Deeply Bound Composite Fermion with a Dirac Magnetic Moment

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We formulate a model in which a light pointlike composite fermion is obtained dynamically from two heavy relativistic constituents bound by strong, short-range forces with the bound-state mass as low as one-tenth the sum of the constituent masses, while still obtaining the correct Dirac moment for this bound state (a factor of 5 greater than that of the constituents). Other static properties and details of the spectrum are investigated.

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The idea that leptons and quarks are composites of more fundamental, heavier constituents has been recently suggested by *many* authors.[1] These models are constrained by known properties of quarks and leptons. The most stringent dynamical ones, which we shall take as a guide, are as follows: (i) Absence, to a part in 10^8, of any nonelectromagnetic Pauli moment for the electron and muon. (ii) Any form factors for the leptons must have a scale ≥ 100 GeV.[2] (iii) The bound-state spectrum should have no low-lying $j=\frac{3}{2}$ states.[3] (iv) The known leptons couple to the charged weak interactions via a left-handed current such that $g_A/g_V \approx 1$.

Since condition (i) is the most stringent, both as a theoretical requirement and as a constraint on models, we will elaborate on it somewhat. Nonrelativistic intuition leads us to expect that a bound state with mass m_B made out of constituents of mass m_c would have a gyromagnetic ratio of the order of m_B/m_c.[4] For some models this ratio is much smaller than 1, rather than the Dirac value $g=2$. Relativistic effects must play a crucial role in increasing the value of g.[5] It has been *conjectured*[6-8] that if the theory is renormalizable on the constituent level, implying that the constituent fermions have Dirac magnetic moments, then it will be renormalizable on the bound-state level, again implying a Dirac moment for the bound state. Dispersion-relation calculations indicate that the bound state will have a Dirac magnetic moment up to corrections of the order m_B/m_c.[9]

Until now, discussions of composite models have concentrated largely on group-theoretic considerations,[1] magnetic-moment constraints,[4,9] and anomaly conditions.[8] A relativistic dynamic calculation is needed.[10] In this Letter we present such a calculation. We consider a model system which consists of a spin-$\frac{1}{2}$ fermion with mass m_f, a scalar boson with mass m_s, and a heavy gauge field with mass m_g. Both the scalar and fermion carry a "supercharge" which couples them to the gauge boson and in turn to each other. In addition the fermion couples to the electroweak gauge bosons, to the photon with charge e, and to the W's via a left-handed current. Before providing details of the calculation we will state the results and confront them with the four conditions outlined earlier. (i) We can bind the constituents as tight as $m_B \approx 0.1(m_f+m_s)$ while achieving a Dirac moment for the spin-$\frac{1}{2}$ bound state (a factor of 5 larger than the constituent moment). (ii) The size of the bound state is of the order of m_g^{-1}, with m_g as large as four times the typical constituent mass. (iii) No other bound states occur in either the spin-$\frac{1}{2}$ or higher spin configurations. (iv) At the above-mentioned range of m_B the value of g_A/g_V is ~ 0.8.

These results may be viewed both optimistically and pessimistically. On one hand we are able to obtain a deeply bound state with the correct magnetic moment (a nonrelativistic estimate would indicate that it should be a factor of 5 smaller) and a value of g_A/g_V that is close to the expected one, although somewhat small. An acceptable size for the bound state exists and no other unwanted bound states appear. On the other hand, we cannot push this simple model, which neglects multiparticle states, much further in order to obtain a much deeper bound state that has the correct static properties. Another drawback is that even at this state the model is highly unnatural in that its parameters have to be finely tuned.

We now turn to the details of the calculation. The relativistic equation that we use[11] is based on the fermion field equation

$$(\not{p}-m_f)\psi(p)=[g_f/(2\pi)^4]\int d^4q\,\not{A}(p-q)\psi(q)\,, \quad (1)$$

where g_f is the supercharge of the fermion and A_μ is the gauge potential. We study the bound-state wave function $\Psi_{\vec{B},\lambda}(\vec{s})$ by taking the matrix element of Eq. (1) between the bound state, $|\vec{B},\lambda\rangle$, with momentum $\vec{B}$ and z component of spin λ and a state of the scalar particle, $|\vec{s}\rangle$, with momentum $\vec{s}$:

$$\Psi_{\vec{B},\lambda}(s) \equiv \langle \vec{s}\,|\,\psi(B-s)\,|\,\vec{B},\lambda\rangle . \tag{2}$$

From Eq. (1) we obtain

$$[(\not{B}-\not{s})-m_f]\langle\vec{s}\,|\,\psi(B-s)\,|\,\vec{B},\lambda\rangle = [g_f/(2\pi)^4]\sum_n \int d^4q\, \langle\vec{s}\,|\,\not{A}(B-s-q)\,|n\rangle\langle n\,|\,\psi(q)\,|\,\vec{B},\lambda\rangle, \tag{3}$$

where $|n\rangle$ is a complete set of intermediate states. So far this equation is exact; the approximation consists of keeping a select set of intermediate states. As all the constituent particles are heavy, it is reasonable to assume that states consisting of many of these will not be important. As a first approximation we take $|n\rangle$ to be the scalar particle itself. We note that we will obtain an equation linear in $\Psi_{\vec{B},\lambda}(\vec{s})$. Before we write it down we have to specify the matrix element of the gauge potential between scalar states:

$$\langle\vec{s}\,'\,|A_\mu(B-s-q)\,|\,\vec{s}\rangle = (2\pi)^4\delta^4(B-q-s')\frac{g_s(s+s')_\mu}{(s-s')^2-m_g^{\,2}}F(s-s'), \tag{4}$$

where g_s is the supercharge of the scalar particle. We have included a form factor,

$$F(s-s') = -\Lambda^2/[(s-s')^2-\Lambda^2], \tag{5}$$

in order to soften the short-distance behavior. This guarantees that the resulting equations are of the Fredholm type and removes possible restrictions on the size of the coupling characteristic of Dirac equations with potentials having a $1/r$ singularity. Such a form factor arises naturally if the scalar is itself composed of two fermions. The resulting equation is

$$[(\not{B}-\not{s})-m_f]\Psi_{\vec{B},\lambda}(\vec{s}) = +\frac{g_s g_f}{(2\pi)^3}\int\frac{d^3s'}{2\omega_{s'}}\frac{(\not{s}+\not{s}')}{(s-s)^2-m_g^{\,2}}F(s-s')\Psi_{\vec{B},\lambda}(\vec{s}\,'), \tag{6}$$

where $\omega_s^{\,2}=s^2+m_s^{\,2}$ is the energy of the scalar particle.[12]

To solve this equation we first go to the rest frame of the bound state, $B=(m_B,\vec{0})$, and perform a partial-wave analysis. For positive-parity, $j=\frac{1}{2}$ states the four-component spinor Ψ reduces to

$$\Psi_{\vec{0},\lambda}(\vec{s}) = \begin{pmatrix} g(s) \\ \vec{\sigma}\cdot\hat{s}f(s)\end{pmatrix}, \tag{7}$$

and Eq. (6) reduces to two coupled integral equations for g and f. The spectrum at the left-hand side of Eq. (6) determines the region of bound-state masses to be[13]

$$|m_s-m_f| < m_B < m_s+m_f . \tag{8}$$

We shall also need expressions for some of the static properties of the bound state. The charge and magnetic moment are obtained from the matrix element of the electromagnetic current

$$\langle\vec{B}',\lambda'\,|\,\bar{\psi}\gamma_\mu\psi\,|\,\vec{B},\lambda\rangle = \bar{u}_{B'}\left[\gamma_\mu F_1(q^2)+\frac{i\sigma_{\mu\nu}q^\nu}{2m_B}F_2(q^2)\right]u_B \approx (2\pi)^{-3}\int\frac{d^3s}{2\omega_s}\langle\vec{B}',\lambda'\,|\,\bar{\psi}\,|\,s'\rangle\gamma_\mu\langle s'\,|\,\psi\,|\,\vec{B},\lambda\rangle . \tag{9}$$

The last approximate equality is in the same spirit as the approximation that led to Eq. (6), namely, of placing a complete set of states between the $\bar{\psi}$ and ψ and retaining only the scalar particles. We obtain

$$G_E(0)=F_1(0)=(2\pi)^{-3}\int d^3s(2\omega_s)^{-1}(f^2+g^2), \tag{10}$$

which we normalize to 1, and

$$G_M(0)=F_1(0)+F_2(0)=(2\pi)^{-3}\int\frac{d^3s}{2\omega_s}\left\{g^2-\frac{f^2}{3}-\frac{2\omega_s}{s}\left[\frac{2}{3}fg+\frac{s}{3}\left(g\frac{\partial f}{\partial s}-f\frac{\partial g}{\partial s}\right)\right]\right\}. \tag{11}$$

Similarly the matrix element of the axial current yields

$$g_A/g_V=(2\pi)^{-3}\int d^3s(2\omega_s)^{-1}(g^2-\tfrac{1}{3}f^2). \tag{12}$$

The integral equations for f and g were solved numerically by the collocation method. The out-

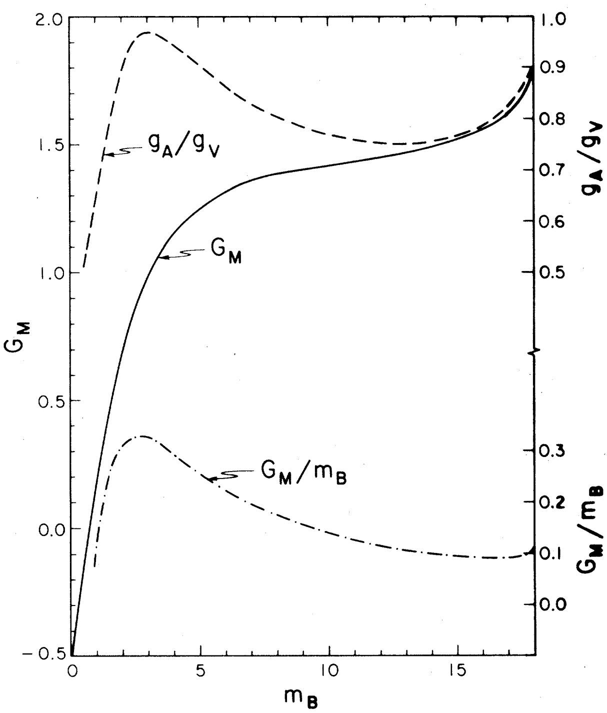

FIG. 1. Static properties of the bound-state fermion as a function of its mass for $m_g = 30$, $\Lambda = m_g$, $m_s = 9.0$, and $m_f = 8.99$. Note that g_A/g_V and G_M/m_B attain maximal values presisely when $G_M = 1.0$.

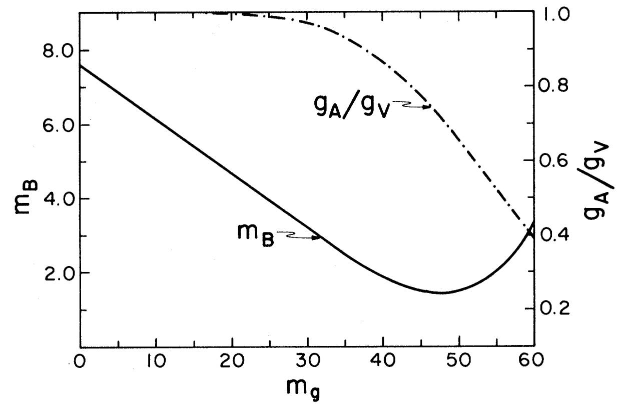

FIG. 2. The value of the bound-state mass at which $G_M = 1.0$ as a function of m_g and $\Lambda = m_g$. The corresponding values of g_A/g_V are also shown.

line of our results was presented above; we shall now discuss some details. Guided by Eq. (8) and desiring to obtain as small an m_B as possible, we chose $m_f \approx m_s$, specifically $m_s = 9.0$ and $m_f = 8.99$ (in arbitrary units). In order to reduce the number of parameters, most of our results are for Λ of Eq. (5) set equal to m_g itself. In the static limit this corresponds to an exponential potential. In Fig. 1 we show the results for $m_g = 30$.

For weak coupling, $m_B \approx 18$ and $G_M \approx m_B/m_f \approx 2$ while $g_A/g_V \sim 1$. As the coupling increases, the binding likewise increases, while both G_M and g_A/g_V decrease. The trend for g_A/g_V reverses and its value starts increasing, reaching a maximum precisely at the point where $G_M = 1$. The value of the magnetic moment $eG_M/2m_B$ is also maximal at this point.[14] The value of the coupling constants at this point is reasonable, $g_s g_f/4\pi \sim 1$. The size of the bound state, as measured by the extent of the wave function in position space, is a local minimum at this point. (All the above observations are true for other values of m_g.) Further increase in binding leads to decreases in G_M and g_A/g_V as well as an increase in the size of the bound state. This is a novel effect in that an increase in binding delocalizes the wave function. An analogous situation exists for a Dirac particle in an attractive square-well potential.[15]

In Fig. 2 we show the value of m_B for which $G_M = 1$ as a function of m_g. We note that m_B may be decreased to $m_B \sim 1.8$ or $0.1(m_s + m_f)$ at $m_g \sim 45$. However the value of g_A/g_V starts decreasing significantly. A further decrease in the value of m_B (keeping $G_M = 1$) may be obtained by changing the value of Λ in Eq. (5). Again the value of g_A/g_V departs further from unity.

We see that the conditions on G_M, g_A/g_V, the spectrum, and particle size, outlined earlier, place severe restrictions on dynamical models of leptons. We have not been able to obtain a bound state with the desired properties whose mass is orders of magnitude smaller than the constituent masses. However, we have been able to obtain a deeply bound state where relativistic effects tend to give the desired pointlike properties in contradiction with nonrelativistic intuition. In addition, Eq. (6) has provided a very useful laboratory for readily studying aspects of deeply bound systems, $m_B \ll \sum m_c$, a subject in which there is essentially no literature.

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[10]Derman, Ref. 1, has done a calculation of a deeply bound state at $m_B = 0$ using the Bethe-Salpeter equation with $m_f \ll m_s$ and a scalar potential. He then uses this solution to estimate the magnetic moment for finite m_B. However, as Derman states, this extension to $m_B \neq 0$ is not justified. Thus his calculations do not represent a dynamical calculation of the magnetic moment.

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[12]We prefer this equation to the Bethe-Salpeter equation for the following reasons: (1) Because of the conservation of the on-shell scalar current, it is gauge invariant. (2) In the limit $m_s \to \infty$ it reduces to the Dirac equation. (3) In the nonrelativistic limit it reduces to the Schrödinger equation with the correct reduced mass. (4) It is easier to solve and interpret.

[13]The absolute value in Eq. (8) results from the scalar field equation.

[14]These coincidences are found numerically to a great accuracy.

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Electric Dipole Moment of the Neutron in a Left-Right-Symmetric Theory of *CP* Nonconservation

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The electric dipole moment of the neutron (μ_n^e) is calculated in the left-right-symmetric theory of Mohapatra and Pati and related to *CP*-nonconserving parameters of the neutral kaon system, yielding the bound $\mu_n^e \gtrsim 10^{-22}|\eta_{+-} - \eta_{00}|$ $e \cdot$cm. Recent measurements of $K_0 \to 2\pi$ amplitudes indicate that μ_n^e may well be of order 10^{-25} $e \cdot$cm which is several orders of magnitude larger than the prediction of the Kobayashi-Maskawa model and only a few times smaller than the current experimental limit $\mu_n^e(\text{expt}) < 6\times 10^{-25} e \cdot$cm.

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The observed asymmetry between left- and right-handed currents in weak interactions has often been speculated to be a low-energy phenomenon.[1] Models of electroweak interactions have been proposed in which the unbroken Lagrangian possesses a left-right symmetry (LRS) based on the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ and the parity nonconservation seen at ordinary energies is implemented by introducing different scales of symmetry breaking for the left- and right-handed gauges.[1] The observed magnitude of *CP* nonconservation has been readily accommodated in such a model by linking it to the suppressed right-handed currents.[2] Unfortunately, the confirmation of these ideas may have to await the construction of accelerators many times more powerful than the ones now available unless one can think of effects that for some reason are suppressed if the underlying interactions are purely left-handed, i.e., $V-A$. One such effect is the