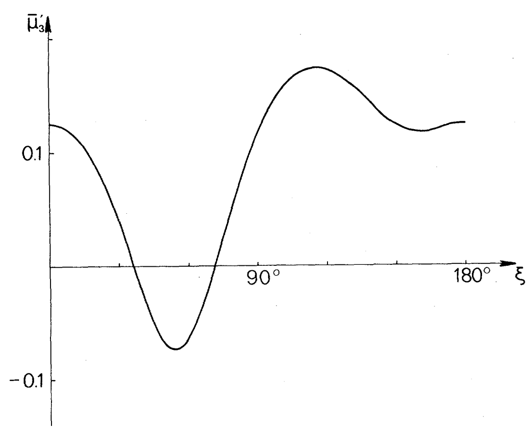

FIG. 2. Effective Poisson's ratio $\bar{\mu}_3'$ for α-quartz as a function of rotation angle ξ as calculated with isothermal compliance constants from Ref. 5 with axes chosen according to IEEE standard on piezoelectricity (1978) (Ref. 6).

The effective Poisson's ratio as introduced above has a direct physical meaning: It is a measure of the relative change in cross section. If we denote the cross section normal to x_3' by A_3' its relative change is given by

$$\Delta A_3'/A_3' = S_1' + S_2'. \tag{6}$$

Hence, from (2) we get

$$\Delta A_3'/A_3' = -2\bar{\mu}_3' S_3'. \tag{7}$$

Thus, whenever the effective Poisson's ratio is negative, this means an increase in cross section under extension.

A check on other trigonal crystals, using data taken from Landolt-Börnstein,[8] revealed that this behavior is not very common. While for several crystals $\bar{\mu}_3'$ closely approaches zero for ξ between 50° and 60°, it nevertheless remains positive in all of them. Although the basic facts have been known for a very long time, this exception for quartz seems not to have been noticed before.

The reason for this difference must be due to the numerical values of the compliance constants. It appears that α-quartz is distinguished by a fairly large value of s_{14} as compared with other crystals and that the behavior described above is dominated by this constant.

Although it is probably just a coincidence, it is interesting to note that throughout (but not confined to) the range of directions of stress resulting in such anomalous lateral expansion α-quartz shows a certain instability. Stress of this orientation easily leads to secondary Dauphiné twinning, i.e., a reversal of the x_1 axis. By this process the crystal reorients itself so as to render lateral expansion normal. Interestingly the twinning tendency is also governed by s_{14}.

One might also raise the question if there is a connection between this unusual mechanical behavior of quartz and its piezoelectricity. The available data give no indication for this: About one third of the trigonal crystals checked by us, which do not show this effect, are piezoelectric, some of them much more strongly than quartz. Their behavior in this respect appears not be distinctly different from nonpiezoelectric ones.

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Multiple Excitation of Lower-Hybrid Drift Waves in the Neutral Sheet

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A numerical simulation of the lower-hybrid drift instability is carried out in which an electric drift toward the neutral sheet is present. It is found that lower-hybrid drift waves, which are excited usually in a transient fashion and decay to a low level in a closed system, are reexcited and execute recurrent growth in the presence of particle drift toward the neutral sheet. Consequently, anomalous resistivity also exhibits a recurrent generation.

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In laboratory and cosmical plasmas, anomalous resistivity associated with microscopic instabilities is an important subject. Computer simulation is known to be a powerful tool to study this type of microscopic process. In fact, a large number of numerical simulations have been carried out and have brought many fruitful results. However, except for a few cases,[1] they are restricted to a conserved system. Not only in a cosmical plasma but also in a laboratory plasma, however, there usually exists an external source that supplies energy and/or particles, and often the presence of such a source plays a crucial role in the ultimate fate of phenomena occurring in the plasma system.

In this Letter, we deal with lower-hybrid drift (LHD) waves which are thought to generate anomalous resistivity. In the neutral sheet of the magnetosphere, the anomalous resistivity could stimulate magnetic reconnection leading to magnetospheric substorms.[2] In a pinch experiment, LHD waves are said to lead to a rapid diffusion in the implosion phase.[3]

Previous simulations of LHD waves in a conserved system have shown that LHD waves are excited near the neutral sheet but subside within several lower-hybrid periods.[4,5] Thus, the associated anomalous resistivity can have power only for a short period. Incidentally, it is said that this happens for drift waves which are thought to cause anomalous diffusion in tokamaks.[6] If so, however, the role of anomalous resistivity would be vanishingly small in practice. Thus, we must abandon the idea that LHD-wave-induced resistivity could play a stimulating role in the magnetic reconnection process in a collisionless plasma. In a real plasma in which we are here interested, such as the magnetospheric neutral

