the source, i.e., to four coherence lengths of the wave packet associated with the lifetime of the intermediate state of the cascade (5 ns), we observed no change in the results.

As a conclusion, our results, in excellent agreement with quantum mechanics predictions, are to a high statistical accuracy a strong evidence against the whole class of realistic local theories; furthermore, no effect of the distance between measurements on the correlations was observed.

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Gravitational and Inertial Effects in Quantum Fluids

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A general theory of the interaction of superfluid helium with a gravitational field is developed within the framework of general relativity by using a covariant generalization of the Gross-Pitaevskii equation. The general relativistic Sagnac effect for the superfluid Josephson interferometer is obtained in the stationary case. The influence of a plane-polarized gravitational wave on a recently proposed superfluid gravitational antenna as well as a new antenna is determined.

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The possibility of using quantum interference effects in superfluid helium to detect gravitational and inertial perturbations, including gravitational radiation, has been proposed recently.[1,2] This raises the exciting possibility of testing general relativity in the laboratory at the quantum mechanical level for the first time. In this Letter, I shall therefore develop the general theory of the influence of gravity and inertia on superfluid helium from a general relativistic point of view. I shall then obtain the phase shift that gives rise to a Josephson current for the specific cases of the stationary Sagnac effect and the interaction of a plane gravitational wave with two simple superfluid gravitational antennas.

The proposed experiments may also be important for the study of superfluid helium because of the absence of a satisfactory microscopic theory for superfluid helium. At present, the superfluid phase of helium is described by an order parameter ψ (a complex function on space-time), which may physically be regarded as an effective wave function of the superfluid. ψ is assumed to satisfy the phenomenological nonrelativistic time dependent Gross-Pitaevskii equation[3]: $i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\nabla^2\psi + g|\psi|^2\psi$, where $2\pi\hbar$ is Planck's constant, m is the mass of the helium atom, and g is a constant. This equation is formally identical to the Ginzburg-Landau equation[4] for superconductivity. The theory that follows, which takes this equation as its starting point, will therefore also be applicable to the superfluid in a superconductor consisting of Cooper pairs.[5] However, in this case it is experimentally difficult to isolate the gravitational effects from the electromagnetic effects, which are much stronger.

A general relativistic generalization of the last equation is

$$\Box\psi + \frac{m^2c^2}{\hbar^2}\psi = -\frac{2mg}{\hbar^2}|\psi|^2\psi, \tag{1}$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, $g^{\mu\nu}$ is the inverse of the metric $g_{\mu\nu}$, ∇_μ is the covariant derivative, and c is the velocity of light. Writing $\psi = \alpha e^{i\varphi}$, where α and φ are real, and $v_\mu = -(\hbar/mc)\partial_\mu\varphi$, it follows that, in the interior of the superfluid,

$$\partial_{[\mu}v_{\nu]} = 0, \tag{2}$$

where the square brackets denote antisymmetrization. Equation (2) incorporates, in addition to the usual relation $\nabla\times\vec{v} = 0$, in a suitable coordinate system,[6] the relation $\partial v_0/\partial x_i - (1/c)\partial v_i/\partial t = 0$.

The real and imaginary parts of (1) are

$$v^\mu v_\mu = 1 + f(\alpha), \tag{3}$$

where $f(\alpha) = (\hbar^2/m^2c^2)\Box\alpha/\alpha + 2g\alpha^2/mc^2$, and

$$\nabla_\mu(\alpha^2 v^\mu) = 0, \tag{4}$$

which is the continuity equation. From (2) and (3),

$$v^\nu\nabla_\nu v^\mu = \tfrac{1}{2}\nabla^\mu f(\alpha). \tag{5}$$

Also $2g\alpha^2/mc^2 = (\lambda_c\alpha/\xi\alpha_0) \sim 10^{-12}$, where the Compton wavelength $\lambda_c = \hbar/mc$ and the coherence length $\xi = (\hbar^2/2m\alpha_0^2 g)^{1/2} \sim 10^{-10}$ m for superfluid helium.[7] Hence in the WKB approximation, $f(\alpha) \ll 1$.

On defining $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, $\eta_{\mu\nu}$ being the Minkowski metric, for weak field ($h_{\mu\nu} \ll 1$) and low superfluid velocity, (3) yields, after removing the rest-mass energy by defining $\theta = \varphi + mc^2t/\hbar$,

$$-\hbar\partial\theta/\partial t = \tfrac{1}{2}mc^2 f(\alpha) + \tfrac{1}{2}mc^2 h_{00} + \tfrac{1}{2}m\vec{u}^2, \tag{6}$$

where $\vec{u} = [(\hbar/m)\nabla\theta - c\vec{h}_0]$, $\vec{h}_0 = (h_{01}, h_{02}, h_{03})$. $m\vec{u}$ is like the kinetic momentum of the superfluid in the coordinate system which defines $h_{\mu\nu}$. On identifying $\frac{1}{2}c^2 f(\alpha) = p/\rho$ (p = pressure, ρ = density) and $\frac{1}{2}c^2 h_{00}$ = Newtonian potential, (6) gives Beliaev's equation[8] in the special case when $\vec{h}_0 = \vec{0}$. It also follows from (6) that

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{c^2}{2}\nabla f + \vec{\mathcal{E}} + \frac{1}{c}\vec{u}\times\vec{\mathcal{B}}, \tag{7}$$

where $\vec{\mathcal{E}} = -\frac{1}{2}c^2\nabla h_{00} - c\,\partial\vec{h}_0/\partial t$ and $\vec{\mathcal{B}} = c^2\nabla\times\vec{h}_0$. Equation (7) represents (5) in the present limit.

The phase difference in going around a closed curve γ beginning and ending at some point p is given by

$$\Delta\varphi(\gamma) = (mc/\hbar)\oint v_\mu dx^\mu. \tag{8}$$

Hence for $\Delta\varphi(\gamma) \neq 0$, γ must be in a multiply connected region or must go around a vortex line at which $\partial_{[\mu}v_{\nu]}$ is singular. If $\psi(p) \neq 0$ then the requirement that ψ be single valued implies the quantization of vortices

$$(mc/\hbar)\oint_\gamma v_\mu dx^\mu = 2\pi n, \tag{9}$$

where n is an integer. This generalizes the Onsager-Feynman quantization condition to an arbitrary closed curve in space-time. If $\psi(p) = 0$, which is the case if there is a barrier or a weak link at p, then (9) need not be satisfied. It follows from (5) and (8) that if γ is dragged along the integral curves of v^μ and f is differentiable on γ then $\Delta\varphi(\gamma)$ will remain constant. Hence, the superfluid does not develop a vorticity whereas the apparatus does, in general, due to the interaction with a gravitational field.[9] But the superfluid can interact with the apparatus and acquire vorticity. By the apparatus developing vorticity I mean that $\oint_\gamma t_\mu dx^\mu$ changes, where t^μ is the four-velocity field of the apparatus.

Consider now superfluid helium contained in a toroidal tube with a Josephson junction, i.e., a weak link at which φ may have a discontinuity and v_μ need not satisfy (9). It can be shown by arguments similar to the superconducting case that there must then be a Josephson current

$$I = I_0 \sin\Delta\varphi(\gamma) \tag{10}$$

at each point p on the world line of the junction, where γ is a closed curve which goes around the tube, beginning and ending at p. Let σ be the two-dimensional submanifold obtained by dragging γ along the integral curves of v^μ. Suppose σ admits a timelike Killing field ξ^μ and v^μ has the same direction as ξ^μ, everywhere on σ. This may be realized in the laboratory, for instance, for superfluid helium in a toroidal tube by giving the tube a uniform rotation or applying a Lense-Thirring field[10] due to another rotating body and waiting till the superfluid is at rest relative to the apparatus as a result of a coupling between the two. Also if $f(\alpha)$ is constant along a given integral curve of v^μ, say near the Josephson junction, then it can be shown using (2) and (3) that

$$\Delta\varphi(\gamma) = (mc/\hbar)\oint_\gamma \lambda^{-1}\xi_\mu dx^\mu, \qquad (11)$$

where $\lambda = \xi_\mu \xi^\mu = [1+f(\alpha)]^{-1}$. Equation (11) is similar in form to the general relativistic Sagnac effect for massive particles,[11] which has been experimentally tested,[12] in the nonrelativistic limit, for neutron interference. The above considerations lead to the prediction of the Sagnac effect in superfluid helium which can be tested by the Josephson current (10) that would flow through the junction. The results of Ref. 11, with $mc/\hbar$ substituted for ω_A, are valid for superfluid helium. It should be possible to detect experimentally, for instance, the effect of Earth's rotation by means of the superfluid interferometer considered here, whose linear dimensions need to be only of the order of a few centimeters.

When the tube interacts weakly with the superfluid, as in the case when zeolite or other porous powders are attached to the tube as proposed by Chiao,[2] I shall postulate that a component of the superfluid represented by the order parameter ψ is dragged by the apparatus while another component, ψ', does not couple to the apparatus. That is, for ψ, $v^\mu = \Lambda t^\mu$ in the submanifold σ defined earlier, where t^μ is the four-velocity field of the apparatus ($t^\mu t_\mu = 1$) and $\Lambda^2 = 1 + f(\alpha)$, from (3). In a coordinate system with its time axes along t^μ (i.e., $t^\mu = g_{00}^{-1/2}\delta_0{}^\mu$), $\Lambda t_\mu = \Lambda g_{00}{}^{-1/2} g_{0\mu} = (1 + \frac{1}{2}h_{00} + \frac{1}{2}f(\alpha),\ h_{01},\ h_{02},\ h_{03}) + O(h^2) + O(f^2) + O(hf)$. So if γ is on a constant-time surface then[13]

$$\Delta\varphi(\gamma) = \frac{mc}{\hbar}\oint h_{0i}dx^i + O(h^2) + O(hf) + O(f^2). \qquad (12)$$

Also in this case $\vec{u} = 0$ in (7) and since $f(\alpha) \ll 1$ it follows that

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\oint h_{0i}\,dx^i\right) \ll 1, \qquad (13)$$

which must hold for (12) to be valid with negligible $O(hf) + O(f^2)$.[14] If one assumes now that the apparatus is radially rigid, i.e., the particles constituting the apparatus are at constant distances and angles from the center of mass, it is convenient to choose the above coordinate system to be Fermi normal around the world line of the center of mass. The phase shift due to the gravitational perturbation is $\Delta\varphi_G(\gamma) = (2mc/\hbar)\oint R_{0lim} \times x^l x^m dx^i$, neglecting $O(x^3)$, $O(h^2)$, $O(hf)$, and $O(f^2)$, where R is the curvature and x^l are the spatial coordinates.[15] It should be noted that when $f(\alpha)$ is negligible, (11) and (12) can be obtained starting from the assumption that the Einstein-Planck law $mc^2 = \hbar\partial\varphi/\partial t$ is valid in the rest frame of the superfluid. Hence the results of the present paper which, at present, have experimental consequences do not really depend on (1) and may be regarded as relying, instead, on the well-tested Einstein-Planck law.

Consider now as the apparatus a "figure-eight" gravitational antenna,[2] which consists of a toroidal tube, containing superfluid helium, wound around N times in the shape shown in Fig. 1(a). Suppose that a plane gravitational wave, with its plane of polarization in the x-y plane and the wave vector in the z direction, is incident on the antenna which is in the x-z plane as shown. The curvature components for this wave near the center of mass, in the chosen Fermi-normal coordinate system, are

$$\begin{aligned} R_{0xxz} = R_{0yzy} = R_{xzxz} = R_{yzzy} = R_{y00y} \\ = (\omega^2/2c^2)A_+ \cos\omega(t - z'/c), \\ R_{x00y} = R_{0xyz} = R_{0yxz} = R_{xzyz} \\ = (\omega^2/2c^2)A_\times \cos\omega(t - z'/c), \end{aligned} \qquad (14)$$

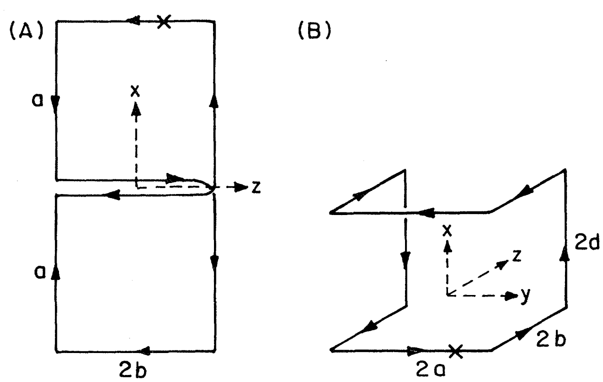

FIG. 1. A schematic representation of the "figure-eight" superfluid gravitational antenna and the "box" antenna. The arrows indicate the direction in which vorticity is set up in the apparatus, which is then transferred to the superfluid when a plane-polarized gravitational wave, with its wave vector in the z direction, passes the apparatus. x denotes the Josephson junction. When $2b$ = half wavelength, the antennas are ideally tuned.

all other components which are independent of the above being zero. With reference to Fig. 1(a), if $a, b \ll c/\omega$, then (13) is satisfied and hence

$$\Delta\varphi_G(\gamma) = (2m\omega^2 Na^2 b/\hbar c)A_+ \cos\omega t. \qquad (15)$$

For the gravitational antenna in the form of a "box" shown in Fig. 1(b), the phase shift due to the same gravitational wave is

$$\Delta\varphi_G(\gamma) = (8m\omega^2 Nabd/\hbar c)A_\times \cos\omega t. \qquad (16)$$

The mass current (10) can be detected by the recoil it produces at the Josephson junction which can be picked up by an electromagnetic transducer.[2] By having the natural frequency of the recoil mechanism close to the frequency of the gravitational wave, the amplitude of oscillation can be made to grow as a result of resonance.

The sensitivity of the proposed interferometer depends critically on the value of I_0 and on thermal fluctuations. Details of this analysis will be contained elsewhere[16] and here I shall only describe it briefly. The energy associated with the Josephson current is $(\hbar I_0/m)(1-\cos\Delta\varphi) \simeq \hbar I_0 \Delta\varphi^2/2m$ for small $\Delta\varphi$. Hence the minimum $\Delta\varphi$ that can be measured in the presence of thermal fluctuations satisfies $\hbar I_0 \Delta\varphi_m^2/2m \sim \frac{1}{2}kT$ where k is Boltzmann's constant and T is the temperature. Now I_0 must ultimately be determined by experiment. But a theoretical estimate of it can be made by using the work of Mamaladze and Cheishvili.[17] If the area of cross section of the tube is 1 cm^2 and the Josephson junction is made by packing together zeolite pores, then this theoretical estimate of I_0 yields $\Delta\varphi_m \sim 10^{-7}$ when $T \sim 10^{-3}$ °K.

It is easy to show from (11) or (12) that the phase shift due to Earth's Lense-Thirring field is $\Delta\varphi_e = \frac{8}{5} NGMm\Omega_n A/\hbar c^2 R$ where M, R, and Ω_n are, respectively, Earth's mass, radius, and the component of the angular velocity normal to the area of the interferometer. It follows that for an interferometer whose size is about 10 m and the number of turns $N \sim 10^3$, $\Delta\varphi_e \sim 10^{-5}$ which is of measurable magnitude. Earth's Sagnac effect can be nullified by mounting the apparatus on a platform, which can be rotated relative to Earth, so that, with use of telescopes rigidly attached to the platform, it could be made nonrotating relative to the distant stars. Taking atmospheric fluctuations, etc., into account, it can be shown that this could be accomplished to the desired accuracy.[16] It also follows from (15) and (16) that for a supernova ($\omega \sim 10^3$, $A_+, A_\times \sim 10^{-18}$) if $N \sim 10^3$, $a, b, d \sim 10$ m, then $\Delta\varphi_G \sim 10^{-7}$ which is of measurable magnitude. But it is important to note that (15) and (16) were obtained based on the assumption of quasirigidity, and it has not been demonstrated that this can be achieved with actual materials in the nonstationary case of the gravitational wave. But the above proposed stationary experiment to detect Earth's Lense-Thirring field seems to be feasible.

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[6]The following conventions are adopted: Greek indices take values 0, 1, 2, and 3. Latin indices take values 1, 2, and 3. $g_{\mu\nu}$ has signature $(+---)$.

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over, in Ref. (2) a Hamiltonian which is valid for low velocities has been used to construct a Feynman path amplitude for a spacelike path which corresponds to infinite velocity.

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