Strong Coupling and the Superfluid ^{3}He A-B Transition in a Magnetic Field

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The difference in chemical potential $\delta\mu$ between the A and B phases of superfluid ^{3}He has been obtained from measurements of the nuclear susceptibility and of the A-B transition temperature in fields up to 4.4 kG. The effect of strong coupling is found to increase rapidly with pressure.

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Manifestations of strong coupling, i.e., deviations from BCS theory in the behavior of superfluid ^{3}He, are seen in properties such as $\Delta C/C_N$, the size of the specific-heat discontinuity at T_c, and ρ_n, the normal density.[1] These properties yield inconsistent interpretations of the pressure dependence of strong coupling. Archie *et al.*[2] have shown that ρ_n can be fitted by a pressure-independent renormalization of the BCS gap. In contrast, measurements of $\Delta C/C_N$ by Alvesalo *et al.*[3] show a pronounced pressure dependence, with strong coupling diminishing with decreasing pressure. In this Letter we discuss the difference in chemical potential $\delta\mu$ between the A and B phases; $\delta\mu$ is very sensitive to strong-coupling effects, which are found to diminish with decreasing pressure. The data extend down to $T \sim \frac{1}{2}T_c$, so that they can be used to test strong-coupling theory outside the Ginsburg-Landau region; in fact, with use of a simple model, they can be extrapolated to $T = 0$. The results are not affected by present uncertainties in the absolute temperature scale.[3]

The data were obtained from measurements of the transition temperatures, T_c and T_{AB}, and the nuclear susceptibility, χ, of ^{3}He at temperatures down to 0.5 mK, pressures p up to 21 bars, and magnetic fields H up to 4.4 kG. The normal-^{3}He–A transition temperature T_c was deduced from the discontinuity in the warming or cooling rate caused by the specific-heat jump. The susceptibility was measured by integrating the absorption signal in field-swept transverse cw NMR. The A-B transition at T_{AB} was detected by the discontinuity in χ. Thermodynamic analysis of the measurements yields the difference in chemical potential $\delta\mu_0 = \mu_A - \mu_B$ in zero magnetic field, as a function of p and T.

We first describe the experimental arrangement and the temperature scale. The ^{3}He is cooled by a sintered-Pd heat exchanger attached to a copper nuclear demagnetization stage.[4] There are two thermometers in the ^{3}He: the primary thermometer, a 250-kHz pulsed Pt NMR spectrometer, and a high-resolution secondary thermometer, lanthanated cerium magnesium nitrate (LCMN) in a SQUID susceptibility bridge. The Pt Curie constant was determined from nearly simultaneous measurements near ~ 10 mK of the Pt susceptibiltiy and of the relaxation time τ_1, assuming a Korringa constant $\tau_1 T = 29.8$ msec K. The Pt susceptibility was then used to calibrate the LCMN (95% La, 5% Ce) over the range of 0.5 to 10 mK, with an accuracy (apart from uncertainties in $\tau_1 T$) of 0.2% of T. The temperature scale defined in this way yields for $T_c(p)$: 0 bar, 0.882 mK; 3.12 bars, 1.229 mK; 9.03 bars, 1.670 mK; 15.05 bars, 1.967 mK; and 21.06 bars, 2.163 mK. These T_c are very accurately proportional to the T_c reported by Alvesalo *et al.*,[3] our T_c being a factor 1.183 lower than theirs. The Alvesalo *et al.* values of $\gamma = C_N/T$ can be changed to our scale by multiplying them by $(1.183)^2$. They then agree with those given by Wheatley.[5] Furthermore, values of C_N at 3 bars measured[4] on our scale agree with Wheatley's. Since it is not known which specific heats are correct, we regard the temperature scale as uncertain by an unknown constant factor. However it seems plausible that the scale-independent energy γT_c^2 is known quite accurately as a function of pressure.

The three coils used for ^{3}He NMR were contained in an epoxy tower surrounded by a shielded superconducting solenoid, with the LCMN thermometer in its base. The solenoid was not persistent and the chief uncertainties in the ^{3}He susceptibility are random and due to noise and fluctuations in the solenoid current. Errors due to textural effects were negligible. The B-phase susceptibility at all pressures except 0 and 3 bars was found to be fitted by the expression[1]

$$\frac{\chi_B}{\chi_N} = \frac{(1+Z_0/4)[2/3+(1/3)Y(T/T_c)]}{1+(Z_0/4)[2/3+(1/3)Y(T/T_c)]} \equiv f\left(\frac{T}{T_c}\right), \quad (1)$$

where $Y(T/T_c)$ is the BCS Yosida function.[1] It

was not necessary to use the more complicated expression[6] involving Z_2, the next relevant Fermi-liquid parameter. At 0 or 3 bars the susceptibility is field dependent[7] but Eq. (1) is still sufficiently accurate for the present analysis. The fitted Z_0, with a similarly derived point at 19 bars due to Ahonen, Krusius, and Paalanen,[8] agree very well with the Z_0 values deduced by Wheatley[5] from measurements on the nonsuperfluid liquid. However we cannot really discriminate between Wheatley's Z_0 and those based on the new specific heats,[3] since we have not included the effect of Z_2, and the Yosida function has not been corrected for strong coupling. In the following we use Eq. (1) and the fitted Z_0 as a convenient empirical representation of χ.

The measurements of T_{AB} as a function of H and p are shown in Fig. 1. At a given pressure

$$\mu_A(T_{AB},H)=\mu_B(T_{AB},H). \tag{2}$$

Using $d\mu=-S\,dT-v\chi\boldsymbol{H}\,d\boldsymbol{H}$, and the fact that χ_A is very close to χ_N, the normal susceptibility, one easily obtains

$$\delta\mu_0(T_{AB})=\tfrac{1}{2}v\chi_N H^2[1-f(T_{AB}/T_c)], \tag{3}$$

where $\delta\mu_0=\mu_A-\mu_B$ in zero field. The two phases are assumed[1,5] to have the same T_c, where $\delta\mu_0=0$. The atomic volume v and the normal susceptibility are known functions of pressure.[5] Note that $\delta\mu_0$ from (3) is unchanged by a constant factor change in the temperature scale.

Equation (3) was fitted by least squares to the T_{AB} and χ data, assuming that $\delta\mu_0$ can be represented by a polynomial in $\epsilon\equiv 1-T/T_c$ and $\pi\equiv 1-p/p_c$, where p_c is the polycritical pressure.

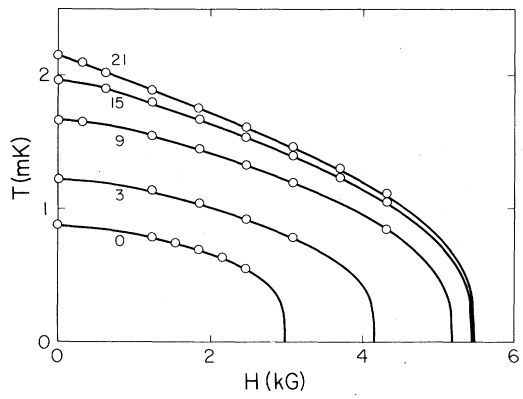

FIG. 1. A-B transition temperature vs magnetic field. The numbers give the nominal pressures in bars. The curves are a fit to the experimental data.

The polynomial was of the form

$$\delta\mu_0/\epsilon^2 k_B=\sum_{i=0}^{N} g_i\epsilon^i \tag{4}$$

with the g_i polynomials in π. The lowest-order term in $\delta\mu_0$ is ϵ^2, since in zero field, S_A-S_B vanishes[5] at T_c. To make our empirical equation more accurate for $p>p_c$, seven values from Ref. 3 of $\epsilon_{AB}=1-T_{AB}/T_c$ in zero field, corresponding to $\delta\mu_0/\epsilon^2=0$, were included in the fit. These additional data made no significant quantitative change except to reduce the calculated uncertainty in extrapolations to high pressure.

An excellent fit to the data was obtained with $N=2$ in Eq. (4). A more interesting and useful result includes terms up to $N=5$, with the extra coefficients g_3, g_4, and g_5 determined so that the entropy difference, $S_A-S_B=(\partial\delta\mu_0/\partial\epsilon)/T_c$, behaves properly as $T\to 0$. The entropy of the A phase is expected[1] to vary as T^3 and that of the B phase to drop exponentially as $T\to 0$. This means that the coefficients of $(1-\epsilon)^2$, $(1-\epsilon)^1$, and $(1-\epsilon)^0$ in S_A-S_B must vanish. With this constraint on g_3, g_4, and g_5 we find $g_0=1.721\pi-1.311\pi^2-0.217\pi^3$, $g_1=3.228-6.591\pi+3.739\pi^2$, and $g_2=-2.628+6.669\pi-5.886\pi^2$, all in microkelvins. This fit, in Eq. (3), gives the curves through the data in Fig. 1. The estimated uncertainty in H_{AB} at $T=0$ is about 0.1 kG. The fitted polycritical pressure is 21.7 ± 0.2 bars. Using $p_c=21.2$ bars from Paulson, Krusius, and Wheatley[9] gives a different set of g_i and a slightly inferior fit, but negligible quantitative change in our conclusions.

The latent heat at the A-B transition in zero field is given by

$$L_{AB}=[(1-\epsilon)\partial\delta\mu_0/\partial\epsilon]_{\delta\mu_0/\epsilon^2=0}.$$

The results from the fit are compared with the recent direct measurements of Alvesalo *et al.*,[3] and a measurement at the melting curve,[10] in Fig. 2. The agreement between the direct measurements and ours is within the errors and it is independent of the temperature scale. This confirms[10] that NMR correctly measures the true thermodynamic magnetization, despite past disagreement with purely static measurements.[11] Near T_c, static magnetization measurements indicate that $\chi_A-\chi_B$ is ~ 1.5 times larger than the values from NMR and, as Eq. (3) shows, the derived L_{AB} would then be increased by the same factor.

The quantity $(\delta\mu_0/\epsilon^2)_{\epsilon=0}$ is related to the difference between the zero-field specific heats of

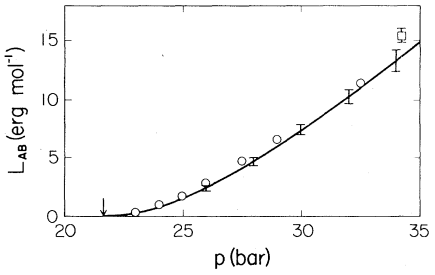

FIG. 2. Latent heat at the A-B transition in zero field vs pressure. The curve is extrapolated from a fit to the present experiment. The points are from Ref. 3 (circles) and Ref. 10 (the square).

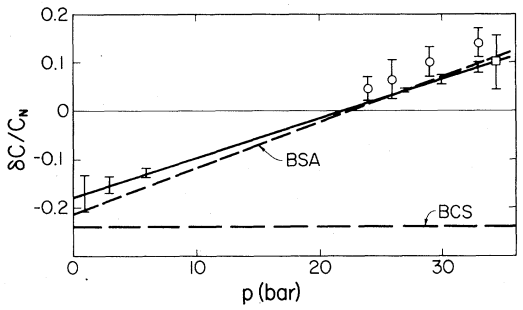

FIG. 3. The difference between the A and B specific heats at T_c vs pressure. The curve is the present work, the circles from Ref. 3, and the square from Ref. 10.

A and B at T_C:

$$\frac{\delta C}{C_N} \equiv \frac{C_A - C_B}{\gamma T_C} = -\frac{2(\delta\mu_0/\epsilon^2)_{\epsilon=0}}{\gamma T_C{}^2}. \quad (5)$$

Note that $\delta C/C_N$ from (5) is independent of a constant factor change in the temperature scale, provided that $\gamma T_C{}^2$ does not change. Both $(\delta\mu_0/\epsilon^2)_{\epsilon=0}$ and $\delta C/C_N$ appear in fourth-order Ginsburg-Landau theory[1] which retains only the ϵ^2 term in $\delta\mu_0$.

We compare our values of $\delta C/C_N$ with other measurements and with theory in Fig. 3. The results of Alvesalo *et al.*,[3] obtained by extrapolating the B-phase specific heat to T_C, are slightly higher than our results, shown by the full curve, but the agreement is satisfactory.

The horizontal broken line in Fig. 3 shows the prediction of BCS theory, the pressure-independent result $\delta C/C_N = (C_A - C_B)/C_N = -0.238$. At low pressure this is not far from the data, but above the polycritical pressure, δC becomes positive and $\delta\mu_0/\epsilon^2$ negative. It is clear that strong-coupling effects increase rapidly as pressure and T_C increase. The dependence of $\delta C/C_N$ on pressure is similar to that found for $\Delta C_B/C_N$ in Ref. 3. However, although $\Delta C_B/C_N$ agrees with the BCS value near $p = 0$, $\delta C/C_N$ is still significantly different from the weak-coupling value at low pressure. This is consistent with the observation of Eisenstein, Swift, and Packard,[12] that the ratio of ϵ_{AB} to H^2 near T_c does not obey the weak-coupling formula at $p = 0$. A similar effect is seen in the difference in ground-state energy, $(\delta\mu_0)_{\epsilon=1}$. The dimensionless ratio $(\delta\mu_0)_{\epsilon=1}/(\gamma T_C{}^2)$ varies almost linearly from 0.011 ± 0.001 at 33 bars to 0.022 ± 0.001 at 3 bars. This is approaching, but has not yet attained, the weak-coupling value 0.028.[13]

The effect of strong coupling on $\delta C/C_N$ can be approximated by the Brinkman-Serene-Anderson[14] (BSA) theory which attributes strong coupling to spin fluctuations in the nearly ferromagnetic liquid. Their prediction for the deviation from the BCS value of $\delta C_0/C_N$ depends on the pressure-dependent parameter

$$\delta = -140\alpha^{-1/2}(T_C/T_F)|Z_0/(4+Z_0)|^{3/2}. \quad (6)$$

Here T_F is the Fermi temperature corresponding to the normal specific heat γT, and α is a number which is $\frac{1}{3}$ for a free quasiparticle gas. The BSA theory can be scaled by adjusting α. Choosing $\alpha = 6.1$ makes $C_A = C_B$ at the polycritical point, producing the curve marked BSA in Fig. 3. Although the adjusted theory accounts for the differences $\mu_A - \mu_B$ or $C_A - C_B$ quite well, as has been noted by other authors,[15] it fails to agree with the individual values of C_A or C_B. More elaborate theories require information about the quasiparticle scattering amplitude which is difficult to obtain from experiment.[16] However they agree with the BSA theory that strong-coupling effects should be proportional to the small parameter T_C/T_F which appears in (6).

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New Interpretation of Mechanical and Electrical Relaxation Peaks in β-Alumina

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Internal friction, ultrasonic attenuation, and electric modulus measurements on β-alumina, which are inconsistent with simple Debye theory, are found to be in good agreement with each other when analyzed with use of the "universal" dielectric response theory. It is suggested that this theory may be used to analyze mechanical relaxation data from other ionic conductors.

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Internal friction[1-3], ultrasonic attenuation,[4] and electric modulus measurements[5] on single-crystal β-alumina show peaks which have been attributed to the diffusion of Na^+ ions in the conduction planes. These peaks have been taken to occur when the condition $\omega\tau = 1$ is satisfied, where ω is the angular frequency of the measurement and τ the Na^+ relaxation time. The phenomena are assumed to adhere to Debye theory in which losses, a, vary with $\omega\tau$ as

$$a \propto \omega\tau/(1+\omega^2\tau^2). \quad (1)$$

In all cases, however, the loss peaks are much wider than predicted by (1) and it has become necessary to postulate a distribution of relaxation times $g(\tau)$ to fit the data. The interpretation is then that the peaks are the sums of a number of Debye peaks, corresponding to a range of relaxation times in the sample. A modified form of a, a', given by

$$a' \propto \int g(\tau)\omega\tau\, d\tau/(1+\omega^2\tau^2), \quad (2)$$

is used to fit the data. Similar techniques have been widely used to explain the non-Debye-like dielectric response of solids. However, Jonscher[6] has suggested that there is often no reasonable

