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Monopole Condensation and the Lattice–Quantum-Chromodynamics Crossover

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We propose the condensation of dynamical monopoles and their strings as the mechanism responsible for the rapid crossover from weak to strong coupling in SU(2) lattice gauge theory. By adding to the Wilson action a chemical potential for monopole formation, we are able to study the crossover in an extended phase plane. We find a critical value for the potential, related to the transition in a dual Ising model, above which monopoles are absent. Support for this picture is garnered from Monte Carlo simulations.

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Recently, there has been considerable interest in the SU(2) lattice gauge theory crossover from strong to weak coupling. Both strong-coupling methods[1] and Monte Carlo simulations[2] provide evidence for a fairly sharp transition at around $\beta = 2.2$. It is at this point that the theory changes its behavior and begins to exhibit the asymptotically free nature of the continuum limit. It is therefore important to understand the mechanism responsible for this crossover.[3]

We propose that the SU(2) crossover is caused by the condensation of dynamical Z_2 monopoles and their associated strings. We will first discuss the nature of these topological objects and show how their density provides a good signal for this transition. Then, we consider a wider class of models by introducing a chemical potential for the monopoles and study the behavior of the theory in the resulting phase plane. In particular, we find a critical value of the chemical potential, determined by the $d=4$ Ising transition, above which monopoles never condense. Finally, we relate these models to the phase transition in an SO(3) model.

Consider the standard four-dimensional SU(2) lattice gauge theory defined by the Wilson action

$$Z = \prod_l \int dU_l \exp \tfrac{1}{2}\beta \sum_p \operatorname{tr}(U_p), \quad U_p = \prod_{l \in \partial p} U_l, \tag{1}$$

where the integration is over SU(2). This theory contains field configurations such that the product of the trace of all the plaquettes bounding a given cube may have either sign. Specifically, define[4]

$$M[c] = \tfrac{1}{2}(1 - \sigma_c), \tag{2}$$

where $\sigma_c = \operatorname{sgn} \prod_{p \in \partial c} \operatorname{tr}(U_p)$. We will refer to $M[c] = +1$ configurations as having a dynamical monopole with its world line passing through the cube c. Obviously, this monopole has a Z_2 character; $M[c]$ is either 0 or 1.

Notice that an isolated monopole has a Dirac string corresponding to a path of plaquettes with $\operatorname{sgn}(\operatorname{tr}U_p) = -1$ extending to infinity. Clearly, the only cube with $M[c] \neq 0$ will be the one at which the string ends; i.e., at the monopole source. These strings, of course, are dynamical objects in their own right. It is also possible to have closed strings (a co-closed set of "flipped" plaquettes), or vortices, as they are also called.

These topological objects have a number of important features. First, Z_2 monopoles are present in an SO(3) gauge theory.[5] They are in fact a result of $\pi_1(\mathrm{SO}(3)=\mathrm{SU}(2)/Z_2)$ being Z_2. The difference between SU(2) and SO(3) is that in SO(3) the string is invisible, as the action goes like $(\mathrm{tr}U_p)^2$. In SU(2), however, the action is sensitive to the sign of the trace, and thus there is no continuum analog of these monopoles.

Conversely, in Z_2 gauge theory, it is the monopoles that do not exist, because every link occurs twice, in opposite orientations, in the product around the cube (2). Thus, any Abelian factor cancels, leaving only closed vortex strings ($d=3$) or closed vortex sheets ($d=4$). The Abelian monopoles occurring in the double Coulomb gas description[6] of Z_2 are not topological in nature; they are in fact topologically trivial combinations of two of these monopoles, with zero flux modulo 2.

We can exhibit these objects more precisely by rewriting, following Mack and Petkova[7] and Tomboulis,[8] the Wilson action as follows:

$$Z=\int \prod_l dU_l \sum_{S_p=\pm 1} \prod_c (1+S_c\sigma_c) \times \exp \tfrac{1}{2}\beta \sum_p S_p |\mathrm{tr}(U_p)|, \qquad (3)$$

where

$$S_c=\prod_{p\in\partial c} S_p, \quad \sigma_c=\prod_{p\in\partial c}\sigma_p,$$

and the integration is restricted to $\mathrm{SO}(3)=\mathrm{SU}(2)/Z_2$. Thus, the SU(2) theory is equivalent to an SO(3) theory coupled via the Z_2 string variables, S_p, to the density of flipped cubes, our monopoles. We note, in passing, that the 't Hooft loop[9] acts by flipping the sign $(1+S_c\sigma_c \to 1-S_c\sigma_c)$ on each of its cubes and so can be considered an external source of monopoles.

The expression (3) can be used to define a more general class of SU(2) models by introducing a chemical potential for dynamical monopoles. This is just an extra term, λS_c, in the action. The $\lambda\to\infty$ limit corresponds to a modified SU(2) model introduced by Mack and Petkova (MP).[4] It is important to note that the weak-coupling expansion is insensitive to the parameter, as a result of the fact that monopoles have energy $\sim\beta$ and disappear in the naive continuum limit.

To investigate the role of these objects, we ran Monte Carlo simulations on a 4^4 lattice. We sweep through the lattice 100 times for each value of β and λ. Thus, a naive estimate of the statistical accuracy of our results yields $1/(4^4\times 100)^{1/2}\sim 1\%$. Of course, near any second-order transitions, the large fluctuations will degrade this accuracy.

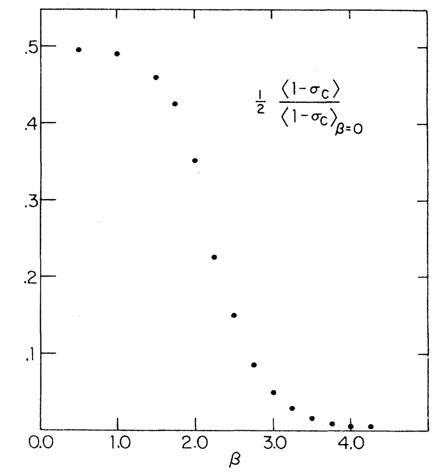

FIG. 1. Monopole density for unmodified Wilson theory.

In Fig. 1, we have displayed the monopole density for the usual ($\lambda=0$) theory. We see a dramatic crossover from zero to its limiting value for strong coupling. From the maximum gradient, we estimate $\beta_{\mathrm{tran}}=2.20\pm0.05$, which agrees precisely with the point at which the average plaquette undergoes its most rapid transition. We are thus led to the idea that the monopoles (and their ubiquitous strings) are responsible for the transition.

To further test this idea, we examine the average plaquette, $1-\frac{1}{2}\langle \mathrm{tr}U_p\rangle$, in the MP model ($\lambda=\infty$) as a function of β. This is presented in Fig. 2, with the curve for $\lambda=0$ from Lautrup and Nauenberg[2] for comparison. We see that the monopoleless version follows the weak-coupling curve well past $\beta\approx 2.2$, where the standard Wilson theory curves up to meet strong coupling. There is now, however, a sharper transition at $\beta\approx 0.9$, which can be easily understood. It is due to the onset of the vortices and is a remnant of the phase transition in the Z_2 theory. Since the MP model is just a Z_2 model with fluctuating coupling given by $\beta_{\mathrm{eff}}=\frac{1}{2}\beta|\mathrm{tr}U_p|$, which varies between 0 and β and is about $\beta/2$ on average at the transition, we can get a rough estimate of this transition by taking $\beta_{\mathrm{tran}}/2=\beta_{\mathrm{crit}}$ or $\beta_{\mathrm{tran}}\approx 0.88$.[4,10] This is in good agreement with the data. Furthermore, the vortex string density, $V[p]=\frac{1}{2}(1-\mathrm{sgn\,tr}U_p)$, undergoes a rapid drop at the transition. We could fur-

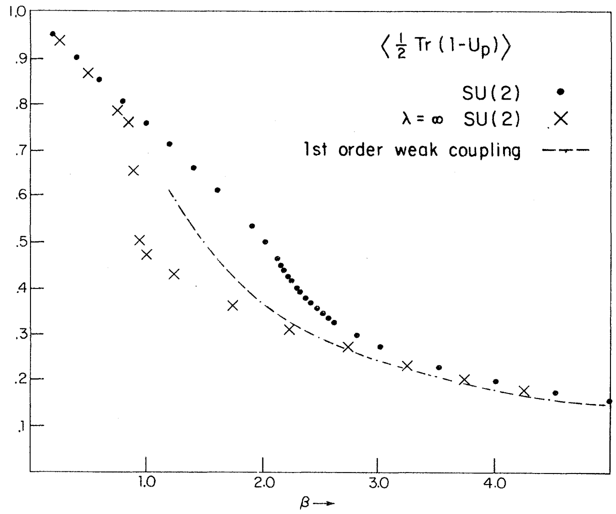

FIG. 2. Average plaquette for MP model (×).

ther modify the action by introducing a constraint requiring $V[p]=0$ for all plaquettes. Doing this removes any hint of a transition, lending additional support to the picture.

We now examine the model at intermediate λ. At $\beta=0$, Eq. (3) for the partition function reduces to

$$\prod_{p}\sum_{S_p=\pm 1}\exp\lambda S_c + A'(S_c),$$

where

$$\exp A' = \int \prod_{l} dU_l \prod_{c} (1+S_c\sigma_c).$$

If we ignore A', this theory is exactly dual to the four-dimensional Ising model with coupling $2K = -\ln\tanh\lambda$. This model has a second-order phase transition at $K\approx 0.15$, or $\lambda\approx 0.96$. We may approximate A' to leading order by $\langle\sigma_c\rangle S_c$ where

$$\langle\sigma_c\rangle = \int \prod_{l\in\partial c} dU_l\,\mathrm{sgn}(\prod_{p\in\partial c} U_p).$$

This is simply related to the limiting value for the monopole density at $\beta=0$ for the Wilson ($\lambda=0$) theory, $\langle M\rangle = \frac{1}{2}-\frac{1}{2}\langle\sigma_c\rangle$. We compute this term[11] using the character expansion of the sign function and find $\langle\sigma\rangle\approx 0.023$. This implies a $\lambda_{\mathrm{eff}}=\lambda+0.02$, or $\lambda_{\mathrm{crit}}\approx 0.94$. One may also verify that higher corrections to the effective action will be small.

In Fig. 3, we show a graph of the monopole density $\langle M\rangle$ vs λ for $\beta=0$. The data provide direct evidence for the second-order phase transition at a value of $\lambda\approx 1$. Thus, there is a critical λ_c above which the monopoles are frozen out.

We can also perform a strong-coupling expansion around this Ising system. The result of this

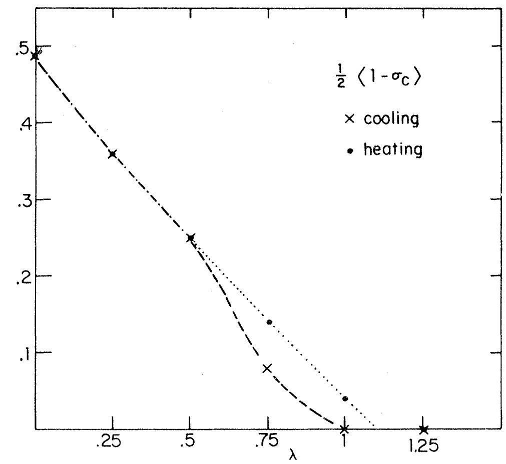

FIG. 3. Monopole density on Ising line ($\beta=0$).

calculation is,[11] to lowest order, a renormalization of λ_c: $\lambda_c(\beta)\approx 0.94-8.3\times 10^{-3}(\beta/2)^6$. Thus, the critical line is rather flat, and moves downward, as depicted in Fig. 4.

The picture of the β-λ phase plane is then roughly as follows. At $\lambda=\infty$, there is a rapid transition due to vortices. This transition remains at $\beta\approx 0.9$ until $\lambda\approx 1$, where the monopoles start making themselves felt. The effect of the monopoles is to push the transition to larger β and also to weaken the transition, turning it into a nonsingular crossover at around $\lambda=0.4$. This then extends to the $\lambda=0$ axis, where the crossover is the well-known one at $\beta=2.2$. Preliminary Monte Carlo data support the λ-β phase boundaries sketched in Fig. 4. In a future publication,[11] we will understand the detailed physics of this phenomenon by relating our combined monopole-vortex dynamics to the familiar[12] Z_2-Higgs theory.

As mentioned previously, these monopoles are also present in an SO(3) theory. Recently, there has been evidence of a strong phase transition in SO(3),[5,13] which is also believed to be related to monopole condensation. We may interpolate between SU(2) and SO(3) by introducing the Z_2 breaking by the replacement

$$\prod_{c}(1+\sigma_c S_c)\to\prod_{c}\exp\eta(S_c\sigma_c-1),$$

reducing the energy of the string. As $\eta\to 0$, the theory becomes the Villain form of SO(3) considered by Halliday and Schwimmer.[5] Thus, the monopole condensate is directly connected to the one discussed in Ref. 5. The β-λ-η space is depicted in Fig. 4, with a reasonable guess for the behavior of the phase boundary (solid lines). In the $\lambda=0$ plane, the crossover should change from

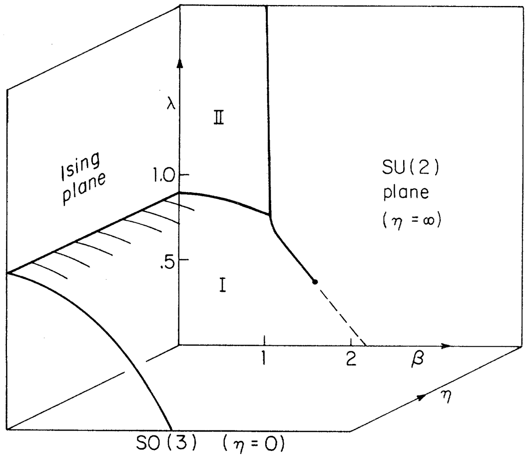

FIG. 4. Monopole (I) and vortex (II) regions in the phase space.

a real (probably first-order) transition to a softer, presumably singularity-free, one as the vortex strings gain mass with $\eta \to \infty$.

It is crucial to realize that we are not saying that the modified models are nonconfining. Mack and Petkova[14] and Yaffe[15] have advanced arguments claiming that it is the flux spreading on all distance scales that is relevant for confinement (which is a statement about the behavior of asymptotically large Wilson loops) and that therefore our local changes to the action will not destroy the area-law behavior of the theory. In fact, our results suggest that the weak-coupling regime is insensitive to the presence of nonzero λ. This would imply that the Wilson-loop area law, shown by Creutz to persist into weak coupling, is equally valid for our models.

Thus, strictly speaking, our results do not touch at all on the problem of confinement, and imply that the crossover at $\beta \approx 2.2$ is a lattice artifact. However, since analogs of our monopoles and vortices exist on larger scales ("fat" vortices, etc.), one might hope that a deeper understanding of their dynamics would shed light on the behavior of the theory at these larger scales. Such questions are currently under investigation.[11]

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