Confinement of High-Energy Trapped Particles in Tokamaks

R. J. Goldston, R. B. White, and A. H. Boozer
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544
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The banana orbits of high-energy trapped particles in tokamaks are found to diffuse rapidly in the radial direction if the toroidal ripple exceeds a low critical value. During this diffusion the energy, the magnetic moment, and the value of the magnetic field strength at the banana tips are conserved.

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The confinement of high-energy trapped particles in a tokamak is important for thermonuclear ignition and a number of plasma heating scenarios. Toroidal ripple due to the finite number of toroidal field coils is well known to have a deleterious effect on trapped-particle confinement. The condition generally found for large ripple effects on particle orbits is that the ripple δ be strong enough to form secondary magnetic wells[1,2]

$$\delta > (\epsilon/Nq)|\sin\theta_t|, \tag{1}$$

with N the number of toroidal field coils, ϵ the inverse aspect ratio, q the safety factor, and θ_t the poloidal angular position of the turning point. In this paper we find a more stringent limit on ripple for particles with large banana orbits

$$\rho_b \gtrsim \sqrt{2}r/\pi^{3/2}(Nq)^{1/2}, \tag{2}$$

with r the local radius, and ρ_b equal to $(2\epsilon)^{1/2}$ times the gyroradius in the poloidal magnetic field. Typically this includes all banana-trapped fusion-product α particles, and in some machines may also include some beam-injected particles or radio-frequency–generated suprathermal particles as well. The fraction of banana-trapped particles is of course $(2\epsilon)^{1/2}$ of the total. This limit on the ripple is approximately

$$\delta \lesssim [(\pi Nq/\epsilon)^{3/2}\rho q']^{-1}, \tag{3}$$

with ρ the gyroradius in the full magnetic field and $q' = dq/dr$. It is generally much more restrictive than Eq. (1), and presents a serious design consideration for future tokamaks.

To understand this ripple limit, consider the model magnetic field

$$B = B_0(1 + \epsilon\cos\theta + \delta\cos N\varphi), \tag{4}$$

with B_0 a constant and $\epsilon = r/R$ the local inverse aspect ratio. We assume $Nq \gg 1$ and that the ripple is too weak to form secondary wells. In this limit one finds that the dominant effect of the ripple is to cause the ordinary banana-trapped particles to have a small radial jump near each turning point.[2,3]

The radial motion of a particle which is dominantly following the nearly axisymmetric magnetic field, and also experiencing the usual axisymmetric ∇B and curvature B drifts, is given by

$$\partial r/\partial\theta = RqV_{\mathrm{dr}}/v_{\parallel}, \tag{5}$$

where $V_{\mathrm{dr}} = (v_{\parallel}^2 + \frac{1}{2}v_{\perp}^2)\sin\theta/(\Omega_{ci}R)$. In an axisymmetric system this drift averages to zero along an orbit. In a nonaxisymmetric system, however, as a particle moves along a field line $v_{\parallel}$ oscillates as a result of the ripple in B, giving rise to an additional oscillating radial excursion. The variation in $v_{\parallel}$ is given by

$$\tilde{v}_{\parallel} = -(v_{\perp}^2/2v_{\parallel})\delta\cos N\varphi. \tag{6}$$

Along most of the particle orbit the small radial excursions due to $\tilde{v}_{\parallel}$ are self-canceling because of the oscillation in $\cos N\varphi$. Near the turning points, however, where $v_{\parallel}$ goes to zero, the excursions are largest and do not necessarily cancel, because of the presence of an end point. Making the simplifying assumption that most of the radial excursion does occur close to the turning point, one can integrate along unperturbed trajectories analytically to find that the displacement after a single bounce is given by

$$\Delta r = \Delta\cos N\varphi_t, \tag{7}$$

where

$$\Delta = (N\pi/\sin\theta_t)^{1/2}(q/\epsilon)^{3/2}\rho\delta, \tag{8}$$

with θ_t and φ_t the poloidal and toroidal position of the turning point. As the banana orbit moves about, the particle conserves energy

$$E = \tfrac{1}{2}mv_{\parallel}^2 + \mu B$$

and magnetic moment μ. Consequently the turning point is always at the same value of magnetic field strength, B, which limits the range of inward radial motion for some particles. The toroidal angle between the two turning points, φ_b

$=2q\theta_t$, must have the radial derivative

$$\frac{d\varphi_b}{dr}=2\theta_t\frac{dq}{dr}+\frac{2q}{r}\frac{\cos\theta_t}{\sin\theta_t} \tag{9}$$

to conserve B at turning points. If the radial jump Δ is large enough to change φ_b by a significant fraction of a ripple phase,

$$N\varphi_b'\Delta \gtrsim 1, \tag{10}$$

where primes denote differentiation with respect to r, then the ripple phase at the turning point becomes stochastic and the banana orbit diffuses at the rapid ripple plateau rate[2,3] $D=\Delta^2/\tau_b$ with τ_b the time to execute a complete banana orbit. This rate is generally much faster than the loss rate for the bulk plasma. The particle would be lost typically in a few thousand bounces, i.e., in a shorter time than required for thermalization.

To treat the problem more precisely, consider the map of a banana orbit using the radii and toroidal angles of its turning points as coordinates. Let $\varphi_p(r)$ be the amount the banana orbit precesses toroidally while moving between turning points. An area-preserving map of the banana orbit motion is given by (see Fig. 1)

$$r_{j+1}=r_j+\Delta\cos N\varphi_j,\quad \varphi_{j+1}=\varphi_j+\varphi_b(r_{j+1})+\varphi_p(r_{j+1}); \tag{11}$$

$$r_{j+2}=r_{j+1}+\Delta\cos N\varphi_{j+1},\quad \varphi_{j+2}=\varphi_{j+1}-\varphi_b(r_{j+2})+\varphi_p(r_{j+2}). \tag{12}$$

Note that the toroidal angle between turning points

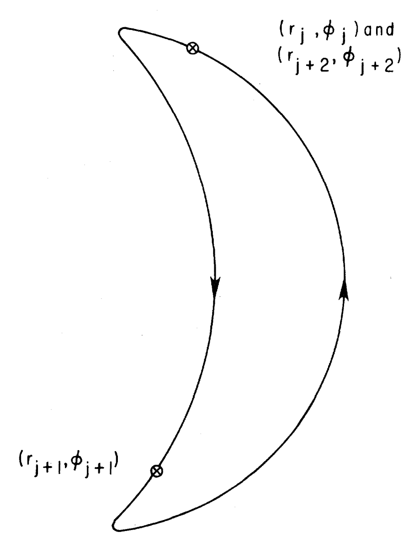

FIG. 1. Banana orbit coordinate system.

φ_b enters with opposite sign in the second set of map equations. Otherwise, they are identical.

We have studied this map numerically to determine the transition to stochastic behavior. We find that the transition value of Δ, $\Delta_{\rm crit}$, varies periodically in the quantities φ_b and φ_p in a complicated way which also depends on φ_b' and φ_p'. We have not been able to find an analytic expression which describes this dependence. We have verified numerically, however, that the transition value is always given, within a factor of 2, by a composite of the Chirikov stochasticity criteria[4] for the two parts of the map. From the first half of the map this criterion gives

$$N|\varphi_p'+\varphi_b'|\Delta<1, \tag{13}$$

and from the second half

$$N|\varphi_p'-\varphi_b'|\Delta<1. \tag{14}$$

The composite condition is then

$$N(|\varphi_p'|+|\varphi_b'|)\Delta<1. \tag{15}$$

We find numerically that the transition to stochasticity occurs for Δ even smaller than this, i.e., the periodic oscillations in the transition value reduce $\Delta_{\rm crit}$ below the value given by Eq. (15) by as much as a factor of 2 for particular values of φ_b and φ_p.

In a tokamak in general $|\varphi_p'|<|\varphi_b'|$, and thus using Eq. (9) we find for the majority of trapped banana particles ($\theta_t\approx\pi/2$) the approximate condition given by Eq. (3). In the extreme limit $\varphi_p'\to 0$, our map is mathematically equivalent to one studied by Karney[5] in connection with lower hybrid heating. He found similar results for the transition to stochasticity.

In conclusion, we have found a new restriction on the magnitude of the ripple in a tokamak, which is generally significantly more severe than that given by the requirement that there be only small regions of ripple trapping. The confinement of fusion-product, banana-trapped α-particles requires a ripple magnitude $\delta\lesssim 0.3\%$ in tokamak reactor designs presently being considered, such as the proposed International Tokamak Reactor, INTOR. This proves to be a serious design consideration.

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Influence of Neutral-Beam Injection on Impurity Transport in the ISX-*B* Tokamak

R. C. Isler, L. E. Murray, S. Kasai,[a] D. E. Arnurius, S. C. Bates, E. C. Crume, J. L. Dunlap, P. H. Edmonds, E. A. Lazarus, M. Murakami, V. K. Pare, M. J. Saltmarsh, D. W. Swain, and C. E. Thomas

Fusion Energy Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

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Observations of radiation from iron and from argon used as a test gas indicate that co-injection inhibits impurity accumulation in the interior of ISX-*B* (impurity study experiment) tokamak discharges, but counter-injection enhances accumulation. These results agree qualitatively with recent theoretical calculations.

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One of the major impediments to achieving net power production from a fusion device may be the presence of impurities than can adversely affect the plasma. Because of this potential problem, a great deal of attention has been focused on methods of controlling the generation of impurities or of minimizing the transport of contaminants into the plasma. Some novel theoretical ideas within the framework of neoclassical theory have emerged recently showing that co-injection of neutral beams (beam current in the same direction as plasma current) can reduce the usual inward transport, whereas counter-injection can produce the opposite behavior.[1,2] These effects should be strongest near the center of the plasma where the momentum transfer from the beams and the plasma rotation are the largest. In this paper we discuss experimental results indicating that impurity behavior consistent with these theories is seen in the ISX-*B* (impurity study experiment) tokamak, although close quantitative comparisons are not yet possible.

Impurity transport in tokamaks is a notoriously complicated problem. In addition to the well understood classical and neoclassical processes,[3] which by themselves would cause impurities to accumulate in the interior of Ohmically heated plasmas, it is usually necessary to invoke an empirical, anomalous diffusion process to explain why accumulation proceeds slowly[4] or why it does not occur at all. If the anomalous transport is large enough, it is not expected that neoclassical, beam-induced effects can be detected. But in the ISX-*B* tokamak impurities do accumulate in Ohmically heated deuterium discharges,[5] and as a result, distinct changes of transport resulting from neutral-beam injection can be observed.[6] Similar changes take place in hydrogen discharges, but they are less distinctive. Anomalous transport is apparently more important in hydrogen discharges than in deuterium ones because accumulation does not occur in Ohmically heated hydrogen plasmas; i.e., the impurity confinement time is shorter than the duration of the discharge.

Typical results for the radiation of argon introduced as a test gas during Ohmically heated discharges are shown in Fig. 1. These data provide a reference for later comparison with the results obtained when using neutral-beam injection. The

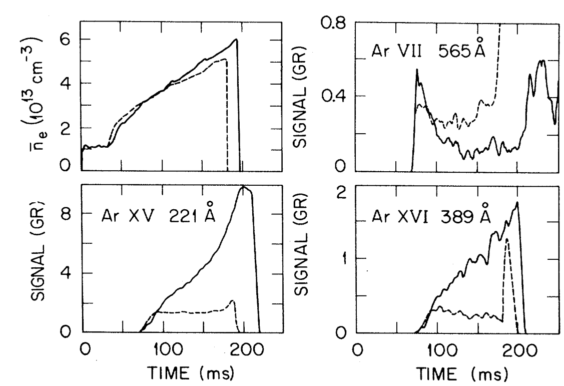

FIG. 1. Line-averaged electron densities and argon line radiation for Ohmically heated discharges in deuterium (solid lines) and in hydrogen (dashed lines). Argon is introduced in a 4-ms pulse at 60 ms.