Reduced Quantum Fluctuations in Resonance Fluorescence

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We demonstrate that reduced quantum fluctuations or squeezing are present in both the atomic observables and the radiation field produced by a two-level atom undergoing resonance fluorescence.

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There is currently considerable interest in creating quantum states which give a reduced uncertainty in the measurement of a particular observable at the expense of increased uncertainty in the measurement of a second noncommuting observable. Interest in such states is stimulated by the current efforts to detect gravity waves.[1] The measurement of such weak forces will be affected by quantum fluctuations in the measurement process. In order to reduce the effect of quantum fluctuations ingenious techniques known as quantum nondemolition measurements have been suggested.[2-5] For example, if one prepares the detector in a state where one observable has reduced fluctuations then, by making measurements on this observable only, the effect of quantum fluctuations may be circumvented.

One scheme presently used in the search for gravity waves is based on the differential change of length of the arms of a Michelson interferometer. Such measurements will ultimately be affected by quantum fluctuations in the radiation field, as discussed by Caves,[6] who subsequently suggested[7] that such fluctuations may be reduced by applying a special state of the radiation field with reduced fluctuations in one quadrature to the second port of the interferometer. Such states of the radiation field where fluctuations in one quadrature are reduced below that of a coherent state have been discussed in the literature[8-12] under various names, e.g., squeezed states, or two-photon coherent states. The reduction of noise in one quadrature in these states has potential applications in optical communications, as has been discussed by Yuen and Shapiro.[13]

An observation of squeezing would demonstrate a quantum mechanical effect of fundamental importance in the detection of weak forces. For example, in the detection of gravitational radiation with use of massive bars the gravitational waves are expected to change the complex amplitude of the fundamental mode of oscillation of the detector by less than the width of a coherent state. In one quantum nondemolition technique suggested,[4] the "back-action evading" method, the measuring device monitors only one mode of the complex amplitude. This device automatically forces that mode into a state similar to a squeezed state enabling changes in amplitude less than the width of a coherent state to be detected.

While it is clear that such squeezed states would find useful applications, there has been no experimental observation of such states. Several schemes in nonlinear optics have been suggested, such as a degenerate parametric oscillator.[10,14-16] A fully quantum mechanical analysis of this device recently made by Milburn and Walls[17] indicates that the maximum squeezing is obtained in the vicinity of the threshold of such a device. However, to date no experimental observation has been made.

In this paper we wish to suggest a scheme which exhibits reduced quantum fluctuations and which should be observable with current experimental techniques. The scheme we refer to is resonance fluorescence from a two-level atom. This classic system has already been the source of one of the major triumphs of quantum optics —the observation of photon antibunching. This uniquely quantum mechanical effect was produced theoretically by Carmichael and Walls[18] and experimentally observed by Kimble, Dagenais, and Mandel[19] and Leuchs, Rateike, and Walther.[20] The phenomenon of photon antibunching in resonance fluorescence is reviewed by Walls.[21]

The observation of photon antibunching indicated that the fluorescent light has reduced intensity or photon-number fluctuations. The calculations and subsequent measurements were concerned with photon-number correlation functions and did not give any information on the phase-dependent properties of the scattered electromagnetic field. It is precisely this point, which has heretofore not been investigated, that we wish to address in this paper.

It is necessary for us first to give a general definition of what we mean by squeezing or reduction of quantum fluctuations. For two arbitrary operators A and B which obey the commutation relation $[A, B]=C$, the product of the uncertainties in determining their expectation values is given by

$$\Delta A \Delta B \geq \tfrac{1}{2}|\langle C\rangle|, \tag{1}$$

where we are only interested in observables having the same physical dimension, such as the real and imaginary parts of the electric field amplitude. We shall define squeezing if the uncertainty in one of the observables satisfies the relation

$$\Delta A^2 < \tfrac{1}{2}|\langle C\rangle|. \tag{2}$$

(a) *Reduced fluctuations in the atomic operators.*—A two-level atom may be described by the algebra of the Pauli spin matrices $\sigma_1, \sigma_2, \sigma_3$ (σ_i is $\frac{1}{2}$ the Pauli matrix) which obey the commutation relations

$$[\sigma_1, \sigma_2] = i\sigma_3. \tag{3}$$

Hence we define squeezing in the atomic observables if

$$\Delta\sigma_{1,2}{}^2 < |\langle\sigma_3\rangle|. \tag{4}$$

We now consider resonance fluorescence from a two-level atom driven with a coherent driving field. The steady-state expectation values of the slowly varying atomic operators are[22]

$$\begin{aligned}\langle\sigma_1\rangle &= -(\sqrt{2}/2)z\delta/(1+\delta^2+z^2),\\ \langle\sigma_2\rangle &= (\sqrt{2}/2)z/(1+\delta^2+z^2),\\ \langle\sigma_3\rangle &= -\tfrac{1}{2}(1+\delta^2)/(1+\delta^2+z^2),\end{aligned} \tag{5}$$

where $z=\sqrt{2}\,\Omega/\gamma$, $\delta=2(\omega-\omega_0)/\gamma$, $(\omega-\omega_0)$ is the detuning between the atom and the driving field, Ω is the Rabi frequency, and γ is the natural linewidth of the atom. The phase of the driving field was chosen as zero. $\langle\sigma_1\rangle$ and $\langle\sigma_2\rangle$ denote the in-phase and out-of-phase components of the amplitude of the atomic polarization.

The variances in σ_1 and in σ_2 are calculated to be

$$\begin{aligned}\Delta\sigma_1{}^2 &= \tfrac{1}{4}-\tfrac{1}{2}z^2\delta^2/(1+\delta^2+z^2)^2,\\ \Delta\sigma_2{}^2 &= \tfrac{1}{4}-\tfrac{1}{2}z^2/(1+\delta^2+z^2)^2,\end{aligned} \tag{6}$$

since $\sigma_i{}^2$ is $\frac{1}{4}$.

From Eq. (4) the conditions are

$$\begin{aligned}&\delta^2 > z^2+1,\\ &\delta^2 > z^2 < 1,\end{aligned} \tag{7}$$

for squeezing in σ_1 and σ_2, respectively. Thus for an appropriate choice of detuning and driving field intensities, squeezing may be found in either one of the components of the atomic observables. We note, however, that the state produced is not a minimum-uncertainty state.

(b) *Reduced fluctuations in the radiation field.*—If we define a normally ordered variance of the electric field components, the condition for squeezing in the radiation field is that

$$(:\Delta E_{1,2}:)^2 = \langle :E_{1,2}{}^2:\rangle - \langle E_{1,2}{}^2\rangle - \langle E_{1,2}\rangle^2 < 0. \tag{8}$$

For a single-mode field this condition may be written

$$\begin{aligned}(:\Delta a_{1,2}:)^2 &= \int(\alpha_{1,2}-\langle\alpha_{1,2}\rangle)^2 P(\alpha_1,\alpha_2)\,d\alpha_1 d\alpha_2\\ &= (\Delta a_{1,2})^2 - \tfrac{1}{4} < 0,\end{aligned} \tag{9}$$

where $P(\alpha_1, \alpha_2)$ is the Glauber P representation for the single-mode field. Hence we see that a necessary condition to obtain squeezing is that $P(\alpha_1, \alpha_2)$ may not be a well-behaved positive definite function as required for a classical probability function. Thus squeezing is a purely quantum mechanical effect.

The variance of the fluctuations in the fluorescent field may be derived with use of the following relation between the radiation field and the atomic operator in the far-field limit[23]:

$$E^{(+)} = E_{\text{free}}{}^{(+)} + \psi(\vec{x})\,\sigma^-\!\left(t-\frac{r}{c}\right)\exp[-i\omega(t-r/c)], \tag{10}$$

where

$$\sigma^- = (\sigma_1-\sigma_2)/2i$$

and

$$\psi(\vec{x}) = (\omega_0{}^2/2\pi\epsilon_0 c^2)[\vec{x}\times(\vec{\mu}\times\vec{x})/r^3].$$

$E^{(+)}$ is the positive-frequency part of the radiation field. We shall consider the variance of fluctuations in the in-phase (E_1) and out-of-phase components (E_2) of the scattered field amplitude.

From Eq. (8) and using the commutation relations of the atomic operators we find for the scattered field

$$\begin{aligned}(:\Delta E_1:)^2 &= \psi^2(\vec{x})[\tfrac{1}{4}(2\langle\sigma^+\sigma^-\rangle - 4\langle\sigma_1\rangle^2] \\ &= \psi^2(\vec{x})[(\Delta\sigma_1)^2 + \tfrac{1}{2}\langle\sigma_3\rangle], \\ (:\Delta E_2:)^2 &= \psi^2(\vec{x})[\tfrac{1}{4}(2\langle\sigma^+\sigma^-\rangle - 4\langle\sigma_2\rangle^2] \\ &= \psi^2(\vec{x})[(\Delta\sigma_2)^2 + \tfrac{1}{2}\langle\sigma_3\rangle].\end{aligned} \tag{11}$$

Now in the stationary state $\langle\sigma_3\rangle < 0$, thus $\langle\sigma_3\rangle = -|\langle\sigma_3\rangle|$, hence the condition for squeezing in the radiation field,

$$(:\Delta E_{1,2}:)^2 = \psi^2(\vec{x})[(\Delta\sigma_{1,2})^2 - \tfrac{1}{2}|\langle\sigma_3\rangle|] < 0, \tag{12}$$

is seen to be identical to the solution (4) we had previously derived for squeezing in the atomic observables. Thus squeezing in the two components E_1 and E_2 of the radiation field may be observed under the conditions (7) in the steady-state regime of resonance fluorescence. This reduction in fluctuations or squeezing may be enhanced in the transient regime, where, for example, on resonance we find: $(:\Delta E_2:)_{\min}{}^2 = -\frac{1}{16}\psi^2$ compared with $(:\Delta E_2:)_{\min}{}^2 = -\frac{1}{32}\psi^2$ on resonance in the steady-state regime.

We conclude that squeezing or a reduction in fluctuations in either quadrature of the radiation field is present in resonance fluorescence from a coherently driven two-level atom. This should be observable under similar experimental conditions to those used for the observation of photon antibunching though phase-sensitive detection of the radiation field is required. Potential schemes to measure the variances in the quadrature phases of the electromagnetic field $(\Delta a_i{}^2)$ have been proposed.[24,25]

The type of measurement described here yields further information on the uniquely quantum nature of the radiation field produced by a two-level atom undergoing resonance fluorescence. Like photon antibunching, squeezing is quantum mechanical in origin and may not be produced by a classical description of the radiation field. However, some quantum states may exhibit antibunching but not squeezing, e.g., a pure number state exhibits antibunching but not squeezing because of the complete lack of phase information. Hence while a measurement of the photon-number correlation function cannot distinguish the fluorescent field from a number state, a measurement of the variances ΔE_1, ΔE_2 would indicate that the fluorescent field carries phase information which is nonclassical in origin.

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[1]For a review, see K. S. Thorne, Rev. Mod. Phys. 52, 285 (1980).

[2]V. B. Braginsky, Yu. I. Vorontsov, and F. Ya. Khalili, Zh. Eksp. Teor. Fiz. 73, 1340 (1977) [Sov. Phys. JETP 46, 705 (1977)].

[3]W. G. Unruh, Phys. Rev. D 18, 1764 (1978), and 19, 2888 (1979).

[4]C. M. Caves, K. S. Thorne, P. W. P. Drever, V. D. Sandberg, and M. Zimmerman, Rev. Mod. Phys. 52, 341 (1980).

[5]S. Chaturvedi, P. Meystre, and D. F. Walls, to be published.

[6]C. M. Caves, Phys. Rev. Lett. 45, 75 (1980).

[7]C. M. Caves, Phys. Rev. D 23, 1693 (1981).

[8]D. W. Robinson, Commun. Math. Phys. 1, 159 (1965).

[9]D. Stoler, Phys. Rev. D 1, 3217 (1970).

[10]E. Y. C. Lu, Lett. Nuovo Cimento 3, 585 (1972).

[11]H. P. Yuen, Phys. Rev. A 13, 2226 (1976).

[12]J. N. Hollenhorst, Phys. Rev. D 19, 1669 (1979).

[13]H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory 26, 78 (1980).

[14]H. Takahasi, Adv. Commun. Syst. 1, 227 (1965).

[15]M. T. Raiford, Phys. Rev. A 2, 1541 (1970).

[16]D. Stoler, Phys. Rev. Lett. 23, 1397 (1974).

[17]G. Milburn and D. F. Walls, to be published.

[18]H. J. Carmichael and D. F. Walls, J. Phys. B 9, L43, 1199 (1976).

[19]H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. A 18, 201 (1978).

[20]G. Leuchs, M. Rateike, and H. Walther, cited by D. F. Walls, Nature 280, 451 (1979).

[21]Walls, Ref. 20.

[22]D. F. Walls, J. Phys. B 13, 2001 (1981); L. Mandel, J. Opt. (Paris) 10, 51 (1979).

[23]H. J. Kimble and L. Mandel, Phys. Rev. A 13, 2131 (1976).

[24]J. H. Shapiro, H. P. Yuen, and J. H. Machado Mata, IEEE Trans. Inf. Theory 25, 179 (1979).

[25]H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory 26, 78 (1980).