Diffractive Excitation in Quantum Chromodynamics

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Hadronic collision models based on quantum chromodynamics predict remarkably large cross sections for diffractive scattering of hadrons on a nuclear target. The diffraction arises from the transparency of a nucleus to the portion of the projectile wave function having small transverse separation between its constituents. Correspondingly, the typical transverse momentum within the diffracted system is significantly enhanced. This quantum-chromodynamics–based picture leads to large cross sections for diffractive charm production.

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Diffractive dissociation, as explained in the classic papers of Feinberg and Pomeranchuk[1] and of Good and Walker,[2] is a distinctive quantum effect which makes up a significant part of the total hadron-hadron cross section at high energy. In a typical dissociation event, the target recoils intact with small momentum transfer, while the projectile is excited to a state with constituents having low transverse momentum. The diffractive excitation arises from the variability of the absorption amplitude as a function of the internal coordinates of the projectile wave function. We wish to focus attention on the enhanced possibility of studying hadronic wave functions by diffractive dissociation on nuclear targets. In a quantum-chromodynamics–based picture of hadron-hadron total cross sections,[3,4] the components of a color-singlet projectile wave function with small transverse separation pass freely through the nucleus while the large–transverse-separation components are nearly totally absorbed.[5] In short, a large target will act as a filter, removing from the beam all but the short-range components of the projectile wave function. The cross section for diffractive production of inelastic states is then equal to the elastic scattering cross section of the projectile on the target, multiplied by the probability that sufficiently small transverse separation configurations are present in the wave function.

The relevant portions of the wave function can be calculated in quantum chromodynamics (QCD), because the short-distance behavior is in the perturbative domain.[6,7] The QCD wave function is represented in terms of Fock components defined at equal time on the light cone; internal coordinates are the transverse position $r_{\perp i}$ and the light-cone momentum fraction $x_i=(k_0+k_3)_i/(p_0+p_3)$ of each constituent, where p is the projectile momentum in a frame with $p^1=p^2=0$. For the interaction between hadrons we make use of the gluon-exchange model of Low and Nussinov.[3] An immediate consequence[4] of the model is that the interaction depends strongly on the configuration of the constituents of the hadron. For Fock components with many constituents, it is very improbable for all the transverse separations to be small. Only the valence Fock configuration has a wave function $\xi(x,r_\perp)$ which is large as $r_\perp\to 0$,[6,7] and hence a wave-function component which is totally noninteracting.[8]

At high energy the S matrix may be written as a function of impact parameter b, and internal projectile coordinates $\xi=(x_i,r_{\perp i})$. The projectile wave function ψ is normalized, $\int d\xi\,|\psi(\xi)|^2=1$. After filtering by the target, the surviving hadron wave function is

$$S(b,\xi)\psi(\xi)=\psi'(b,\xi). \tag{1}$$

Defining $\bar{S}(b)=\int d\xi\,|\psi(\xi)|^2 S(b,\xi)$, the expectation value of S for the incoming hadron, we decompose ψ' as

$$\psi'(b,\xi)=\bar{S}(b)\psi(\xi)+[S(b,\xi)-\bar{S}(b)]\psi(\xi). \tag{2}$$

The first term is simply the survival amplitude of the incident state, while the second term is a superposition of excited hadron states orthogonal to the incident state. If for certain values of ξ there is weak absorption, $S(b,\xi)\simeq 1$, then the cross section for producing such ξ's is given by

$$d\sigma/d\xi=\int d^2b\,|1-\bar{S}(b)|^2|\psi(\xi)|^2 = \sigma_{el}|\psi(\xi)|^2, \tag{3}$$

where σ_{el} is the elastic scattering cross section. Thus the diffractive cross-section ratio $(\sigma_{el})^{-1}\times d\sigma/d\xi$ directly measures the original wave function.

As an example, consider a pion projectile. We treat the interaction of the $q\bar{q}$ Fock component, and assume that the higher Fock components are completely absorbed. The asymptotic form of the $q\bar{q}$ Fock-state wave function can be calculated in perturbative QCD.[6] With the normalization taken from the $\pi \to \mu\nu$ decay rate, the wave function at $r_\perp = 0$ is given by

$$\psi(x, r_\perp = 0) = (4\pi)^{1/2}\sqrt{3}x(1-x)f_\pi \qquad (4)$$

with $f_\pi = 93$ MeV. The associated cross section is then

$$\left.\frac{d\sigma}{dx\,d^2r_\perp}\right|_{r_\perp \to 0} \simeq \sigma_{\rm el}{}^\pi\, 12\pi f_\pi{}^2 x^2(1-x)^2. \qquad (5)$$

If the typical $r_\perp$ which survived the nuclear filtering were small enough the q and $\bar{q}$ would materialize as jets in the final state. In the center of mass system of the $q\bar{q}$ jet the x variable would be related to the jet angle θ (relative to the incoming beam direction) by $x = (1+\cos\theta)/2$, and the internal momentum $k_\perp$ of the q (or $\bar{q}$) would determine the mass of the diffractive system through $M^2 = k_\perp{}^2/x(1-x)$. However, we shall see that the nuclear filter does not generally produce states with large enough $k_\perp$ that a jet structure can be expected to appear.

To make a quantitative estimate for nuclear targets we shall assume that the absorption is strong for impact parameters less than the nuclear radius, i.e., $\bar{S}(b) \simeq 0$ for $b < R \simeq 1.2A^{1/3}$. Then the wave function of the diffractive excitation is

$$\psi'(x, r_\perp) = S(b, r_\perp)\psi(x, r_\perp), \quad b < R. \qquad (6)$$

We shall now relate $S(b, r_\perp)$ to the pion-nucleon total cross section. In vector-exchange models such as the Low-Nussinov model, the derived cross section depends only on the $r_\perp$ distribution and not on the x distribution,[4] and so we have dropped the x variable. We shall also find for large nuclei that $S(b, r_\perp)$ is damped rapidly with increasing $r_\perp$, and we may neglect the $r_\perp$ dependence of ψ in computing the momentum spectrum. The cross section is then given approximately by

$$d\sigma/dx\,d^2r_\perp \simeq \pi R^2 |\psi(x, r_\perp = 0)|^2 S_A{}^2(r_\perp), \qquad (7)$$

where $S_A{}^2(r_\perp)$ is the mean S^2 for the nucleus. Transforming ψ' to momentum space, the cross section is

$$\frac{d\sigma}{dx\,d^2k_\perp} \simeq \pi R^2 |\psi(x, r_\perp = 0)|^2 \frac{\tilde{S}_A{}^2(k_\perp)}{(2\pi)^2}, \qquad (8)$$

where $\tilde{S}_A(k_\perp) = \int \exp[ik_\perp \cdot r_\perp S_A(r_\perp)]\, d^2r_\perp$.

We now estimate $S_A(r_\perp)$. In the Low-Nussinov model the S matrix for scattering on a single nucleon has the following limit as $r_\perp \to 0$:

$$S_{\pi N}(b, r_\perp) \simeq 1 - \mu(b) r_\perp{}^2. \qquad (9)$$

The expression for $\mu(b)$ is complicated and, in any case, its exact normalization can only be determined by reference to an observed cross section. We use the total pion-nucleon cross section to estimate the integrated $\mu(b)$,

$$\sigma_{\pi N}{}^T = 2\int d^2b\,(1 - {\rm Re}S) = 2\langle r_\perp{}^2\rangle_\pi \int d^2b\,\mu(b). \qquad (10)$$

We can then treat the scattering from nuclei with an eikonalized pion-nucleon scattering operator,

$$S_A \simeq \exp[-\int dz\,d^2b'\,\mu(b - b')\rho(b') r_\perp{}^2]$$
$$\simeq \exp[-(r_\perp{}^2/2\langle r_\perp{}^2\rangle_\pi)\int dz\,\rho_0\sigma_{\pi N}{}^T]. \qquad (11)$$

We evaluate this expression using the vector-dominance model of the pion radius, $\langle r_\perp{}^2\rangle = 4/m_\rho{}^2$. With the further estimates of nuclear target parameters $\rho_0 = 0.16/\text{fm}^3$, $\int dz\,\rho = \frac{4}{3}\rho_0 R$, and pion-nucleon total cross section $\sigma_{\pi N}{}^T \simeq 25$ mb, we obtain

$$S_A(b, r_\perp) \simeq \exp[-(r_\perp{}^2 A^{1/3}/20)\,(1\ \text{GeV}^2)], \quad b < R, \qquad (12)$$

otherwise independent of R. Combining this with Eqs. (4) and (8) yields a cross section in the momentum representation of

$$\frac{d\sigma_A}{d^2k_\perp} = \pi R^2 \int dx\,|\psi(x,0)|^2 (2\pi)^{-2}\left(\frac{20\pi}{A^{1/3}(1\ \text{GeV}^2)}\right)^2 \exp\left(-\frac{10k_\perp{}^2}{A^{1/3}(1\ \text{GeV}^2)}\right)$$
$$\simeq 50\ \text{mb GeV}^{-2}\exp[-10k_\perp{}^2/A^{1/3}(1\ \text{GeV}^2)]. \qquad (13)$$

For the largest nuclei, $A^{1/3} \simeq 6$, and the $k_\perp$ of the q and $\bar{q}$ is quite large, $\langle k_\perp{}^2\rangle \simeq 0.6$ GeV^2. One expects that this large $\langle k_\perp\rangle$ will be reflected in a $p_\perp$ spectrum for the hadrons which materialize that is considerably harder than the typical spectrum for diffractive hadron-nucleon collisions, $\langle p^2\rangle \simeq 0.1$ GeV^2. The integrated diffractive excitation cross section from Eq. (13) is

$$\sigma_A \simeq (16\ \text{mb})A^{1/3}, \qquad (14)$$

which should be compared with the diffractive cross section due to single πN collisions on the periphery of the nucleus. The latter may be estimated in terms of the exponential tail of the nuclear density distribution, $\rho(r) \sim e^{-r/a}$, with $a \sim 0.55$ fm. The single-collision cross section $\sigma_{\pi A}^{(1)}$ is roughly given by

$$\sigma_{\pi A}^{(1)} \simeq 2\pi R a, \tag{15}$$

and the diffractive component is

$$\sigma_{\pi A}^{\mathrm{diff}} \simeq 2\pi R a \sigma_{\pi N}^{\mathrm{diff}} / \sigma_{\pi N}^{T}. \tag{16}$$

Taking $\sigma_{\pi N}^{\mathrm{diff}} \simeq 3$ mb, we find

$$\sigma_{\pi A}^{\mathrm{diff}} \simeq (5\ \mathrm{mb}) A^{1/3}, \tag{17}$$

considerably smaller than the transmitted component. Thus, if the absorptive interaction is color sensitive, the diffractive cross section on nuclear targets should be dominated by the transparency of the nucleus to small-$r_\perp$ components of the pion wave function. A conventional geometrical picture of hadron-hadron scattering, in which the absorption is proportional to the hadronic density overlap, would not predict this dramatic effect.

The masses typical of the diffractive excitation are determined by the relation

$$M^2 = k_\perp^2 / x(1-x) \geq 4 k_\perp^2. \tag{18}$$

With $A^{1/3} \simeq 6$, $\langle k_\perp^2 \rangle \simeq 0.6\ \mathrm{GeV}^2$, we have

$$M^2 \simeq 3\ \mathrm{GeV}^2. \tag{19}$$

The diffractive condition, that the nucleus remains unexcited, requires that the momentum transfer to the nucleus be smaller than the inverse nuclear radius. In terms of the mass of the diffractive excitation this condition is[9] $M^2 R / p_{\mathrm{lab}} < 1$, which is easily satisfied with present high-energy accelerators.

Because substantial masses can be coherently produced by diffractive excitation, it is clear that the $r_\perp$ filtering may play an important role in the production of diffractive states containing charm. It has been suggested[10] that the proton contains heavy quarks in some of its intrinsic Fock states. If these states were characterized by small transverse dimension,[10] then they would be able to pass through the nucleus without interaction, and Eq. (3) could be used as a guide to the magnitude of the expected cross section. Integrating Eq. (3) over all ξ for such states we then obtain

$$\sigma_{\mathrm{charm}}^{\mathrm{diff}} = P_c \sigma_{\mathrm{el}}, \tag{20}$$

where P_c is the probability of an intrinsic charm Fock state in the projectile hadron. It has been estimated[11] that for nucleons $P_c \simeq 2\%$; with $\sigma_{\mathrm{el}} \simeq \pi R^2 \simeq 50 A^{2/3}$ mb we obtain

$$\sigma_{\mathrm{charm}}^{\mathrm{diff}} \simeq (1\ \mathrm{mb}) A^{2/3} \tag{21}$$

for nuclear targets. In collisions with a single nucleon, diffractive charm production may also be significant, though we see no reason for it to be dominant as in the case of a nuclear target. Naively employing Eq. (21) for charm production on a nucleon target yields $\sigma_{\mathrm{charm}}^{\mathrm{diff}} \simeq 0.1$ mb, i.e., of the same order as the cross section observed at the CERN intersecting storage rings.[12] *Thus, in comparing Fermilab with intersecting storage ring charm production, one should assume at most an $A^{2/3}$ dependence on the nuclear target.*[13]

While the nuclear-target filtering creates diffractive production of large M^2 final states, it is clear that the softness of the filter places an effective cutoff on the masses. The cross section for asymptotically large M^2 and $k_\perp^2$ can, however, be estimated by gluon exchanges between target and projectile. For diffraction, the Low-Nussinov model requires two-gluon exchange between the projectile and each struck nucleon. The predicted high-$k_\perp^2$ behavior (modulo logarithms) of $d\sigma/dk_\perp^2$ is $1/k_\perp^6$ for pion beams, $1/k_\perp^8$ for proton beams, and $1/k_\perp^4$ for photon beams.[14] Each is a single power of $k_\perp^2$ more suppressed than the high-$k_\perp^2$ tail of the intrinsic projectile wave function because of color cancellations. *The exponential suppression we obtained in Eq. (13) is of course a many-body effect, with validity limited to nuclear targets and low to moderate $k_\perp$.* In the regime where the jet structure is visible, the pion-induced jets will have an angular distribution in the jet c.m. frame of $d\sigma/dM^2 d\cos\theta \simeq \sin^2\theta$, reflecting the x dependence of Eq. (4).

In conclusion, we emphasize that all our results rely crucially upon the assumption that the absorptive amplitude is color sensitive, as in the Low-Nussinov model, so that Fock states of small intrinsic size interact weakly. The diffractive excitation spectrum then provides an unusual measure of the short-range constituent structure of hadrons.

Note added.—An experiment showing some indication of a transparency effect may be found in Ref. 15.

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