Vortex-Antivortex Pair Dissociation in Two-Dimensional Superconductors

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The nonlinear I-V characteristics below the superconducting transition of high-sheet-resistance Hg-Xe films have been analyzed with use of a model in which the nonlinearity is a consequence of current-induced vortex-antivortex unbinding. The characteristics can be fitted by $V \sim I^{a(T)}$, where $a(T)$ is a measure of the areal superelectron density $n_s(T)$. The universal relations predicted for the Kosterlitz-Thouless-Berezinskii transition in two dimensional superconducting films are obeyed.

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A thin superconducting film with high normal-state sheet resistance $R_N^\square$ is expected to exhibit a vortex-antivortex unbinding transition at a temperature T_c somewhat below the usual BCS critical temperature T_{c0}.[1-3] Such a topological phase transition was first discussed in relation to other two-dimensional systems by Kosterlitz and Thouless[4] and by Berezinskii,[5] and has been experimentally verified for superfluid ^{4}He films.[6] The basic picture is that just below T_{c0} there are a large number of quantized vortices, even in the absence of an external magnetic field, which are free to flow in the film and hence produce resistance. If a pair of vortices with opposite helicity are closer than the transverse penetration depth $\lambda_\perp = \lambda(T)^2/d$ [$\lambda(T)$ is the usual bulk magnetic penetration depth and d is the thickness of the film], their interaction energy is logarithmic in the separation, and the theory predicts that below T_c each such vortex pair will be bound, leading to topological long-range order.

Just below T_c, the areal superelectron density $n_s \equiv n_s^{\text{bulk}} d$ follows the universal[7] behavior

$$z(T) \equiv \pi n_s(T)\hbar^2/4mk_B T = 2 + \pi[(T_c - T)/b(T_{c0} - T_c)]^{1/2}, \qquad (1)$$

where the constant b is of order unity.[3] Evaluating Eq. (1) at T_c using the theory of dirty superconductors leads to the relation

$$T_c/T_{c0} = 1/(1 + 0.173 R_N^\square/R_c), \qquad (2)$$

where $R_c = \hbar/e^2 = 4108\ \Omega$.[1]

Several experiments have recently been carried out to test both universal[8] and nonuniversal[9-11] predictions of the theory. These have provided qualitative and in some cases rough quantitative agreement with several aspects of the theory. Although alternative interpretations have also been suggested,[11] disagreements with theory have generally been attributed in part to effects related to granularity and inhomogeneity in the films under consideration,[11] and in part also to the finite size of the sample and of $\lambda_\perp$.

In this Letter we report measurements of the nonlinear I-V characteristics of these "amorphous" Hg-Xe alloy films below T_c. In this regime, the current breaks up pairs of bound vortices, causing a current-dependent resistance. The curves have been analyzed in terms of the theory of Halperin and Nelson,[3] which allows the determination of $n_s(T)$, T_c, and T_{c0}. The values thus inferred from a number of samples provided an excellent fit to the universal equations (1) and (2) of the theory.

The films studied in the present investigation were condensed onto glazed alumina substrates held at a temperature of 4.2 K, with use of a molecular-beam oven as a vapor source. Detailed procedures as well as the composition-dependence of the metal-insulator transition in the Hg-Xe alloy have been given elsewhere.[12] On the conducting side, Hg(Xe) films exhibit two general classes of superconducting transitions. In one class (discussed separately[13]) the resistance R above T_c exhibits a power-law dependence on reduced temperature and can be explained by a current-scaling analysis.[14] Although structural determination has not been possible for our Hg(Xe) films, such films are presumably granular since their I-V characteristics are similar to those reported for NbN films on which electron microscopy has been carried out.[14] The second class, which includes the films analyzed in the present paper, exhibit an "exponential" dependence of R on T. The success of the analysis which will be presented below argues that the films are likely homogeneous over a substantial range of sheet resistances. Although we have not yet carried out a systematic study, the "granular" films had compositions very close to the metal-insulator transition and were significantly annealed, whereas the "homogeneous" films contained somewhat

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more metal and were quench condensed at 4.2 K or slightly annealed.

The I-V characteristics of a typical homogeneous sample for a range of temperatures near and below T_c are given in the log-log plot of Fig. 1. These data were obtained on a nominally 70 mole% metal Hg(Xe) film with thickness $d = 150$ Å, length $L = 3$ mm, width $w = 1$ mm, and resistance $R_N = 13.8$ kΩ ($R_N = R_N^{\square} L/w$). For $T < T_c$ the curves can be fitted by straight lines over a wide range of voltages and temperatures, corresponding to $V \sim I^a$ with a temperature-dependent exponent $a(T)$. An expression of this form has been derived by Halperin and Nelson for superconducting films,[3] by transcription of the free-vortex nucleation theory for superfluid ^{4}He films.[15] The result can be written in the form

$$V/IR_N \approx x(T)(I/I_0)^{z(T)}, \tag{3}$$

where $z(T)$ is the dimensionless quantity defined in Eq. (1), and $x(T) \equiv 2[z(T) - 2]$. $I_0(T)$ is essentially the Ginzburg-Landau critical current and takes the form near T_c

$$I_0(T_c) = wek_B T_c/\hbar\xi_c, \tag{4}$$

where $\xi_c \equiv \xi(T_c) = \xi(0)[1 - T_c/T_{c0}]^{-1/2}$ is the Ginzburg-Landau coherence length evaluated at T_c.

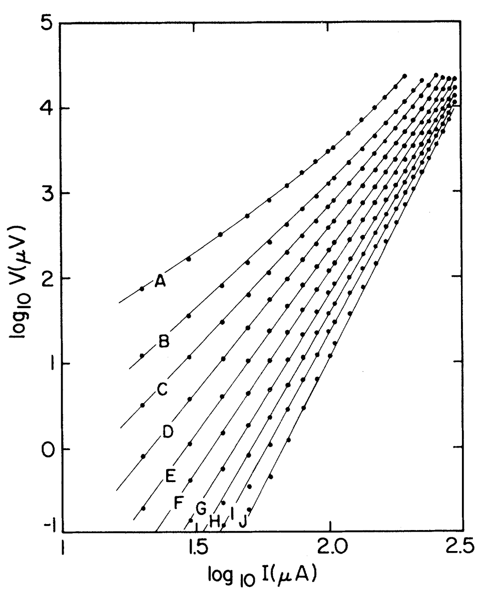

FIG. 1. log V vs log I for sample II for various temperatures: curve A, 2.435 K; B, 2.29 K; C, 2.20 K; D, 2.09 K; E, 2.00 K; F, 1.905 K; G, 1.815 K; H, 1.71 K; I, 1.62 K; and J, 1.5 K. The lines are a guide for the eye. The straight-line fits were made neglecting regions in the upper right of the figure. The lines at temperatures above $T_c = 2.26$ K are curved.

Outside a critical region of order $\Delta T \approx T_{c0} - T_c$ Eq. (3) should still hold, but with the Ginzburg-Landau expression for $n_s(T)$ which results in

$$z_{\rm GL}(T) = 2(T_{c0} - T)/(T_{c0} - T_c). \tag{5}$$

Equation (3) should hold when $I \ll I_0$. The inferred values of I_0 can only be approximate, but from Fig. 1, curve E, one obtains $I_0 \approx 3$ mA which implies $\xi_c \sim 150$ Å and $\xi(0) \sim 60$ Å. These are quite reasonable values. For larger currents or very close to T_c there are logarithmic corrections to Eq. (3).[3]

Although the detailed derivation is rather complicated, one can develop a heuristic explanation of Eq. (3) based on Ref. 15. The internal energy $U(r)$ of a vortex-antivortex pair separated by a distance r such that $\xi \ll r \ll \lambda_\perp$ is given by

$$U(r) = 2q^2 \ln[r/\xi(T)], \tag{6}$$

where $q^2 \equiv (\Phi_0/4\pi)^2 \lambda_\perp^{-1} = \pi n_s \hbar^2/4m$, $\Phi_0 = hc/2e$ is the flux quantum, and the coherence length $\xi(T)$ represents the diameter of the vortex core. In the presence of a supercurrent I_s unbinding results if $r > r_c \approx \hbar/2mv_s \sim 1/I_s$, corresponding to a "saddle point" in the internal energy. Since vortex depairing occurs by thermal activation, the rate of such depairing will go as

$$\exp[-U(r_c)/k_B T] \approx \exp[-2z(T)\ln(r_c/\xi)]$$
$$\approx (r_c/\xi)^{-2z(T)} \approx (I_s/I_0)^{2z(T)}. \tag{7}$$

In steady state the rate of production of free vortices must match the recombination rate, which goes as n_f^2 (n_f is the density of free vortices), recombination being a two-body process. Therefore the current-dependent resistance should go as

$$R = V/I \sim n_f \sim (I_s/I_0)^{z(T)} \tag{8}$$

which is essentially the dependence of Eq. (3). The above analysis breaks down when $r_c < \xi_-(T) \equiv \xi_c \exp[1/x(T)]$, which occurs for large currents or near T_c.[3,15]

Using the obvious association $a(T) = 1 + z(T) = 3 + x(T)/2$, the experimental values of $a(T)$ are plotted in Fig. 2 for three samples. From Eq. (1) values of T_c are chosen by the condition $a(T_c) = 3$. Likewise T_{c0} is determined with use of Eq. (5) by a simple linear extrapolation of $a(T)$ from the "Ginzburg-Landau regime" just outside the critical regime to the axis as shown in Fig. 2 by straight lines. Alternatively, T_c could be determined by a linear fit of $x^2(T)$ with use of Eq. (1) in the "critical region" close to T_c as indi-

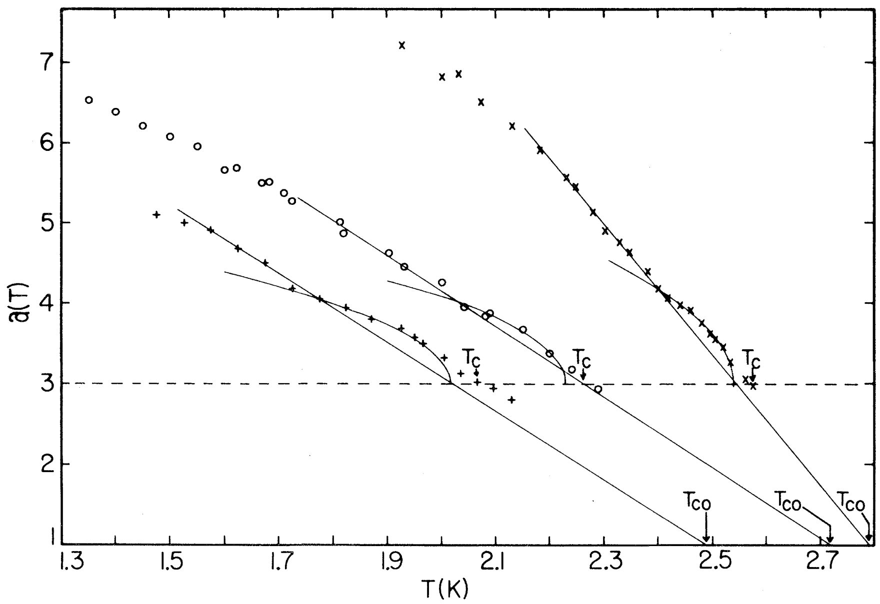

FIG. 2. $a(T) = 1 + z(T)$ for three as-deposited samples with $R_N^\square = 4.4$ kΩ, plusses (sample I); 4.55 kΩ, circles (sample II); 1.6 kΩ, crosses (sample III). The straight lines are fits by Eq. (5) and the curved lines by Eq. (1).

cated by the curved lines in Fig. 2. Similarly, T_{c0} could be determined by the slope rather than the intercept of the straight line fit. For both alternative methods the values are similar.

A striking feature of Fig. 2 is the well-defined character of the critical region characterized by $(T_c - T)^{1/2}$ behavior and the sharpness of the transition to a linear Ginzburg-Landau (GL) regime, rather specific indications of a topological phase transition at T_c. Furthermore, the dimensionless constant b from Eq. (1) is ≈ 4 for all three samples in Fig. 2. The fact that $z(T)$ curves away from the straight line for low temperatures suggests that the GL approximation $T \approx T_c \approx T_{c0}$ is no longer valid. The precise way to correct this expression for low temperatures and possible strong-coupling effects is unclear. Nevertheless, the fact that the theory works as well as it does over such a wide range of temperature indicates that the basic picture is correct.

An important additional check on the consistency of this picture is obtained by comparing T_c/T_{c0} resulting from the above analysis to predictions based on the generalization of Eq. (2) for a wider temperature range as derived by Beasley, Mooij, and Orlando.[1] (This prediction does not take into account the possible correction factors due to the renormalization of the superfluid density.) In Fig. 3 the values of $\tau_c \equiv (1 - T_c/T_{c0})$ from the three samples of Fig. 2 are plotted together with the theoretical line. In addition, each of these samples was annealed at a low temperature one or more times to decrease $R_N^\square$, and the data were treated in the same way as in Figs. 1 and 2. Although some of the resultant samples did not exhibit the classic shape of the critical region as

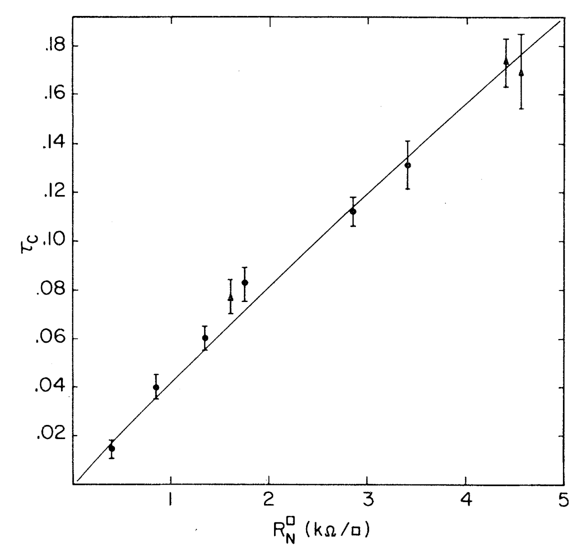

FIG. 3. $\tau_c \equiv 1 - T_c/T_{c0}$ vs $R_N^\square$ (kΩ/□). The triangles represent as-deposited films while circles denote films which were annealed. The line is a theoretical result from Ref. 1. Error bars reflect the uncertainty in the determination of T_{c0}.

shown in Fig. 2 (perhaps related to inhomogeneity or to strong-coupling effects), it was always possible to determine T_c from $a(T_c) = 3$ and T_{c0} by extrapolation to $a(T_{c0}) = 1$ from the GL regime. Values of τ_c from samples treated this way are also plotted in Fig. 3.

So far we have referred only to the regime for $T < T_c$. Other experiments have examined the regime for $T > T_c$, and have had some difficulty obtaining qualitative agreement with the theory for reasonable values of the parameters.[9-11] Unfortunately, this was the case with the present experiment as well. The theoretical expressions[2,3] for the linear resistance above T_c are far sharper than data in every case. The excess resistance for $T \approx T_c$ can be easily understood as due to edge effects, stray fields, etc. However, in the regime $T \approx T_{c0}$, the excess *conductance* (by orders of magnitude) is difficult to understand. One might speculate about the possibility of a percolating network of one-dimensional Hg filaments with a higher T_{c0}, which gets saturated in the large-current nonlinear resistance regime below T_c. However, at present the matter remains open.

There are a number of reasons why the nonlinear resistance regime below T_c studied here is likely to be a good place to test the vortex-antivortex unbinding transition. Excess free vortices due to whatever source are likely to be overwhelmed by the large number of free vortices resulting from pairs broken by the current. In addition, in the nonlinear resistance regime below T_c, edge nucleation and annihilation of vortices are not expected[15] to change the form of Eq. (3), in contrast to the linear regime above T_c where edge effects are crucial.[16]

In conclusion, we have found that analysis of the nonlinear resistance regime for $T < T_c$, for quench-condensed (and slightly annealed) Hg(Xe) films, is in excellent agreement with the model of Halperin and Nelson.[3] Values of T_c/T_{c0} thus inferred follow the predicted dependence on $R_N^{\square}$. Together these provide compelling evidence that the Kosterlitz-Thouless-Berezinskii theory is indeed applicable to the superconducting transitions of high-sheet-resistance films.

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