PHYSICAL REVIEW
LETTERS

VOLUME 47 | 13 JULY 1981 | NUMBER 2

Mean Free Path in a Nucleus

J. W. Negele
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

K. Yazaki
Department of Physics and Faculty of Science, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan
(Received 11 May 1981)

It is shown that proper treatment of the nonlocality of the nuclear optical potential resolves much of the apparent discrepancy between previous theoretical calculations and empirical values of the nucleon mean free path.

PACS numbers: 21.10.−k, 24.10.Ht, 24.90.+d

One of the most fundamental properties characterizing the propagation of a particle in the nuclear medium is its mean free path. Hence, it has been a serious problem that past theoretical calculations have appeared to yield a mean free path of roughly 3 fm for nucleons of energy 50–150 MeV whereas the experimental value in this energy range is approximately 5 fm.[1-3] Since the simple physical argument that λ should be $(\sigma\rho)^{-1}$ times a correction factor for Pauli blocking[4] yields qualitative agreement with detailed numerical calculations, it has been puzzling that such a large discrepancy with experiment exists.

In this note, we show that the bulk of the apparent discrepancy is resolved by taking proper account of the nonlocality of the optical potential. Since our purpose is primarily pedagogical, no new detailed calculations are presented, and numerical results are taken from the extensive review article of Jeukenne, Lejeune, and Mahaux.[5] Furthermore, for simplicity, since the magnitude of the imaginary potential is small compared to the relevant energies, we shall expand all results to first order in the imaginary potential.

The propagation of a nucleon in a nucleus is specified by the proper self-energy Σ of the one-particle Green's function, which is equivalent to the optical potential.[6] In translationally invariant nuclear matter, the self-energy is a function of the energy E and momentum k, and the propagation is described by the dispersion relation

$$E = k^2/2m + \Sigma(k, E) . \tag{1}$$

The self-energy is in general complex, and it will be useful to write

$$\Sigma(k, E) \equiv U(k, E) + i\, W(k, E) , \tag{2}$$

where W is negative for k above the Fermi momentum. In the case of scattering theory, the dispersion relation (1) is solved for real E, k is necessarily complex

$$k \equiv k_R + ik_I, \tag{3}$$

and the imaginary part yields the mean free path

$$k_I = 1/2\lambda. \tag{4}$$

A first approximation to solving the full dispersion equation (1) is obtained by neglecting the imaginary part as in Ref. 5, in which case one obtains

$$E = k_0^2/2m + U(k_0, E) \tag{5}$$

with the familiar result

$$k_0 = \{2m[E - U(k_0, E)]\}^{1/2}. \tag{6}$$

Assuming W is small relative to $E - U$, which is well satisfied for the energies under consideration, the next approximation is obtained by expanding around k_0 and retaining corrections of first order in W

$$E \approx \frac{k_0^2 + 2k_0(k_R - k_0 + ik_I)}{2m} + U(k_0, E) + \frac{\partial U}{\partial k}\bigg|_{k_0}(k_R - k_0 + ik_I) + W(k_0, E) + \cdots, \tag{7}$$

with the result

$$k_R = k_0 = \{2m[E - U(k_R, E)]\}^{1/2}, \tag{8a}$$

$$k_I = -W(E, k_R)\left(\frac{k_R}{m} + \frac{\partial U}{\partial k}\bigg|_{k_R}\right)^{-1}. \tag{8b}$$

At this point, it is useful to recall the definitions of three distinct effective masses from Ref. 5. The total effective mass m^* is defined to reproduce the density of states specified by Eq. (5)

$$\frac{k}{m^*} \equiv \frac{dE}{dk} = \frac{k}{m} + \frac{\partial U}{\partial k} + \frac{\partial U}{\partial E}\frac{dE}{dk} = \frac{k}{m}\left(1 + \frac{m}{k}\frac{\partial U}{\partial k}\right)\left(1 - \frac{\partial U}{\partial E}\right)^{-1}. \tag{9}$$

Since m^*/m is given by the product of terms involving partial derivatives with respect to energy and momentum, it is useful to define an E mass

$$m_E/m \equiv (1 - \partial U/\partial E) \tag{10}$$

and a k mass

$$\frac{m_k}{m} \equiv \left(1 + \frac{m}{k}\frac{\partial U}{\partial k}\right)^{-1}. \tag{11}$$

Then, the total effective mass is the product

$$\frac{m^*}{m} = \left(\frac{m_E}{m}\right)\left(\frac{m_k}{m}\right). \tag{12}$$

It is shown in Ref. 5 that in lowest-order Brueckner theory, for $E > k_F$, m_E/m is signficantly greater than 1 and m_k/m is much less than 1, yielding a product m^*/m that is somewhat less than 1. From Eqs. (4), (8b), and (11) it follows that

$$\lambda = 1/2k_I = -k_R/2m_k W(E, k_R). \tag{13}$$

Since at low energy, m_k/m is of the order of 0.6, the mean free path in Eq. (13) is 67% larger than the conventional result for the mean free path obtained by ignoring the $\partial U/\partial k$ term in Eq. (7) and thus replacing m_k in Eq. (13) by m. If one wishes to compare a theoretical $W(E, k_R)$ with the imaginary part of a phenomenological *local* optical potential, $W^L(E)$, it is evident from Eq. (13) and the preceding arguments that $(m_k/m) \times W(E, k_R)$, rather than $W(E, k_R)$, should be compared with $W^L(E)$.

A different, but equivalent, way to obtain the result in Eq. (13) is to consider the dispersion relation, Eq. (1), with real k and consequently complex E. This case arises, for example, if one inserts a complete set of states in the Green's function to obtain the Lehmann representation, and the imaginary part of E specifies the quasiparticle lifetime. Expanding to the same order, one obtains

$$E_0 - \frac{i\Gamma}{2} \approx \frac{k^2}{2m} + U(k, E_0) - \frac{i\Gamma}{2}\frac{\partial U}{\partial E}\bigg|_{E_0} + iW(k, E_0) \tag{14}$$

with the result

$$\Gamma = -2W\left(1 - \frac{\partial U}{\partial E}\bigg|_{E_0}\right)^{-1}. \tag{15}$$

If the quasiparticle lifetime is Γ, the mean free path is

$$\lambda = v/\Gamma, \tag{16}$$

where the group velocity v is

$$v \equiv \frac{dE}{dk} = \frac{k}{m^*} = \frac{k}{m}\frac{m}{m_k}\frac{m}{m_E}. \tag{17}$$

Combining Eqs. (10), (15), (16), and (17), the formula (13) for the mean free path is again obtained.

To appreciate the quantitative effect of various contributions to the mean free path in nuclear matter, it is useful to compare three distinct approximations to the nuclear mean free path with the experimental data in Fig. 1 determined from reaction cross sections.[2] The first approximation, $(\bar{\sigma}\rho)^{-1}$, where $\bar{\sigma}$ is the average of the n-p and p-p cross sections is much too small because of the neglect of the Pauli principle.

The local expression for the mean free path

$$\lambda_L = -k_R/2mW(E, k_R) \tag{18}$$

includes the Pauli principle but neglects the factor m_k/m in Eq. (13) which accounts for the nonlocality of the optical potential. The values for λ_L plotted in Fig. 1 are calculated according to Eq. (18) using lowest-order Brueckner theory results for $W(E, k_R)$ from Fig. 23 of Ref. 5. These values display the usual problem of yielding roughly 3 fm instead of the 5-fm result determined empirically. Finally, our nonlocal result, Eq. (13), is plotted in Fig. 1, using values of m_k/m from Fig. 18 of Ref. 5 and one observes that inclusion of the nonlocality produces qualitative agreement. Given the limitations of lowest-order Brueckner theory, our expansion to first order in W, and the ambiguities in extracting a mean free path for nuclear matter from scattering from finite nuclei, it is not surprising that some discrepancies remain, especially in the energy dependence. Nevertheless, we believe that Fig. 1 clearly demonstrates that the effect of nonlocality in increasing the mean free path is qualitatively as important as the Pauli principle, and resolves the bulk of the discrepancy between theory and experiment.

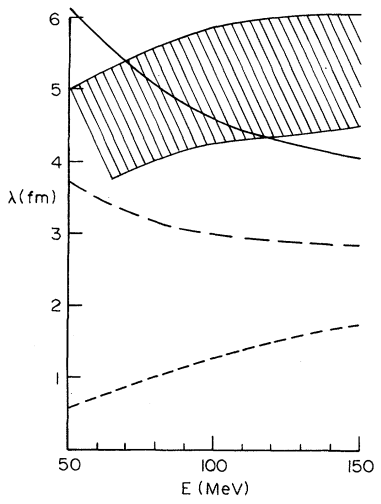

FIG. 1. Comparison of three approximations to the nucleon mean free path with the range of values (shaded band) compatible with reaction cross sections for Ca, Zr, or Pb in Ref. 2. The short-dashed curve denotes $(\bar{\sigma}\rho)^{-1}$, the long-dashed curve represents the local result λ_L of Eq. (18), and the solid curve indicates the correct nonlocal expression λ of Eq. (13).

In the special case of purely quadratic momentum dependence in the real and imaginary parts of the optical potential, the dispersion relation may be written

$$E = k^2/2m + U_0 + iW_0 + (U_2 + iW_2)k^2. \tag{19}$$

In this case, the k mass defined in Eq. (11) is

$$1/2m_k = 1/2m + U_2 \tag{20}$$

and the mean free path calculated from k_I is

$$\lambda = \frac{-k_R}{2m_k W_0 + 4m_k^2 (E - U_0) W_2}, \tag{21}$$

where

$$k_R = [2m_k(E - U_0)]^{1/2}. \tag{22}$$

One can show, as for example in Ref. 5, that W_2 is positive so that the two terms in the denominator of Eq. (21) have opposite signs. Because $(E - U_0)$ is very large, the second term can significantly affect the mean free path and thus Eq. (21) superficially appears to differ from our general result, Eq. (13). The apparent inconsistency is resolved by expanding the optical potential around k_R instead of $k = 0$.

In finite nuclei, the microscopically derived nonlocal optical potential may be related to the phenomenological local potentials used by experimentalists through a suitably defined equivalent local potential. Of particular interest to us is the value of the imaginary part of the equivalent local potential in the nuclear interior, which reflects how the nonlocality increases the mean free path. For simplicity, we consider the case in which the nonlocality is approximated by quadratic momentum dependence, as in Eq. (19):

$$\{\nabla[-1/2m - U_2(r) - iW_2(r)]\nabla + U_0(r) + iW_0(r)\}\psi = E\psi. \tag{23}$$

Since this form of nonlocality is utilized in a variety of effective interactions of the Skyrme form[7] and may be derived by the density matrix expansion,[8] we believe it is sufficiently accurate to elucidate the essential physics. Defining the quantity f by

$$-\frac{1}{2mf(r)} \equiv -\frac{1}{2m} - U_2(r) - iW_2(r) \tag{24}$$

and an equivalent local wave function φ by

$$\psi(r) \equiv [f(r)]^{1/2}\varphi(r), \tag{25}$$

Eq. (23) becomes

$$\left\{-\frac{1}{2m}\nabla^2 - \frac{1}{2m}\left[\frac{1}{2}\frac{\nabla^2 f}{f} - \frac{3}{4}\left(\frac{\nabla f}{f}\right)^2\right] + E(1-f) + f(U_0 + iW_0)\right\}\varphi = E\varphi \tag{26}$$

and f may be expanded to first order in W_2 to obtain

$$f(r) = \frac{m_k(r)}{m} - i\frac{2m_k(r)^2 W_2(r)}{m}, \tag{27}$$

where $m_k(r)$ is defined by Eq. (20). The $\nabla^2 f$ and $(\nabla f)^2$ terms in Eq. (26) contribute to the surface potential and are presumably adequately parametrized by the phenomenological surface terms included in experimental analyses. The real and imaginary parts of the volume term of the equivalent local optical potential are, therefore, specified by

$$U^{EL} = \mathrm{Re}[E(1-f) + f(U_0 + iW_0)] = E\left(1 - \frac{m_k}{m}\right) + \frac{m_k}{m}U_0 \tag{28}$$

and

$$W^{EL} = \mathrm{Im}[E(1-f) + f(U_0 + iW_0)] = (E - U_0)\frac{2m_k{}^2 W_2}{m} + \frac{m_k}{m}W_0. \tag{29}$$

Thus, one observes directly that W^{EL}, which is to be compared with empirical data, is reduced relative to W_0 by the k-mass factor m_k/m as well as the extra term appearing in Eq. (21) which is characteristic of a quadratic expansion about $k=0$. Substitution of Eqs. (28) and (29) in the formula for the mean free path in a local potential precisely reproduces Eq. (21) as it should given the equivalence of Eqs. (19) and (23).

It is a matter of taste how one interprets the physics of the equivalent local potential. If one prefers to think in terms of wave-function suppression, then Eq. (25) requires that the nonlocal wave function ψ be suppressed relative to φ, so that a large nonlocal W_0 is required to accomplish the same absorption as W^{EL}. Alternatively, one may view the phenomenological analyses as a totally artificial transform of the real nonlocal physics, in which case Eqs. (28) and (29) show how to properly invert the transform. In either case, one sees that the equivalent local potential leads to the same conclusions as the nuclear matter analysis, and this feature will naturally occur in more general nonlocal potentials for which the construction of the equivalent local potential is more complicated.

Our interest in this problem was greatly stimulated by discussions with John Schiffer and his emphasis of the fundamental disagreement between theoretical and empirical mean free paths. We wish to acknowledge helpful correspondence from Claude Mahaux.

One of us (J. W. N.) is grateful for a fellowship by the Japan Society for the Promotion of Science which made this collaboration possible. This work was supported in part through funds provided by the U. S. Department of Energy under Contract No. DE-AC02-76ER03069.

Note added.—Subsequent to the completion of this work, we became aware of related work by Fantoni, Friman, and Pandharipande.[9]

[1]J. P. Schiffer, Nucl. Phys. A335, 348 (1980), and private communication.

[2]A. Nadasen, P. Schwandt, P. P. Singh, W. W. Jacobs, A. D. Bacher, P. T. Debevec, M. D. Kaitchuck, and J. T. Meek, Phys. Rev. C 23, 1023 (1981).

[3]A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969).

[4]K. Kikuchi and M. Kawai, *Nuclear Matter and Nuclear Reactions* (North-Holland, Amsterdam, 1968), p. 33.

[5]J. P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rep. 25, 83 (1976).

[6]J. S. Bell and E. J. Squires, Phys. Rev. Lett. 3, 96 (1959).

[7]D. Vautherin and D. M. Brink, Phys. Rev. C 5, 626 (1972).

[8]J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972).

[9]S. Fantoni, B. L. Friman, and V. Pandharipande, to be published.