also assure us that it makes remarkably good sense, at least for spherical nuclei, to construct microscopic collective potentials $V(\beta,\gamma)$, as we have done, for example, in calculating rotational states for ^{20}Ne, and to think of them picturesquely in terms of the hydrodynamical collective model.[2] In a following paper we shall investigate the liquid limit of the $\lambda_0 \neq 0$, $\mu_0 \neq 0$ sp(3,R) representations and their application to the collective states of deformed nuclei. For these nuclei, low-lying collective states in addition to the giant resonances are obtained as coherent mixtures of $0\hbar\omega$ Elliott u(3) wave functions with high-lying states.

We would like to acknowledge support by the National Science Foundation and the Science and Engineering Research Council of Canada.

[1]A. Bohr, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 27, No. 14 (1952); A. Bohr, B. Mottelson, and J. Rainwater, Rev. Mod. Phys. 48, 365 (1976), and references therein.

[2]G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. 38, 10 (1977), and Ann. Phys. (N.Y.) 126, 343 (1980).

[3]F. Arickx, Nucl. Phys. A284, 264 (1977), and A268, 347 (1976); F. Arickx, J. Broeckhove, and E. Deumens, Nucl. Phys. A318, 269 (1979).

[4]K. T. Hecht and D. Braunschweig, Nucl. Phys. A295, 34 (1978).

[5]D. R. Peterson and K. T. Hecht, Nucl. Phys. A344, 361 (1980).

[6]J. P. Elliott, Proc. Roy. Soc. London, Ser. A 245, 128, 562 (1958); M. Harvey, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 1.

[7]S. Goshen and H. J. Lipkin, Ann. Phys. 6, 301 (1959).

[8]T. Suzuki, Nucl. Phys. A217, 182 (1973), and A220, 569 (1974).

[9]G. Rosensteel and D. J. Rowe, Ann. Phys. 104, 134 (1977).

Nuclear Saturation from Two-Nucleon Potentials

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Results of nuclear-matter calculations are presented for several nucleon-nucleon potentials that fit scattering data and deuteron properties. The results suggest that two-body potentials cannot account quantitatively for nuclear saturation.

PACS numbers: 21.30.+y, 21.65.+f

A long-standing question in nuclear physics is whether nuclear saturation can be quantitatively understood in terms of two-nucleon potentials that are consistent with scattering data and deuteron properties.[1,2] Calculations of sufficient accuracy to answer this question have recently become available. In this note I report results for several nucleon-nucleon potentials. These results form a pattern that tends to confirm the suspicion that no two-nucleon potential can account quantitatively for nuclear saturation.

Nuclear saturation properties are conveniently studied by treating nuclear matter at uniform density. The ground state of this system at any density can be represented by a Fermi-gas state modified by amplitudes for the excitation of n correlated particle-hole pairs ($n=2,3,\ldots$).[3-5] If the many-body Hamiltonian involves only two-body interactions, an exact expression for the energy involves only the two-particle–two-hole excitation amplitude. I have developed a method for solving the Schrödinger equation for this amplitude with adequate accuracy. The details of the method are given in Ref. 6 and tests of its accuracy are reported in Refs. 7 and 8.

The results are shown in Fig. 1, where energy per particle E/A is plotted against Fermi momentum k_F, which is related to the density ρ by $\rho = 2k_F{}^3/3\pi^2$. The solid circles are saturation points calculated in the lowest-order approximation with various two-body potentials that are fitted to scattering data and the deuteron. The potentials and their designations in Fig. 1 are as follows. The Reid[9] soft-core potential in two-body channels with $j \leq 2$, augmented by the potentials of Ref. 6 in higher partial waves (for which Reid's paper gives no potential), is called the modified Reid soft-core potential and designated MRSC. The MRSC potential with the $^1P_1$, $^3P_1$, and $^3P_2$-$^3F_2$ potentials replaced by those of Table I, Model II of Bethe and Johnson[10] is designated MRBJ. The remaining authors and designations

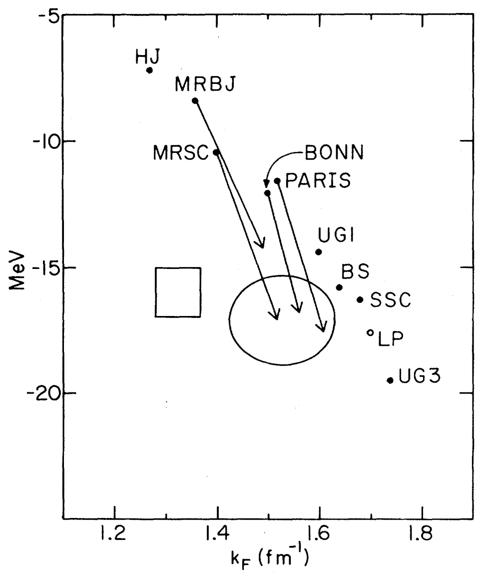

FIG. 1. Calculated saturation points for nuclear matter, as described in the text.

in Fig. 1 are Holinde and Machleidt[11] (BONN), Lacombe *et al.*[12] (PARIS), Hamada and Johnston[13] (HJ), Ueda and Green[14] (UG1 and UG3), Bryan and Scott[15] (BS), and de Tourreil and Sprung[16] (SSC).

The lowest-order approximation is not adequate. For four potentials the arrows show the shift in saturation point when higher-order terms are included using the method of Ref. 6. The estimated calculational uncertainty in the MRSC saturation point is indicated by the oval, and the uncertainties are similar for the MRBJ, Bonn, and Paris points. The open circle labeled LP is the variational result of Lagaris and Pandharipande[17] using their nucleon-nucleon potential.[18] The empirical saturation point lies inside the rectangular box. It is obtained by calculating nuclear matter with a variety of soft, effective two-body interactions that are fitted to binding energies and radii of nuclei (see Table II of Ref. 19).

The lowest-order results lie on a narrow band[20] called the Coester band, which misses the empirical region. The position of a potential along the band is primarily determined[21,22] by the strength of the tensor force, which is measured by the deuteron D-state probability P_D, and secondarily by the strength of the short-range repulsion. Potentials with larger P_D and stronger short-range repulsion saturate at lower binding energy and density. The MRSC and MRBJ potentials have $P_D = 6.5\%$, Bonn and Paris have $P_D = 5.8\%$, and LP has $P_D = 5.2\%$.

The MRSC and MRBJ potentials reduce to the one-pion-exchange potential at distances $r > 2$ fm but are otherwise phenomenological. The Bonn potential includes theoretical contributions from exchange of π, σ, η, δ, φ, ρ, and ω mesons, modified by phenomenological form factors at high momentum (i.e., at short distance). The fictitious σ meson mocks the intermediate-range attraction due to two-pion exchange. The Paris potential is calculated theoretically for $r > 0.8$ fm and is phenomenological for smaller r. The long-distance behavior is given by one-pion exchange. The remaining theoretical components of the Paris potential are obtained by using empirically determined pion-nucleon and pion-pion scattering amplitudes in conjunction with dispersion relations. In the Paris and Bonn potentials the tensor force, which comes mainly from π exchange, is weakened by ρ exchange. The MRSC and MRBJ potentials do not include ρ exchange. An interpretation of certain few-body reaction data suggests that the tensor force of MRSC and MRBJ may be too strong.[23]

When MRSC was found to saturate at too high a density, MRBJ was tried, in the hope that by starting in lowest order further up the Coester band, one might reach the empirical region. However, this clearly does not happen. The MRBJ, Bonn, and Paris saturation points suggest a new band that is parallel to the Coester band and still quite far from the empirical region. The reason that MRSC lies off this new band may be that it gives rather different phase shifts from the others in the $^1P_1$ channel. For example, the $^1P_1$ contribution, in lowest order, is more repulsive for the Paris potential than for MRSC. If this difference in lowest-order results is added to the full MRSC saturation curve, the saturation point moves higher by 2.3 MeV with no change in density. The higher-order terms, for MRSC with the $^1P_1$ potential replaced by that of the Paris potential, have not been calculated. Under the assumption that they are similar to those for MRSC, the tip of the MRSC arrow would move higher by about 2 MeV and lie on the band suggested by the MRBJ, Bonn, and Paris points.

If the calculational uncertainties in these four potentials were of random sign and as large as the oval in Fig. 1, they would not define a new band with any precision. However, since the calculations are all done in the same way, it is likely that the errors are of the same sign and order of magnitude in all four cases, thus allowing a new band to be seen. The LP variational result lies somewhat off this band. Comparison of results from the present method with variational results, for various test potentials, has been extremely helpful in testing the accuracy of both methods. There may be a tendency for the variational calculation to predict a higher saturation density (see Fig. 10 of Ref. 7). The difference is within the expected uncertainty, and it is not known what effect the use of a more sophisticated variational wave function would have on the LP saturation point.

The systematic trend of results with a variety of two-body potentials strongly suggests that two-body potential models cannot quantitatively account for nuclear saturation. The discrepancy is displayed in a different way in Fig. 2, which shows an empirical saturation curve in comparison with the MRSC saturation curve, and their difference ΔE. The empirical curve is a parabola saturating at $k_F = 1.33$ fm^{-1}, $E/A = -16$ MeV. Its curvature is determined by requiring $k_F{}^2 d^2(E/A)/dk_F{}^2 = 210$ MeV at the saturation point, in accord with the analysis of breathing-mode data by Blaizot, Gogny, and Grammaticos.[24] The empirical curve is expected to be reliable in the neighborhood of the minimum. Its extrapolation all the way to $k_F = 1.8$ fm^{-1} is not supported by data but is made in order to show the qualitative nature of the discrepancy with the MRSC calculation.

According to Fig. 2, in order to repair the discrepancy ΔE between the MRSC and empirical saturation curves, we need a physical effect that gives more binding at low density and less binding at high density. The effects of three-body

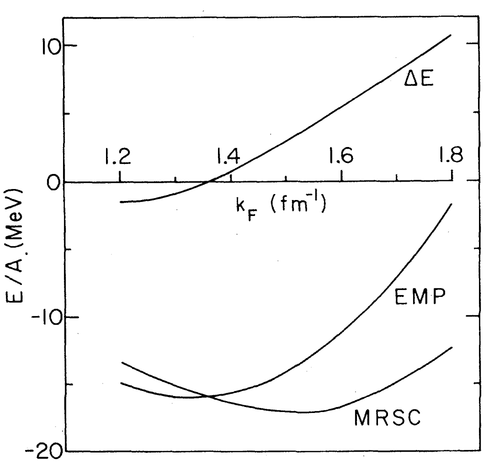

FIG. 2. Empirical (EMP) and calculated (MRSC) saturation curves for nuclear matter, and their difference (ΔE).

forces[25] and of virtual isobars[26] in nuclear matter are being actively investigated in the search for an explanation.

The fact that two-body potential models give too little binding at lower densities is in agreement with earlier calculations of light nuclei. The average density of light nuclei is below the saturation density of nuclear matter, and the work of Kümmel, Lührmann, and Zabolitzky[4] has shown that ^{4}He, ^{16}O, and ^{40}Ca are all underbound by the Reid soft-core potential, which also underbinds the triton.[27]

In conclusion, it is unlikely that any two-body potential fitted to scattering data and deuteron properties can account quantitatively for nuclear saturation. An additional effect must be found that gives more binding below the empirical saturation density and less binding at higher densities.

I am indebted to F. Coester and J. G. Zabolitzky for discussions of nuclear-matter theory, to S. Pieper for advice on problems of computing, to R. V. Reid for supplying potentials in higher partial waves for the MRSC potential, to R. Machleidt and K. Holinde for discussion of the Bonn potential and for a computer program to evaluate its matrix elements, to B. Loiseau for formulas used in programming the Paris potential, and to T.-S. H. Lee for the use of several potential and phase-shift programs. This work is supported under Contract No. W-31-109-ENG-38 by the U. S. Department of Energy.

[1]B. D. Day, Rev. Mod. Phys. 50, 495 (1978).

[2]V. R. Pandharipande and R. B. Wiringa, Rev. Mod. Phys. 51, 821 (1979).

[3]F. Coester, in *Lectures in Theoretical Physics: Quantum Fluids and Nuclear Matter*, edited by K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1969), Vol. XI B.

[4]H. Kümmel, K. H. Lührmann, and J. G. Zabolitzky, Phys. Reps. 36C, 1 (1978).

[5]B. D. Day, in Proceedings of the Varenna Summer School "From Nuclei to Particles," June 1980 (to be published).

[6]B. D. Day, Phys. Rev. C, to be published.

[7]B. D. Day, in *The Meson Theory of Nuclear Forces and Nuclear Matter*, edited by D. Schütte, K. Holinde, and K. Bleuler (Bibliographisches Inst., Zurich, 1980).

[8]B. D. Day, in Proceedings of the International Conference on Recent Progress in Many-Body Theories, Oaxtepec, Mexico, 1981 (to be published).

[9]R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[10]H. A. Bethe and M. B. Johnson, Nucl. Phys. A230, 1 (1974).

[11]K. Holinde and R. Machleidt, Nucl. Phys. A247, 495

(1975).
[12]M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Coté, P. Pirès, and R. de Tourreil, Phys. Rev. C 21, 861 (1980).
[13]T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).
[14]T. Ueda and A. E. S. Green, Phys. Rev. 174, 1304 (1968).
[15]R. Bryan and B. L. Scott, Phys. Rev. 177, 1435 (1969).
[16]R. de Tourreil and D. W. Sprung, Nucl. Phys. A201, 193 (1973).
[17]I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. A359, 349 (1981).
[18]I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. A359, 331 (1981).
[19]J. L. Friar and J. W. Negele, Adv. Nucl. Phys. 8, 219 (1975).
[20]F. Coester, S. Cohen, B. Day, and C. M. Vincent, Phys. Rev. C 1, 769 (1970).
[21]M. I. Haftel and F. Tabakin, Phys. Rev. C 3, 921 (1971).
[22]I. R. Afnan, D. M. Clement, and F. J. D. Serduke, Nucl. Phys. A170, 625 (1971).
[23]E. L. Lomon, in *Nucleon-Nucleon Interactions—1977*, edited by H. Fearing, D. Measday, and A. Strathlee, AIP Conference Proceedings No. 41 (American Institute of Physics, New York, 1977), p. 554.
[24]J. P. Blaizot, D. Gogny, and B. Grammaticos, Nucl. Phys. A265, 315 (1976).
[25]V. R. Pandharipande, in Proceedings of the International Conference on Recent Progress in Many-Body Theories, Oaxtepec, Mexico, 1981 (to be published).
[26]R. B. Wiringa, to be published.
[27]Y. E. Kim and A. Tubis, Annu. Rev. Nucl. Sci. 24, 69 (1974).

Λ Production near Threshold in Central Nucleus-Nucleus Collisions

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Λ's produced in central collisions of ^{40}Ar + KCl at 1.8-GeV/u incident energy were detected in a streamer chamber by their charged-particle decay. For central collisions with impact parameters $b < 2.4$ fm the Λ production cross section is 7.6 ± 2.2 mb. A calculation in which Λ production occurs in the early stage of the collision qualitatively reproduces the results but underestimates the transverse momenta. An average Λ polarization of -0.10 ± 0.05 is observed.

PACS numbers: 25.70 Bc

In the study of high-energy nucleus-nucleus collisions it is difficult to extract information about the initial stage of the reaction where high baryon densities may occur. Studies of nucleon and cluster emission[1] are consistent with a development towards chemical equilibrium in the final stages of the reaction preempting information about the primary stages. In this Letter we report the results of Λ production in central nucleus-nucleus collisions, just above the *NN*