Are Quantum-Chromodynamic Scaling Violations in Deep-Inelastic Lepton-Hadron Scattering Really Observed?

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We present evidence that the observed scaling violations in deep-inelastic scattering are likely to be low-Q^2 phenomenon, as predicted by the massive-quark model.

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Are quantum-chromodynamic (QCD) scaling violations in deep-inelastic lepton-nucleon scattering really observed? That this question is a highly nonacademic one, and should be answered in a nonambiguous way, is obvious to everybody. For it is clear that, should the answer be definitely negative, it would cast grave doubts upon all the theoretical developments that are based on the notion of asymptotic freedom (AF), like perturbative QCD and the more ambitious program of grand unified theories.

In order to avoid misunderstanding let us state with all clarity that this Letter does not pretend to give a definitive answer to the above mentioned question, but rather to suggest that the available experimental information provides support for a pattern of scaling violations different from AF predictions.[1] We propose to substantiate our contention by showing that an approach to deep-inelastic phenomena,[2] which predicted the subasymptotic nature of scaling violations well before their first discovery, does provide a remarkably successful and economical description of the data from $Q^2=3$ GeV^2 up to values as high as $Q^2=200$ GeV^2.

The main points of the massive-quark model (MQM),[2] that we shall adopt to describe deep-inelastic lepton-nucleon scattering, are the following: (i) The quark degrees of freedom exist only in *finite* space-time domains (bags). (ii) In the bag domain quarks have the same behavior as low (effective) hadrons: In particular, high-energy quark Green's functions are Regge behaved. (iii) Quarks, as in the quark-parton model, have a point coupling to electromagnetic and weak currents. The points (i) and (iii) are true properties of the only confined theories that we know today, i.e., the two-dimensional (one-space, one-time) gauge theories,[3] while (ii) contends that Regge behavior at high energy for the hadrons originates from such a behavior at the constituents' level.

It has been shown[4] that the above three points constrain rather tightly the behavior of structure functions in the deep-inelastic region. In particular, (a) the behavior of structure functions in the Bjorken limit, for both $x\to 0$ and $x\to 1$, is completely fixed, and (b) one can predict the structure of the $O(1/\sqrt{Q^2})$ subasymptotic corrections to the structure functions. A preliminary analysis of deep-inelastic lepton-nucleon scattering based on (a) and (b) has been performed in Ref. 4, giving a satisfactory description of recent νN and eN data for $Q^2>6$ GeV^2. If we want, however, to describe data at lower values of Q^2, and in particular the very precise data for eN scattering that are available from the Stanford Linear Accelerator Center–Massachusetts Institute of Technology (SLAC-MIT) experiment,[5] our analysis must be refined. This we have done in this work. The results which we give below have been obtained in the following way.

According to (a) and (b) we have determined the $x\to 0$ and $x\to 1$ behavior for both the scaling term and the $O(1/\sqrt{Q^2})$ corrections. We have then linearly interpolated the structure functions between the two limits ($x=0$ and $x=1$), constraining the interpolation by the Adler[6] sum rule. In this way we obtain a description of *all* observable deep-inelastic lepton-nucleon scattering processes in terms of only six parameters, that we can express in the following way [$F(x,Q^2)$ denotes either $F_2(x,Q^2)$ or $xF_3(x,Q^2)$]:

$$F(x,Q^2)=\beta(Q^2)\sum_{n=0}^{6}a_n(Q^2)\xi_n x^{\alpha_n}(1-x)^{\beta_n}, \qquad (1)$$

TABLE I. The α_n, β_n exponents of Eq. (1) as determined from the MQM analysis. Terms with $n=0,1,2$ dominate in the double Regge region ($x\to 0$), whereas terms with $n=3,4,5,6$ dominate in the triple Regge region ($x\to 1$).

n	0	1	2	3	4	5	6
α_n	0	0.5	0.5	1	1	1.5	1.5
β_n	9	4.5	5.5	3	7	2.5	6.5

TABLE II. The ξ_n coefficients for the different deep-inelastic processes. In order to compute $F_2(x, Q)$ for νN scattering, we must set $\epsilon = +1$, while $\epsilon = -1$ yields $xF_3(x, Q)$.

ξ \ Process	ep	en	νp	νn
ξ_0	2	2	$3(1+\epsilon)$	$3(1+\epsilon)$
$\xi_1 = \xi_2$	3	2	6	12
ξ_3	$0.52 + 0.2R(x)$	$0.18 + 0.3\,R(x)$	$0.24 + 1.2\,R(x)$	$2.28 + 0.6\,R(x)$
ξ_4	$5/9 + S(x)/9$	$5/9 + S(x)/9$	2ϵ	2ϵ
ξ_5	1	$4/9$	0	$50/9$
ξ_6	3	2	6ϵ	12ϵ

where $\beta(Q^2)$ is the overall factor ($m_\rho^2 = 0.6$ GeV^2)

$$\beta(Q^2) = Q^2/(Q^2 + m_\rho^2) \tag{2}$$

which tends to 1 in the Bjorken limit and correctly reproduces the small-Q^2 behavior[7] of the structure functions. $a_n(Q^2)$ for $n = 0, \ldots, 4$ are constants satisfying the linear constraint of the Adler sum rule

$$0.47 = 1.09\,a_1 + 0.42\,a_3 + a_2. \tag{3}$$

The $O(1/\sqrt{Q^2})$ subasymptotic scaling violations are described by $a_5(Q^2) = a_5/\sqrt{Q^2}$ and $a_6(Q^2) = a_6/\sqrt{Q^2}$. The exponents α_n and β_n in Eq. (1) have been fixed by the MQM analysis of the $x \to 0$ and $x \to 1$ limits and are given in Table I. Finally the "Clebsch-Gordon" coefficients ξ_n for the different deep-inelastic processes are given in Table II.

A few comments are now in order. The exponents appearing in Table I have been determined following the treatment of Ref. 4, to which we refer the interested reader, except for the interpolating term $n = 2$, which appears with a factor

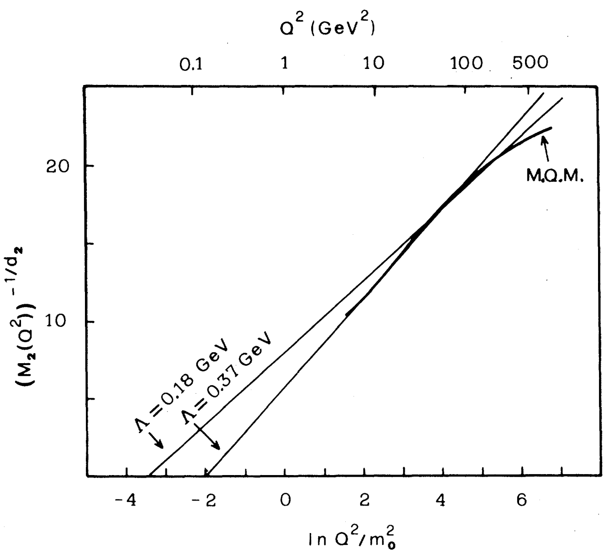

FIG. 1. Plot of M_2^{-1/d_2} as a function of $\ln Q^2/m_0^2$ ($m_0 = 1$ GeV; see text).

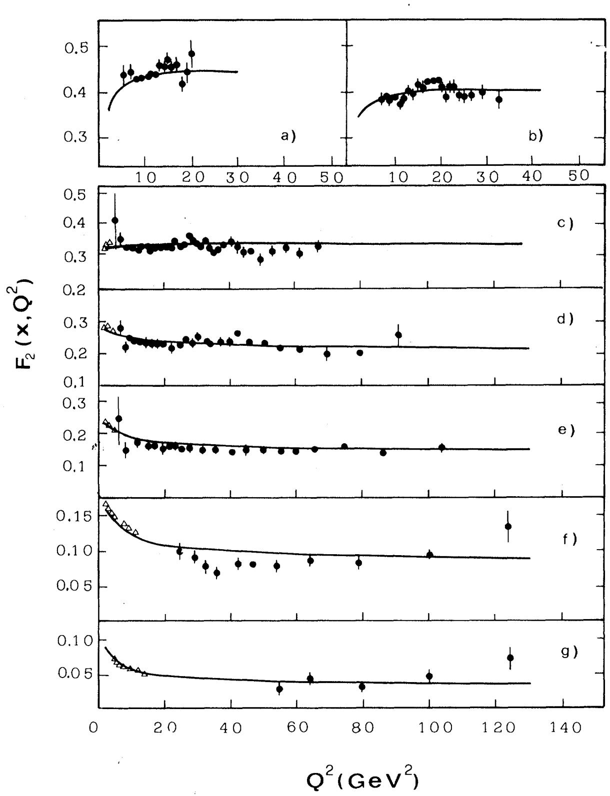

FIG. 2. Comparison between MQM theoretical predictions and $F_2^{\mu N}$ data from MSU-FNAL collaboration (closed circles) and $F_2^{ed}/2$ data from SLAC-MIT collaboration (open triangles) for several x bins: (a) $0.03 < x < 0.06$; (b) $0.06 < x < 0.1$; (c) $0.1 < x < 0.2$; (d) $0.2 < x < 0.3$; (e) $0.3 < x < 0.4$; (f) $0.4 < x < 0.5$; (g) $0.5 < x < 0.7$. The errors shown are statistical only.

$(1-x)$ with respect to the "double Regge term" ($n=1$). As one can see we have followed a "minimal interpolation" strategy by adding to our original parametrization only one term, that we have constrained by the Adler sum rule, which we had previously neglected. As for the Gross-Llewellyn Smith[8] sum rule, our parametrization turns out to violate it by a very small amount, i.e., by 6%.

Turning now to Table II, the "Clebsch-Gordan" coefficients have been calculated again in Ref. 4. The only difference with that calculation originates from a more accurate analysis of the model described in that paper. We have in fact found that, for the terms that are important for large x, one must take into account violations of SU(6) and SU(3) that are far from negligible.[9] This is the reason for the functions $R(x)$ and $S(x)$ appearing in the rows ξ_3 and ξ_4, whose expressions are given by

$$R(x)=\left\{\frac{0.71(1-x)+x[m_8^2-m_N^2(1-x)]}{0.71(1-x)+x[m_{10}^2-m_N^2(1-x)]}\right\}^3, \quad (4)$$

where $m_8^2=1.2\ \text{GeV}^2$ is the baryon octet central mass, and $m_{10}^2=2\ \text{GeV}^2$ is the baryon decuplet central mass; and

$$S(x)=\left\{\frac{0.71(1-x)+x^2m_N^2}{0.71(1-x)+x[m_{\Sigma,\Lambda}^2-(1-x)m_N^2]}\right\}^7, \quad (5)$$

where $m_{\Sigma,\Lambda}=1.18$ GeV is the Σ, Λ average mass. In order to display in a suggestive fashion the difference between the AF predictions of perturbative QCD and the MQM analysis, in Fig. 1 we report the MQM prediction for $M_2(Q^2)^{-1/d_2}$ as a function of $\ln Q^2/m_0^2$ ($m_0^2=1\ \text{GeV}^2$), where

$$M_2(Q^2)=\int_0^1 dx\, xF_3^{\nu N}(x,Q^2) \quad (6)$$

and $d_2=32/81$ is the AF anomalous dimension calculated for three flavors. We clearly see that we do not obtain a straight line as predicted by AF, thus giving a nice explanation why experiments in different Q^2 ranges, when fitted with the AF parametrization, yield different values for the parameter ($\ln \Lambda^2/m_0^2$ is the intercept of the straight line with the axis of the abscissa).

We have fitted our parameters $a_0, \ldots, a_6$, constrained by the Adler sum rule (2) and by the positivity conditions ($a_0>0$, $a_1+a_2>0$, $a_3, a_4, a_5, a_6>0$ as required by our model), by χ^2 minimization of the SLAC-MIT,[5] Michigan State University–

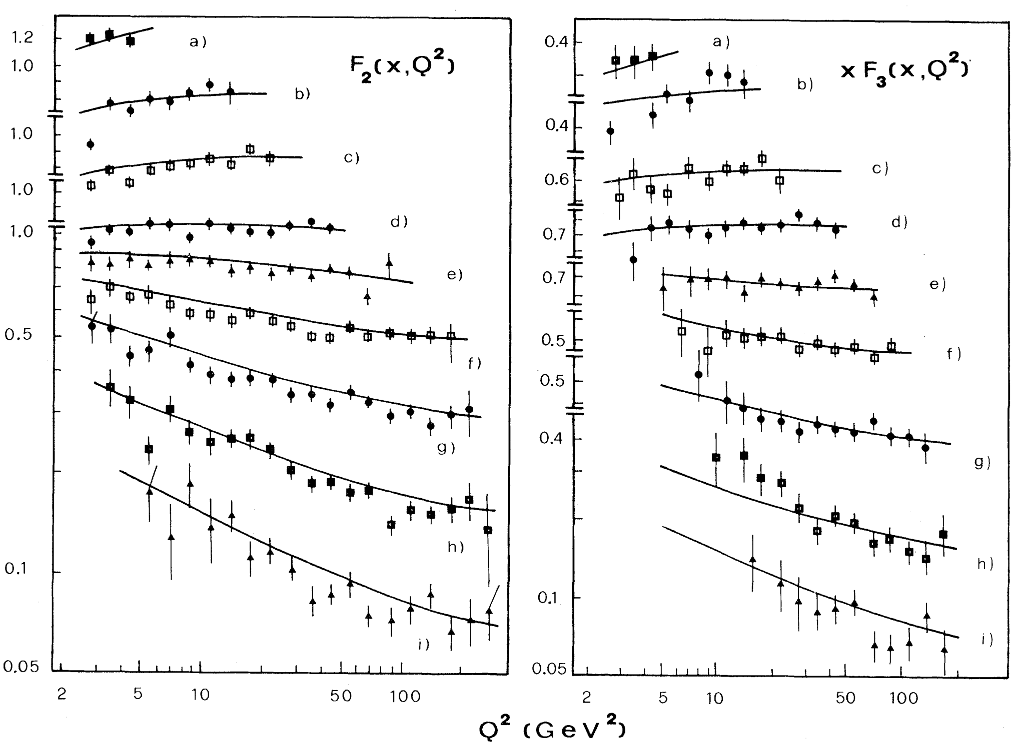

FIG. 3. Comparison between MQM theoretical predictions and CDHS new data for several x bins: a, $0<x<0.03$; b, $0.03<x<0.06$; c, $0.06<x<0.1$; d, $0.1<x<0.2$; e, $0.2<x<0.3$; f, $0.3<x<0.4$; g, $0.4<x<0.5$; h, $0.5<x<0.6$; i, $0.6<x<0.7$.

Fermilab[10] electroproduction, and CERN–Universität Dortmund–Universität Heidelberg–Centre d'Etudes Nucléaires de Saclay (CDHS)[11] ν-production data. The result we obtain is

$$a_0 = 0.21;\quad a_1 = 0.94;\quad a_2 = -0.58;\quad a_3 = 0.07;$$
$$a_4 = 0.10;\quad a_5 = 3.66\text{ GeV};\quad a_6 = 0.8\text{ GeV}; \tag{7}$$

and the corresponding $\chi^2 = 541$ for 395 degrees of freedom $(541/395 = 1.37)$. Notice that our fit does not allow for any normalization uncertainty, while in computing the χ^2 the systematic errors have been quadratically added to the statistical ones (for the MSU-FNAL data the systematic error has been taken to be equal to 7%).

In Fig. 2 we compare our fit with the SLAC-MIT deuterium and the MSU-FNAL iron data. In Fig. 3 a comparison is carried out for the new preliminary CDHS[12] data for the $F_2^{\nu N}(x,Q^2)$ and $x \times F_3^{\nu N}(x,Q^2)$ structure functions (our predictions have been scaled down by 5%, within the stated normalization uncertainty of these data).

Which conclusions can we draw from our work? More good quality data at large Q^2 values are certainly needed before we can answer with reasonable confidence the central question of this paper. If not a definite answer, however, we believe we have given strong suggestions that the scale breaking in deep-inelastic lepton-hadron scattering may well be a low-Q^2 phenomenon as predicted by the massive-quark model.

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