Decay of the Zero-Voltage State in Small-Area, High-Current-Density Josephson Junctions

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(Received 6 May 1981)

We have measured the distribution in current for the onset of voltage in small-area Josephson junctions for temperatures down to ~1.6 K. The lifetime of the $\langle V\rangle = 0$ state for our highest current density junctions becomes temperature independent for $k_BT \lesssim \hbar\omega_0/20$, a T much less than the WKB prediction (ω_0 is the junction resonant frequency). This is consistent with the Caldeira-Leggett theory, which includes damping effects on the quantum tunneling rate.

PACS numbers: 74.50.+r, 05.04.+j, 72.70.+m

Many experiments have studied the ramifications of the classical Langevin equation, but few have probed the low-temperature, high-frequency regime where quantum corrections to the picture must be expected. High-current-density Josephson tunnel junctions[1] at temperatures of a few kelvins make possible experimental studies[2] in this regime. Biased in the $\langle V\rangle=0$ state at currents near the critical current, the junctions are sensitive to fluctuations in the terahertz range, centered about their natural frequency, ω_0. Fluctuation effects may be observed by sweeping the junction current through the critical current and noting the distribution, $P(i)$, of current values at which voltage sets in.[3] $P(i)$ should depend on whether $k_BT \gg \hbar\omega_0/2$ (classical regime) or $k_BT \ll \hbar\omega_0/2$ (quantum regime).[4-6] We have measured the width, σ, of $P(i)$ for junctions in the quantum regime and found that quantum effects do not appear until $k_BT<\hbar\omega_0/20$, in contrast to a WKB limit of $k_BT<\hbar\omega_0/7$.

To interpret our experiments we invoke the usual model of such junctions, a "perfect" Josephson junction with current $i=i_c\sin\varphi$ in parallel with resistance R (which we consider constant) and capacitance C.[7] Here φ is the phase difference across the junction, and i_c is the critical current. The Langevin equation

$$M[d^2\varphi/dt^2+\gamma d\varphi/dt]=-dU/d\varphi+f(t) \qquad (1)$$

then describes the position and momentum coordinates φ and $Md\varphi/dt=2\pi MV/\Phi_0$ where V is the voltage across the junction, $\gamma=(RC)^{-1}$, $M=C(\Phi_0/2\pi)^2$, and $\Phi_0=2.07\times10^{-15}$ Wb the flux quantum. The phase moves in the potential $U(\varphi)=-M\omega_j^2\times[\cos\varphi+x\varphi]$; ω_j is the junction plasma frequency ($\omega_j^2=2\pi i_c/\Phi_0C$), and x is the ratio (i/i_c) of the junction current to its critical current. For the noise term $f(t)$ the fluctuation-dissipation theorem gives the spectral density $\langle f^2(\omega)\rangle =\gamma M\hbar\omega\coth(\hbar\omega/2k_BT)$, usually interpreted classically as the high-temperature limit $\langle f^2(\omega)\rangle =2\gamma Mk_BT$.

Without fluctuations, the phase can be trapped in a local minimum at $\varphi=\sin^{-1}x$ if $x<1$, and is prevented from escape by an energy barrier $\Delta E =2M\omega_j^2[(1-x^2)^{1/2}-x\cos^{-1}x]$. No dc voltage develops across the junctions since $\langle d\varphi/dt\rangle=0$. For $x>1$ the potential wells vanish and the phase increases at a speed determined by the damping and the overall steepness of the potential. This corresponds to the finite voltage state of the junction. We are interested in x slightly less than 1, in the presence of fluctuations. The phase is now restrained by a low barrier (inset, Fig. 1) which, in the absence of fluctuations, would hold $\langle V\rangle=0$ until i exceeds i_c. Fluctuations, however, cause premature transitions to the $\langle V\rangle=0$ state at some distribution of values of i.

Fulton and Dunkleberger[3] studied the case $\hbar\omega_0/2\ll k_BT$ where the classical noise term applies. In the low-damped limit, $\omega_0\gg\gamma$, the phase oscillates in the well with attempt frequency $\omega_0=\omega_j(1-x^2)^{1/4}$ and has probability $\exp(-\Delta E/k_BT)$ of surmounting the barrier on each attempt, switching the junction to the voltage state. The lifetime θ for the $\langle V\rangle=0$ state can be expressed as $\theta =(\omega_0/2\pi)\exp(-\Delta E/k_BT)$. The distribution in i of switching events,

$$P(i)=[\theta(i)(di/dt)]^{-1}[1-\textstyle\int_0^i P(u)du], \qquad (2)$$

can be measured by sweeping the junction bias

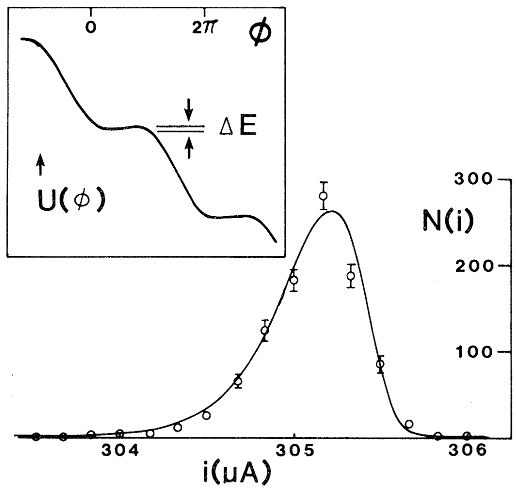

FIG. 1. Measured distribution for $T = 1.6$ K for small high–current-density junction. The solid line is a fit by the CL theory for $R = 20\ \Omega$, $C = 8$ fF, and $i_{c\,\mathrm{FF}} = 310.5\ \mu\mathrm{A}$. The inset is $U(\varphi)$ for $x = 0.8$ with barrier ΔE.

current back and forth many times and recording the events per current interval. The width σ scales like $\sim T^{2/3}$, as observed by Fulton and Dunkleberger.

When damping is not considered and the ground-state energy of the metastable well satisfies $\hbar\omega_0/2 \gg k_B T$, tunneling should take over as the dominant decay mode. A WKB analysis predicts that tunneling should equal thermal escape when $k_B T \cong \hbar\omega_0/7$.[8] (Evidence for phase tunneling in superconducting rings closed by point contacts was reported by den Boer and de Bruyn Ouboter,[9] and Prance *et al.*[10]) According to several recent theories damping hinders quantum tunneling. Caldeira and Leggett (CL)[6] predict a reduction factor $\exp[-AM\gamma(\Delta\varphi)^2/\hbar]$; A is of order unity and $\Delta\varphi$ is the tunneling distance under the barrier. Koch, Van Harlingen, and Clarke[11] (KVC) have used a quasiclassical approach interpreting the quantum limit $\langle f^2(\omega)\rangle = \gamma M\hbar\omega$ of the noise term as real dynamical noise which they simulate with a computer. For underdamped and critically damped junctions, the KVC treatment predicts an effective activation temperature less than $\hbar\omega_0/2k_B$,[12] but quantitative results are not yet available. KVC's Langevin equation can be derived from the equation of motion of the Wigner distribution[13] by neglecting third- and higher-order terms introduced by the anharmonicity of the potential $U(\varphi)$. The neglected terms are independent of damping. However, fluctuations due to the bath increase with damping, and eventually overwhelm the neglected terms.

We looked for departures from classical thermal activation in small Pb-Pb(In) Josephson junctions[1,2] with areas ranging from $\sim 3\times10^{-8}$ to $\sim 5\times10^{-10}$ cm^2, and current densities between 5×10^3 and 5×10^5 A/cm^2. Measurements were made in a carefully shielded environment. To improve noise immunity, an aluminum shield blocked the opening of the stainless steel tube that led to the sample chamber and only two leads were connected to the sample, in series with 1-kΩ metal film resistors in the sample chamber.

Data were collected by a PDP-11/34 computer run in a multichannel analyzer mode. Current supplied to the junction was varied in small increments. The voltage across the sample and the 1-kΩ resistors was amplified and a linear signal, equal to the voltage developed across the resistors, was electronically subtracted. The computer compared this amplified signal to a reference voltage level set between the $\langle V\rangle = 0$ level on the sample and the $\sim$2-mV level that signified a transition to the $\langle V\rangle \neq 0$ state. For each transition detected, a count was added to the memory bin associated with the current level at the transition. The applied current was then swept back through $i = 0$, in opposite polarity and increasing amplitude until a transition was again detected from the $\langle V\rangle = 0$ state to the $\langle V\rangle \neq 0$ state. (The junction remained in the $\langle V\rangle = 0$ state for $\sim$10 msec, longer than the thermal time constant of the sample.) This process was repeated, usually 1000 times, yielding a distribution for positive transitions and one for negative transitions. The widths of the distributions were calculated, and the average apparent critical current of the sample was determined from the difference in current value between the centers of the two distributions. A typical distribution is shown in Fig. 1.

To test the immunity of the system to external noise, we measured $P(i)$ for junctions with low i_c and areas $> 10^{-8}$ cm^2 for which no quantum effects were expected. In this low-damped, classical regime, for small i_c, σ is very sensitive to external noise since it scales approximately as $i_c^{1/3}$. We measured σ as narrow as 0.03 μA. Since most measurements were made for values of i_c where we expected $\sigma > 0.1\ \mu$A, we concluded that external noise was not a problem.

We next measured $P(i)$ for a junction expected even by WKB analysis to behave classically for $T \gtrsim 2$ K. The junction i_c ranged from 111 μA at 4.2 K to 131 μA at 1.6 K, its area $\sim 3\times10^{-8}$ cm^2,

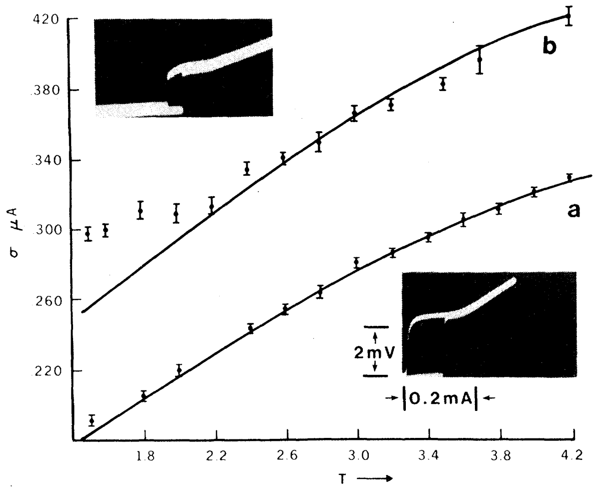

FIG. 2. Measured distribution widths σ vs T for two junctions with current sweep of $\sim 400\ \mu A/\text{sec}$. Curve a is lower current density junction data and curve b is higher density junction data. The traces adjacent to the plots are the corresponding I-V characteristics at 4.2 K. The scales are the same for both traces.

and data on the capacitance of Pb(In)-oxide junctions[14] allow us to estimate $C \sim 200$ fF. These parameters give $\omega_j/2\pi \sim 250$ GHz, so that $\hbar\omega_j/k_B \sim 12$ K. Curve a in Fig. 2 shows σ versus T for this junction. The adjacent trace is its I-V characteristic at 4.2 K. Each datum point on both curves is averaged from ~ 10 distributions of 1000 transitions. The solid line represents classical theory [Eq. (2)]. The only fitting is to the fluctuation-free critical current (i_{cFF}). We used classical theory to calculate the reduction in i_c from fluctuations, and inferred i_{cFF} which was typically 2% to 10% larger than the apparent i_c. It is evident from the data that σ for this junction is in excellent agreement with the classical prediction for T as low as 1.6 K.

Curve b in Fig. 2 shows data for a much smaller, high-current-density junction of area $\sim 5 \times 10^{-10}$ cm^2, for which i_c ranged from 254 μA at 4.2 K to 311 μA at 1.6 K. It is representative of data taken for several such junctions, whose σ versus T plots are qualitatively the same. The adjacent trace shows the I-V characteristic at 4.2 K. From the junction geometry and calculations based on a nonlinear RSJ (resistively shunted junction) model that match I-V characteristics to junction parameters,[15] we estimate 5 fF $< C < 15$ fF. The uncertainty in C is that of the area of the junction itself and its parasitic capacitance. With these parameters $\hbar\omega_0/k_B$ is 70–110 K. At $T \sim 2$ K, x for transitions was ~ 0.98, so that $\hbar\omega_0/k_B \sim 40$ K. Since $\Delta E \sim 20 k_B T$, there is a single bound state in the potential well. The good agreement with the classical theory for $T > 2.2$ K shows that the simple WKB prediction, a broad, temperature-independent width below $T = 8$ K, is not valid.

TABLE I. T at which CL escape equals thermal escape for several values of R and C and for $i_c = 310\ \mu$A and $di/dt = 400\ \mu$A/s. The numbers in brackets are the distribution widths in microamperes.

R (Ω) \ C (fF)	5	8	11
10	1.3 [0.187]	1.2 [0.183]	1.2 [0.179]
20	2.3 [0.316]	2.1 [0.298]	2.0 [0.285]
30	3.2 [0.404]	2.8 [0.372]	2.6 [0.351]
∞	10.9 [0.813]	8.2 [0.683]	6.8 [0.609]

Deviation from the classical prediction is evident below ~ 2.2 K. The data can be fitted by the CL theory for $R = 20\ \Omega$ and $C = 8$ fF, if we use the same prefactor and exponential factor, A, as for the low-damped case. The solid line of Fig. 1 is such a fit of $P(i)$ with $i_{\text{cFF}} = 310.5\ \mu$A and $di/dt = 400\ \mu$A/sec. The agreement of the data with the skewed theoretical distribution is further evidence that σ is not significantly broadened by external noise. (Table I gives the calculated values, σ, of the tunneling distribution, with use of CL, and T at which tunneling and thermal escape rates are equal.) In fitting the data, the relevant damping resistance is the low-voltage quasiparticle resistance which is hard to determine because of the low hysteresis of the I-V characteristic. Similar junctions, measured in a cryostat where a magnetic field suppressed the supercurrent, exhibited a low-voltage resistance between 3 and 7 times the normal-state resistance. Therefore, we estimate R to be between 15 and 35 Ω, which is consistent with a CL interpretation of our data.

In summary, we have measured the distribution for decay of the $\langle V \rangle = 0$ state in Josephson junctions in a regime where quantum tunneling is expected to modify the decay rate. We observed such a modification at $T < 2$ K, a temperature much lower than the quantum energy corresponding to the maximum frequency response of a high-current-density junction. The results are consistent with a prediction that dissipation reduces the rate of tunneling barrier escape.

We are grateful to T. A. Fulton, W. J. Skocpol, and R. H. Koch for important discussions. We particularly thank T. D. Clark whose interest in quantum tunneling motivated this work.

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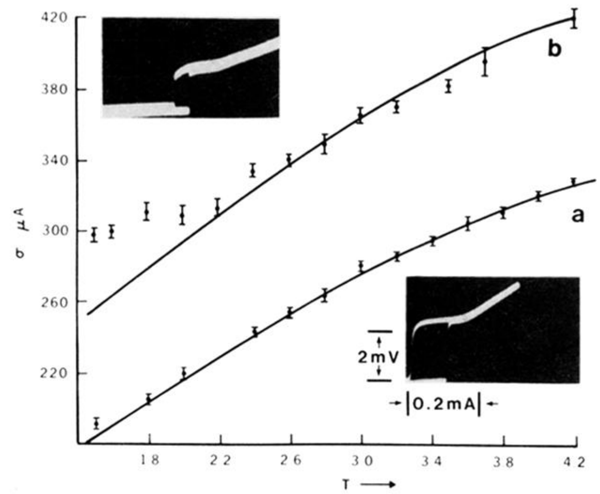
ERRATUM

HOT-CARRIER THERMALIZATION IN AMORPHOUS SILICON. Z. Vardeny and J. Tauc [Phys. Rev. Lett. 46, 1223 (1981)].

We found that the reported value of the depolarization factor ρ smaller than 1 was due to an experimental artifact. The correct value is $\rho = 1$ (no polarization memory). Curves (b) in Figs. 1 and 2 should be multiplied by 4/3 and 1/0.7, respectively; curves (c), which are now equal to (a) − (b), are not changed. This correction does not affect the analysis of the data and the conclusions.



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