Self-Consistent Reduction of the Spitzer-Härm Electron Thermal Heat Flux in Steep Temperature Gradients in Laser-Produced Plasmas

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A simple self-consistent limitation of the anisotropic portion of the local electron distribution function results in an order-of-magnitude reduction from the Spitzer-Härm thermal heat flux in steep temperature gradients, in agreement with recent Fokker-Planck simulations. A new upper bound derived for the local heat flux, substantially lower than that given by a "free-streaming" limit, accounts for most of the "inhibition" previously required in the interpretation of many experiments with high-power–laser-produced plasma.

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Thermal conduction plays an important role in the transport of energy in laser-fusion implosions. The commonly used value of the thermal conductivity was derived by Spitzer and Härm[1] (S-H) assuming that the electron-ion collision mean free path is much smaller than typical temperature scale lengths. In plasmas produced by high-power lasers this assumption fails because of the short scale lengths and high temperatures encountered near the heat front. To avoid nonphysical behavior, the upper limit to the heat flux is often assumed to be the "free-streaming" limit $Q_f = \alpha n_e kT(kT/m)^{1/2}$, where $\alpha \sim 0.65$.[2] Interpretation of many experimental results suggests, however, that α be smaller by about an order of magnitude,[2-4] typically $0.03 \leq \alpha \leq 0.1$. The use of such a small value of α, without a physical basis, is unsatisfactory, and has led to large uncertainties in target design and simulation of experiments.[4] Recent studies[5,6] suggest that classical Coulomb collisions, and not anomalous processes, are dominant in thermal electron transport. A numerical solution[5] to the Fokker-Planck equation on a static system indicates a reduction of the thermal heat flux in steep temperature gradients by roughly an order of magnitude below that given by the S-H description.

In this Letter we present a simple extension to the S-H theory by imposing a physically motivated limit on the anisotropic portion of the electron distribution function, resulting in a local description of the electron thermal conduction in steep temperature gradients. Although a unique expression for the heat flux cannot be obtained when there are both local and nonlocal contributions,[5,6] we show that the local contributions account for most of the reduction in the heat flux inferred from experimental evidence[2,3,7,8] and predicted by the Fokker-Planck simulation.[5] Our results are most applicable to collisional plasmas, such as present short-wavelength interaction experiments,[7] as opposed to hot-electron-dominated experiments. These results are easily incorporated into laser-fusion simulation codes, whereas a Fokker-Planck treatment of thermal electron transport in such codes would be prohibitive.

We follow the derivation of the electron thermal conductivity given by Spitzer and Härm,[1] using for simplicity the limit of high ionic charge (Z) in which electron-ion (e-i) collisions are dominant. To include correctly the electron-electron (e-e) collision requires a numerical treatment of the energy and momentum exchange terms.[1] The e-e collisions are approximated by correcting the e-i collision mean free path by a factor $Z/(Z+1)$ to include the e-e momentum exchange.

The Boltzmann equation describing the electron distribution function, $f(x,v,\mu,t)$, in a one-dimensional planar plasma is given by

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial x} - \frac{eE}{m}\left(\mu \frac{\partial f}{\partial v} + \frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu}\right) = \left(\frac{\partial f}{\partial t}\right)_c, \quad (1)$$

where x is the spatial coordinate, v the velocity, and $\mu \equiv \cos\theta$, where θ is the angle the velocity makes with the x direction. E, e, and m represent the electric field, and the charge and mass of an electron, respectively. We approximate the collision operator[1] as $(f_0 - f)/\tau(v)$, where $\tau(v) = \lambda(v)/v$ is the collision time and $\lambda(v)$ is the mean free path at velocity v.

In the presence of small gradients we assume that the distribution function $f(x,v,\mu,t)$ has a weak angular dependence and can be expressed by $f(x,v,\mu,t) = f_0(x,v,t) + \mu f_1(x,v,t)$, where f_0 and f_1 represent the local isotropic and anisotropic components, respectively. In the case of thermal equilibrium f_0 is the local Maxwellian, and an expression for f_1 can be obtained by taking the first angular moment of Eq. (1). In the steady state f_1 is given by

$$f_1(x,v) = -\lambda(v)\left(\frac{\partial f_0}{\partial x} - \frac{eE}{mv}\frac{\partial f_0}{\partial v}\right). \qquad (2)$$

The electric field, E, is required to preserve charge neutrality, which is equivalent to the zero-current condition given by $J = (4\pi e/3)\int_0^\infty v^3 f_1\,dv = 0$. E is obtained by substituting Eq. (2) into the $J=0$ condition. We assume Coulomb scattering: $\lambda(v) = \lambda_0(v/v_{\rm th})^4$, where $v_{\rm th}$ is the thermal velocity $(2kT/m)^{1/2}$, and λ_0 is the total scattering mean free path for 90° scattering by multiple collisions at kT $\{\lambda_0 = (kT)^2/[\pi n_e(Z+1)e^4 \ln\Lambda]\}$.[1] Using these assumptions in Eq. (2) yields for the ratio f_1/f_0

$$\frac{f_1}{f_0} = \frac{\lambda_0}{L}\left(\frac{v}{v_{\rm th}}\right)^4\left[\left(\frac{v}{v_{\rm th}}\right)^2 - 4.0\right], \qquad (3)$$

where we assume T decreases with increasing x and $L \equiv (T/|dT/dx|)$. Finally, the net heat flux, Q, is defined by $Q = (4\pi m/6)\int_0^\infty v^5 f_1\,dv \equiv \int_0^\infty Q(v)\,dv$, which upon substitution of Eq. (3), yields Fourier's law for heat conduction: $Q = -\kappa\, dT/dx$, where κ is the S-H electron thermal conductivity for high-Z plasmas.

From Eq. (3) it can be seen that f_1/f_0 increases with λ_0/L and at some velocity, depending on λ_0/L, becomes greater than unity. However, the S-H diffusion description is not valid for $f_1 \gtrsim f_0$ because the distribution function, f, becomes negative for some μ.[9] Furthermore, for any transport description the particle flux, $\int d\mu\,\mu f(\mu) \equiv f_1/3$, cannot exceed the free-streaming value $\bar{\mu}_{\max} f_0$, where $\bar{\mu}_{\max}$ is the maximal average μ of the distribution function. For a half-isotropic distribution streaming into a vacuum, $\bar{\mu}_{\max} = 0.25$, resulting in $f_1 \lesssim 0.75 f_0$. At those velocities for $f_1 \lesssim f_0$, the S-H heat flux, $Q(v)$, becomes unphysically large, independent of the assumed transport treatment.

In the present work no attempt has been made to solve the transport equation in order to obtain the actual $f_1(v)$. However, a technique[10] to extend the validity of the diffusion approximation to the range of steep gradients is to physically limit $f_1(v)$ to its upper value $f_{1,m}(v)$ which should be close to $f_0(v)$. By applying this procedure to the S-H diffusion formulation, one obtains an (approximate) extension of the S-H local description to estimate the heat flux in steep temperature gradients. Flux limiting f_1 before calculating the integrated Q is the proper and physical way of applying this technique, in contrast with the commonly used free-streaming estimate obtained by limiting the integrated Q. In order to calculate the maximum *local* heat flux we choose $f_{1,m}$ to be the local Maxwellian $f_0(v)$. This assumption is approximately justified for a collisional plasma, where the gradients are not too steep ($\lambda_0/L \lesssim 0.1$; see the later discussion of Fig. 2). For steeper gradients,[6] the nonlocal contributions due to collisionless electrons will dominate the heat flow and the present local theory is not adequate. In order to carry out this limiting procedure self-consistently, we proceed as follows: (i) Let $f_{1l} = \min[f_1(v), f_{1m}(v)]$, where $f_1(v)$ is given by Eq. (2). (ii) Substitute $f_{1l}(v)$ into the $J=0$ condition to yield an expression for the electric field E:

$$E = -\left[\int_0^{v_c} dv\, v^3 \lambda(v)\frac{\partial f_0}{\partial x} + \int_{v_c}^\infty dv\, v^3 f_{1m}\right]\left[\frac{e}{m}\int_0^{v_c} dv\, v^3 \frac{\lambda(v)}{v}\frac{\partial f_0}{\partial v}\right]^{-1},$$

where v_c is the velocity above which f_1 is limited to f_{1m}. (iii) Finally, the self-consistency of the process is completed by requiring that v_c satisfy the condition $f_1(v_c) = f_{1m}(v_c)$, when the above expression for E is substituted into Eq. (2) for f_1. Having determined v_c (and therefore E), the self-consistent flux-limited $f_{1l}(v)$ gives the new upper bound for the net heat flux. We note that using a limited f_{1l}, without self-consistently determining E, results in nonzero currents, and for $\lambda_0/L \gtrsim 0.05$, negative net Q's.

The results of the above treatment are compared to S-H theory in Fig. 1. Spitzer-Härm theory predicts that the bulk of the energy is carried by electrons with velocity between $2v_{\rm th}$ and $3.5v_{\rm th}$. In Fig. 1(a), $\lambda_0/L = 0.002$, where S-H theory is expected to be accurate, f_1 exceeds its maximum value only at $v \approx 3v_{\rm th}$, and, since Q is insensitive to $Q(v)$ in this range, the flux-limiting procedure does not significantly change Q from the S-H heat flux for this small λ_0/L. In contrast, note that for $\lambda_0/L = 0.1$ [Fig. 1(b)], which violates the assumptions of S-H theory as illustrated by f_1 which exceeds f_0 near $v \approx 2v_{\rm th}$, limiting f_1 sharply reduces the heat flux $Q(v)$. Limiting the positive portion of f_1 also results in a substantial reduction in the return current needed to preserve neutrality, and hence a reduction in the required $\boldsymbol{E}$.

The reduction of the heat flux below the S-H value is illustrated in Fig. 2 as a function λ_0/L.

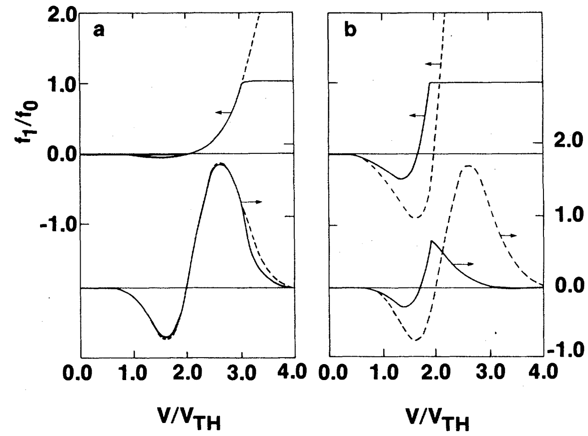

FIG. 1. Spitzer-Härm (dashed curves) and self-consistent flux-limited (solid curves) particle flux, f_1/f_0, and heat flux, $Q(v)$ (in relative units), for (a) $\lambda_0/L = 0.002$, and (b) $\lambda_0/L = 0.1$. The maximum absolute value of $Q(v)$ illustrated in (a) is 0.02 of the value in (b).

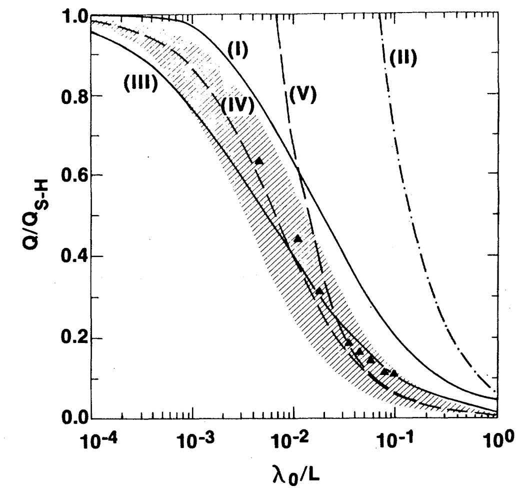

FIG. 2. Reduction of Spitzer-Härm electron thermal flux as a function of λ_0/L for $Z = 4$; Curve I, self-consistent limitation ($f_1 \leq f_0$) with a sharp cutoff (see Fig. 1); Curve II, free-streaming net flux limitation ($\alpha = 0.65$) with a sharp cutoff; Curve III, same as I with $f_1 \leq 0.75 f_0$ with a harmonic cutoff; Curve IV, same as II with $\alpha = 0.06$ and a harmonic cutoff; and curve V, same as IV with a sharp cutoff. The shaded region is bounded by $0.03 \leq \alpha \leq 0.1$ using a harmonic cutoff. The solid triangles are the results from Ref. 5. Note that the λ of Ref. 5 defined at $\frac{3}{2}kT$ corresponds to $2.25\lambda_0$ here.

We choose $Z = 4$ for comparison with Ref. 5, and the e-e contribution to κ was included by using the δ_T of Ref. 1 (for $Z = 4$, $\delta_T \sim 0.5$). The plotted range of λ_0/L extends from 10^{-4}, where S-H theory applies, to unity, where nonlocal transport effects dominate. Curve I shows the reduction obtained from the self-consistent treatment when f_1 is limited to its maximum physical value f_0. *This limitation represents a new upper limit to the local heat flux, which is substantially lower than the free-streaming flux (Q_f with $\alpha = 0.65$, curve II).* Note that the upper bound obtained agrees with the numerical results (the points plotted were obtained from Ref. 5) for the heat fluxes to within a factor of 2 or better over the entire range $0.004 \leq \lambda_0/L \leq 0.1$. For more collisionless plasmas, e.g., those dominated by hot electrons[6] ($\lambda_0/L \gtrsim 0.1$), the present theory must be modified to include deviations of the isotropic portion of the distribution, f_0, from a local Maxwellian prescription.

To obtain the correct net heat flux as a function of λ_0/L the actual dependence of f_1 on v must be obtained. Intuitively one expects f_1 to make a smooth transition to an upper limit (f_{1m}), somewhat below f_0, resulting in a further reduction of the net heat flux. To estimate this effect in analogy with techniques often employed in transport calculations[10] the transition of f_1 to its maximum value (f_{1m}) is obtained by use of a "harmonic" mean, $f_{1l} = (f_1^{-1} + f_{1m}^{-1})^{-1}$. (This procedure requires a numerical search for E.) Curve III (Fig. 2) shows the results obtained by this method for $f_{1m} = 0.75 f_0$. A choice of f_{1m} between $0.5 f_0$ and f_0 is not crucial since Q varies only by 10–25% over this range of f_{1m}. The result of this local treatment (curve III) yields an order of magnitude reduction in the heat flux, in the range $0.03 \leq \lambda_0/L \leq 0.1$, which is typical of the conditions at the top of the heat front where the main thermal inhibition occurs (see Fig. 2 of Ref. 4) and can be seen to agree with the results from Ref. 5. Note that in this region of λ_0/L, the mean free path, λ, of the electrons carrying most of the energy (for $V \sim 2v_{\mathrm{th}}$, $\lambda \simeq 16\lambda_0$) is approximately equal to the temperature gradient scale length, L, supporting our premise that the heat flux there is predominantly local (one might anticipate this result by analogy with the results for the minimum thickness for a strong shock).[11] Our local treatment cannot predict the preheating at the base of the front where nonlocal contributions dominate, and the heat flux cannot be described in terms of local ($\kappa \nabla T$) variables.

The reduction of the heat flux obtained by use of a variety of methods for various values of the flux limiter α are also illustrated in Fig. 2. The shaded area, indicating the "inhibition" obtained for $0.03 < \alpha < 0.1$ from harmonic-mean methods used to fit experimental results,[2,7,8] encompasses both curve III and the Ref. 5 ($\alpha \sim 0.1$) results. Curve IV shows that the harmonic-mean method with $\alpha = 0.06$ is in reasonable agreement with our self-consistent model as a function of λ_0/L. Another method,[4] which computes the minimum of the S-H and Q_f with $\alpha = 0.06$ (curve V) agrees with our results only for $\lambda_0/L \gtrsim 0.02$.

In conclusion we have presented a simple self-consistent model for the thermal heat flux limitation in steep temperature gradients giving a new upper bound much lower than that of the free-streaming limit. An advantage of this limitation model is that the effect of the local contributions to the heat flow are clearly isolated. It also appears that the trend indicated by the particular Fokker-Planck result of Ref. 5 for the reduced flux at the top of the heat front is of general applicability.

Our analysis shows that the need for the anomalously small α's to fit experimental results resulted from a misinterpretation of the free-streaming limit as representative of the maximum heat flux which can be carried by a plasma in extreme conditions. Furthermore, the reduction is related to the conditions at the heat front itself, dependent on λ_0/L, and not directly to physical processes unique to the laser-plasma interaction, and therefore should apply at all stages of pellet implosion involving heat flow.

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