SU(5) and the Invisible Axion

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Dine, Fischler, and Srednicki have proposed a solution to the strong *CP* puzzle in which the mass and couplings of the axion are suppressed by an inverse power of a large mass. We construct an explicit SU(5) model in which this mass is the vacuum expectation value which breaks SU(5) down to SU(3)⊗SU(2)⊗U(1).

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The standard SU(3) ⊗ SU(2) ⊗ U(1) gauge theory appears to be adequate to *describe* all of the phenomenology of the strong, electromagnetic, and weak interactions. Moreover, much of the structure of these interactions is *explained* by the theory in the sense that it follows directly from the form of the gauge interactions. However, there are a number of features which can be *described* in the context of SU(3) ⊗ SU(2) ⊗ U(1) but which are in no sense *explained*. Some of these features, such as charge quantization and the observed value of the weak mixing angle, are *explained* by the extension of SU(3) ⊗ SU(2) ⊗ U(1) to the grand unifying group SU(5).[1] The rest comprise the fundamental puzzles of contemporary particle physics: Why SU(3) ⊗ SU(2) ⊗ U(1) [or SU(5)] and not some other gauge group? How many generations of quarks and leptons exist and why? Why do the quark masses and mixing angles take their observed values? Why is the *CP* nonconservation in the SU(3) strong interactions so small? Finally, in the context of grand unified theories, there is the hierarchy puzzle. Why are the mass scales associated with the electroweak and strong interactions so small compared to the unification mass scale $M_u \simeq 10^{15}$ GeV?[2] Some or all of these questions may not have answers. The world may just be the way it is.

Our penultimate question, the puzzle of the smallness of strong *CP* nonconservation, is particularly tantalizing. Several different mechanisms have been proposed to *explain* the smallness. Soft *CP* nonconservation[3] or a massless up quark[4] might do it at a price in elegance. The Peccei-Quinn[5] symmetry would do it, but the predicted axion[6] is not seen.[7] Some workers[8] have suggested scenarios in which the axion is heavy and hard to see. Dine, Fischler, and Srednicki[9] (DFS) have recently suggested a clever variant of the Peccei-Quinn scheme in which the axion mass and its coupling to normal matter are inversely proportional to a large and arbitrary vacuum expectation value (VEV) of an SU(2) singlet scalar field. If this VEV is large enough, their axion is invisible.

In this paper, we comment on the DFS idea. We first note that the singlet VEV must be greater than 10^9 GeV to satisfy astrophysical constraints.[10] In the SU(3) ⊗ SU(2) ⊗ U(1) theory, such a large mass scale is unnatural. Thus, in the context of SU(3) ⊗ SU(2) ⊗ U(1), the DFS idea is a trade-off. It explains the smallness of strong *CP* nonconservation at the cost of introducing a hierarchy puzzle.

In a grand unified theory, it seems reasonable to imagine that the singlet VEV is of order M_u. Our main purpose in this paper is to describe a model in which it is more than reasonable, it is automatic, because the DFS singlet field is precisely the field whose VEV breaks SU(5) down to SU(3) ⊗ SU(2) ⊗ U(1). In our model, the hierarchy puzzle is still with us, but the strong *CP* puzzle is solved at no addtional cost.

The astrophysical constraints on a light axion have been discussed by Dicus, Kolb, Teplitz, and Wagoner.[10] They find that for a light axion with conventional couplings, the power radiated in axions by the helium core of a red supergiant star would exceed the power in photon emission by about 10^{13}. Consistency with the usual stellar models can only be achieved if the axion couplings are reduced by at least $10^{6.5}$. In the DFS model, the axion coupling is reduced by the ratio of the usual Higgs VEV, $u \simeq 250$ GeV, to the singlet VEV. Thus the singlet VEV must be of order 10^9 GeV or larger.

Our main concern is the construction of an explicit SU(5) model which solves the strong *CP* puzzle. The fermion fields are the usual left-handed $\underline{10}$'s (T_L) and right-handed $\underline{5}$'s (F_R). The spinless fields are two $\underline{5}$'s, represented by column vectors H_1 and H_2, and a *complex* $\underline{24}$, represented by a traceless 5×5 matrix Σ. The Yukawa couplings are (schematically)

$$g_1 \overline{T}_L{}^c T_L H_1 + g_2 \overline{T}_L F_R H_2 , \tag{1}$$

where *c* denotes charge conjugation. These are invariant under the Peccei-Quinn symmetry

$$T_L \to e^{-i\alpha/2} T_L , \quad F_R \to e^{i\alpha/2} F_R , \qquad H_1 \to e^{i\alpha} H_1 , \quad H_2 \to e^{-i\alpha} H_2 . \tag{2}$$

We demand that this be a symmetry of the scalar meson self-interactions with the addition of the following transformation law for the Σ field:

$$\Sigma \to e^{-i\alpha} \Sigma . \tag{3}$$

Then the most general potential for the scalars is

$$V(H_1, H_2, \Sigma) = V_1(\Sigma) + V_2(H) + V_3(H, \Sigma) , \tag{4}$$

where

$$V_1(\Sigma) = -\tfrac{1}{2}\mu^2 \operatorname{Tr}(\Sigma^\dagger\Sigma) + \tfrac{1}{4}a\,[\operatorname{Tr}(\Sigma^\dagger\Sigma)]^2 + \tfrac{1}{2}b \operatorname{Tr}(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma) + \tfrac{1}{4}c[\operatorname{Tr}(\Sigma^2)][\operatorname{Tr}(\Sigma^\dagger)^2] + \tfrac{1}{2}d \operatorname{Tr}(\Sigma\Sigma\Sigma^\dagger\Sigma^\dagger) , \tag{5}$$

$$V_2(H) = -\tfrac{1}{2}\mu_1{}^2(H_1{}^\dagger H_1) - \tfrac{1}{2}\mu_2{}^2(H_2{}^\dagger H_2) + \tfrac{1}{4}\alpha_1(H_1{}^\dagger H_1)^2 + \tfrac{1}{4}\alpha_2(H_2{}^\dagger H_2)^2 + \tfrac{1}{4}\alpha_3(H_1{}^\dagger H_1)(H_2{}^\dagger H_2) + \tfrac{1}{4}\alpha_4(H_1{}^\dagger H_2)(H_2{}^\dagger H_1) , \tag{6}$$

$$V_3(H, \Sigma) = \gamma_1(H_1{}^\dagger H_1)\operatorname{Tr}(\Sigma^\dagger\Sigma) + \gamma_2(H_2{}^\dagger H_2)\operatorname{Tr}(\Sigma^\dagger\Sigma) + \beta_1 H_1{}^\dagger\Sigma\Sigma^\dagger H_1 + \beta_2 H_2{}^\dagger\Sigma\Sigma^\dagger H_2 + \delta_1 H_1{}^\dagger\Sigma^\dagger\Sigma H_1 + \delta_2 H_2{}^\dagger\Sigma^\dagger\Sigma H_2 + gH_2{}^\dagger\Sigma^2 H_1 + g^* H_1{}^\dagger\Sigma^{\dagger 2} H_2 + hH_2{}^\dagger H_1 \operatorname{Tr}(\Sigma^2) + h^* H_1{}^\dagger H_2 \operatorname{Tr}(\Sigma^{\dagger 2}) , \tag{7}$$

where all constants except *g* and *h* are real.

For a range of parameters, the VEV's will take the form

$$\langle\Sigma\rangle = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3-\epsilon & 0 \\ 0 & 0 & 0 & 0 & -3+\epsilon \end{bmatrix} \lambda_0/2 \tag{8}$$

$$\langle H_{1,2}\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \lambda_{1,2}/\sqrt{2} \end{bmatrix} . \tag{9}$$

The $SU(3)\otimes SU(2)\otimes U(1)$ singlet component of Σ is the DFS singlet field in this model. Its VEV, λ_0, must be of order M_u while

$$|\lambda_1|^2 + |\lambda_2|^2 = u^2 . \tag{10}$$

It follows that ϵ is very small:

$$|\epsilon| = O(u^2/M_u{}^2) . \tag{11}$$

The axion is primarily the antihermetian part of the singlet component of Σ. But, it contains a small admixture (of order λ_j/λ_0) of the neutral components of H_j through which it couples to fermions.

One might worry that by enlarging the Higgs structure of our SU(5) theory we may have made the hierarchy puzzle more severe than in the standard SU(5) model. We can quantify this worry by counting the number of unnatural constraints which must be imposed to insure that the VEV's satisfy the desired hierarchy

$$|\lambda_0| \gg |\lambda_{1,2}| \gg |\epsilon\lambda_0| . \tag{12}$$

In the standard SU(5) model there is only one constraint in the sense that only one combination of large numbers must cancel to make the theory work.

The most straightforward way to minimize *V* is to require that (8) and (9) is an extremum, and that the second derivative matrix is positive semidefinite, so that (8) and (9) is at least a local minimum. Alternatively, for a range of the parameters, we can rewrite *V* as a sum of positive semidefinite terms, all of which vanish at the VEV, (8) and (9), which is thus an absolute minimum.

With either method, we find that there is a single unnatural condition which must be satisfied. As in the standard SU(5) model, the condition is that the square of mass of the true Higgs doublet (in the sense of Georgi and Nanopoulos[11]) be small.[12] The true Higgs boson is the SU(2) doublet component of

$$\lambda_1{}^* H_1 + \lambda_2{}^* H_2 . \tag{13}$$

The orthogonal doublet typically has a mass of order M_u. This is very different from the usual Peccei-Quinn scheme in which the extra charged Higgs is light. In our version of the DFS model, the only extra particle with mass small compared to M_u is the invisible axion.

The invisible axion is a curious beast. Although it is very light, it does not really belong to the effective low-energy field theory that describes our world. Because it is a pseudo-Goldstone boson associated with symmetry breaking at M_u, all of its interactions are suppressed by inverse powers of M_u. This solution to the strong *CP* puzzle simply has no other consequences in low-energy particle physics. However, there may be cosmological implications of this idea.

Guth and Pi[13] point out a cosmological problem of conventional SU(5) with no trilinear coupling of the 24. It is associated with the discrete symmetry $\Sigma \to -\Sigma$ which leads to a twice degenerate vacuum. Our model has no trilinear couplings; however, the discrete symmetry $\Sigma \to -\Sigma$ is embedded within the continuous Peccei-Quinn symmetry of the Higgs potential.

The invisibility of our axion is established by the following order-of-magnitude estimates of its properties: axion mass $\sim f_\pi m_\pi / M_u \sim 10^{-8}$ eV; lifetime for 2γ decay $\sim (M_u/f_\pi)^5 \tau_{\pi^0} \sim 10^{56}$ yr; pseudoscalar couplings $\sim f_\pi/M_u \sim 10^{-16}$; scalar couplings $\sim \bar{\theta} f_\pi / M_u \sim 10^{-31}$.

CP-nonconserving scalar couplings of the axion are induced by the nonperturbative breaking of the Peccei-Quinn symmetry. In principle, $\bar{\theta}$ is calculable in our model, and we estimate it to be about 10^{-15}. The scalar couplings will lead to a "long-range" attraction of baryons by axion exchange.[14] The effect is about 10^{-24} of the universal gravitational attraction. The contribution of $\bar{\theta}$ to the electric dipole moment of the neutron is about 10^{-31} $e\cdot$cm.[15] The most disquieting aspect of this solution to the strong *CP* problem is the predicted existence of an almost massless particle which is in practice unobservable.[16]

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Note added.—Our argument that only a single unnatural condition is needed to produce the hierarchy is rather general. It applies, for example, to the SU(5) model in Ref. 10, where a real 24, a complex singlet, and two 5's of Higgs are used. The astrophysical constraints[10] on the SU(2) ⊗ U(1) singlet VEV are also mentioned in Ref. 9.

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