tion energy is shared between the fragments in proportion to their masses, i.e., the composite system is thermalized prior to the separation of the fragments. Figure 4(c) shows the c.m. energy spectrum of neutrons emitted by the H fragment taking PM No. 5 as reference counter. A much higher temperature is obtained as compared with Fig. 4(b), reflecting the effect of the relatively high-energy nonequilibrium neutrons present in PM No. 5.

To conclude, we have shown that the neutron emission in deep-inelastic collisions of ^{86}Kr on ^{166}Er at 12 MeV/amu cannot be accounted for by assuming only isotropic evaporation by fully accelerated fragments. The experimental analysis shows the presence of a nonequilibrium component of neutrons emitted mainly on the side of the L fragment in quasielastic events, and on the side of the H fragment in strongly damped events. In both cases the results are consistent with a broad velocity distribution centered around the beam velocity. Yet our results clearly indicate (see, for example, the spectra of PM No. 3, Fig. 2) that the nonequilibrium neutrons do not have an angular distribution which is symmetric around the beam direction.

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U(6/4) Dynamical Supersymmetry in Nuclei

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We suggest that a supersymmetry scheme based on the supergroup U(6/4) may be useful in describing many properties of nuclei in the Os–Pt region. The bosons and fermions in the fundamental representation of U(6/4) are the low-lying collective (bosonic) and single-particle (fermionic) degrees of freedom. Experimental evidence indicates that the scheme applies to several nuclei in the region within ≈30%. This appears to be the first observed example of a supersymmetry.

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Recently, one of us[1] has suggested that dynamical supersymmetries may be present in the spectra of complex nuclei. This suggestion was based on the comparison of the excitation energies of a pair of nuclei in the Os–Pt region. However, it was neither clear which supergroup was relevant to the problem, if any, nor the extent to which the supersymmetry was experimentally present. We have, therefore, performed a more detailed investigation of the problem and in this Letter we wish to summarize our results. A more elaborate account of this work will appear in a longer publication.

Supergroups are relevant to mixed systems of bosons and fermions. Contrary to normal symmetry operations, which transform either fermions into fermions or bosons into bosons, supersymmetry operations have also pieces that transform bosons into fermions and vice versa. For the problem discussed here, the bosons are the low-lying collective degrees of freedom of a heavy nucleus. It has been shown[2] that these can be well described by six dynamical bosons, divided into a scalar, $J=0$ (called s), and a quadrupole, $J=2$ (called d), boson, and assigned to the six-dimensional representation of U(6). Introducing boson creation and annihilation operators, altogether denoted by $b_\alpha^\dagger$ (b_α), $\alpha=1,\ldots,6$, we can construct the 36 generators of U(6) as $G_{\alpha\alpha'}{}^{(B)}=b_\alpha{}^\dagger b_{\alpha'}$. The bosons can also be thought of as highly correlated pairs of identical nucleons with $J=0$ and $J=2$, respectively, similar to the Cooper pairs of the electron gas. Although in a more detailed model[3] one distinguishes between neutron (ν) and proton (π) pairs (bosons), we shall neglect this difference here and only speak of bosons. The number of bosons in a given even-even nucleus is then taken as the number $N=N_\pi+N_\nu$ of active proton and neutron pairs outside the major closed shells at 8, 20, 28, 50, 82, and 126. This is actually either the number of particle pairs or of hole pairs, whichever is smaller.

In addition, when describing states in odd-even nuclei, or states in even-even and odd-even nuclei with two or more unpaired particles (two or more quasiparticle states[4]), one needs to introduce explicitly fermionic degrees of freedom.[5] The dimension of the fermionic space is $m=\sum_i(2j+1)$, where j_i are the values of the angular momenta contained in a major shell. For example, for the shell 50–82, $j_i=\frac{5}{2},\frac{7}{2},\frac{11}{2},\frac{3}{3},\frac{1}{2}$, and $m=32$. The fermions can be assigned to the m-dimensional representation of the group U(m). Introducing creation and annihilation operators for fermions, altogether denoted by $a_i{}^\dagger(a_i)$, $i=1,\ldots,m$, the m^2 generators of U(m) can be written as $G_{ii'}{}^{(F)}=a_i{}^\dagger a_{i'}$.

The mixed problem of bosons and fermions which we want to study is described by the Hamiltonian

$$H=H_B+H_F+V_{BF},\quad H_B=H_0+\sum_{\alpha,\alpha'}\epsilon_{\alpha\alpha'}G_{\alpha\alpha'}{}^{(B)}+\sum_{\alpha,\alpha',\beta,\beta'}u_{\alpha\alpha'\beta\beta'}G_{\alpha\alpha'}{}^{(B)}G_{\beta\beta'}{}^{(B)},$$
$$H_F=H_0'+\sum_{i,i'}\eta_{ii'}G_{ii'}{}^{(F)}+\sum_{i,i',kk'}v_{ii'kk'}G_{ii'}{}^{(F)}G_{kk'}{}^{(F)},\quad V_{BF}=\sum_{\alpha,\alpha',ii'}w_{\alpha\alpha'ii'}G_{\alpha\alpha'}{}^{(B)}G_{ii'}{}^{(F)},\tag{1}$$

where H_0,H_0' are invariant under $\mathrm{U}^{(B)}(6)$ and $\mathrm{U}^{(F)}(m)$, respectively, and the coefficients $\epsilon_{\alpha\alpha'}$, $u_{\alpha\alpha'\beta\beta'}$, $\eta_{ii'}$, $v_{ii'kk'}$, and $w_{\alpha\alpha'ii'}$ are chosen so as to conserve the angular momenta both of the bosons and of the fermions. In general, the group structure of this problem is $\mathrm{U}^{(B)}(6)\otimes\mathrm{U}^{(F)}(m)$, where we have placed a superscript B and F in order to distinguish bosons from fermions. If there is no further symmetry, spectra of nuclei with different values of the number of bosons, N, and of fermions, M, will have no reason to be related to each other. However, suppose that we assign the dynamical bosons and fermions to the same representation of a group larger than $\mathrm{U}^{(B)}(6)\otimes\mathrm{U}^{(F)}(m)$. Then spectra of nuclei belonging to the same representation of this larger group will be related by the symmetry operations of the larger group. Since these representations contain both bosonic and fermionic states, it cannot be an ordinary Lie group, but a supergroup. Supergroups have been introduced in the context of dual models,[6] supersymmetric field theories,[7] and supergravity.[8]

The supergroup appropriate to our problem appears to be U(6/m). We can explicitly construct the generators of supergroups U(n/m) by means of creation and annihilation operators. The generators are

$$\begin{aligned}&G_{\alpha\alpha'}{}^{(B)}=b_\alpha{}^\dagger b_{\alpha'}\quad (n^2),\\&G_{ii'}{}^{(F)}=a_i{}^\dagger a_{i'}\quad (m^2),\\&F_{\alpha i}{}^\dagger=b_\alpha{}^\dagger a_i\quad (mn),\\&F_{i\alpha}=a_i{}^\dagger b_\alpha\quad [mn/(m+n)^2],\end{aligned}\tag{2}$$

which we can place in a matrix form[9]

$$\begin{pmatrix} b^\dagger b & b^\dagger a\\ a^\dagger b & a^\dagger a\end{pmatrix}.\tag{3}$$

The Bose sector of the algebra is $\mathrm{U}^{(B)}(n)\otimes\mathrm{U}^{(F)}(m)$.

The representation theory of supergroups of the type U(n/m) has been worked out by two of us[10] and by other authors.[11] In our work, representations are characterized by Young supertableaux. These are similar in appearance to normal tableaux but very different in nature. For example, in the completely supersymmetric tableaux of Fig. 1, all bosonic indices are symmetrized but all fermionic indices are antisymmetrized. We have placed a slash through the boxes in order to emphasize the difference.

When only one single-particle level is important in a certain region of nuclei, the dimension m, which in general is rather large, becomes small. For example, in the Os–Pt region, the states with unpaired protons are dominated by the $2d_{3/2}$ level. Thus, $m=2j+1=4$ and the appropriate supergroup is U(6/4). The completely supersymmetric representations of U(6/4) are characterized by an integer $\mathfrak{N}$, the total number of bosons plus fermions $\mathfrak{N}=N+M$. The representation of $\mathrm{U}^{(B)}(b)\otimes\mathrm{U}^{(F)}(4)$ contained in each representation $[\mathfrak{N}]$ of U(6/4) are given by the direct product representations $\sum_k([\mathfrak{N}-k],\{k\})$, where the square brackets denote symmetric representations of $\mathrm{U}^{(B)}(6)$ and the curly brackets, antisymmetric representations of $\mathrm{U}^{(F)}(4)$. The sum stops at $\mathfrak{N}-k=0$ or $k=4$, whichever comes first. As a consequence, we expect the following states to belong to the same representation of U(6/4): $|N=\mathfrak{N},M=0\rangle$, $|N=\mathfrak{N}-1,M=1\rangle$, $|N=\mathfrak{N}-2,M=2\rangle$, $|N=\mathfrak{N}-3,M=3\rangle$, and $|N=\mathfrak{N}-4,M=4\rangle$.

Starting with any even-even nucleus, characterized by a certain number of bosons (pairs) N, we can then construct a supermultiplet by acting with the supergenerators $F=a^\dagger b$. For example, starting with ${}^{190}_{76}\mathrm{Os}_{114}$ ($N=9$), we can construct the supermultiplet shown in Fig. 2. If the supersymmetry scheme applies, all states in the supermultiplet should be described by the same energy formula,

$$E=E_0+E_1'N+E_2'N^2+E_3'\mathfrak{N}+E_4'\mathfrak{N}^2+E_5'\mathfrak{N}N-\tfrac{1}{4}A_1\Sigma(\Sigma+4)$$
$$-\tfrac{1}{4}A_2[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+\sigma_3{}^2]+\tfrac{1}{6}B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+CJ(J+1), \qquad (4)$$

corresponding to the chain of subgroups[1]

$$\mathrm{U}(6/4)\supset\mathrm{U}^{(B)}(6)\otimes\mathrm{U}^{(F)}(4)\supset\mathrm{SO}^{(B)}(6)\otimes\mathrm{SU}^{(F)}(4)\supset\mathrm{Spin}(6)\supset\mathrm{Spin}(5)\supset\mathrm{Spin}(3)\supset\mathrm{Spin}(2). \qquad (5)$$

In Eq. (4), E_0, E_1', E_2', E_3', E_4', E_5', A_1, A_2, B, and C are arbitrary parameters and the quantum numbers N, $\mathfrak{N}$, Σ, σ_1, σ_2, σ_3, τ_1, τ_2, and J label the representations of the groups in (5). We have performed a numerical analysis of the excitation energies of the pair of nuclei ${}^{190}_{76}\mathrm{Os}_{114}$-${}^{191}_{77}\mathrm{Ir}_{114}$, Table I. Excitation energies are in general given in terms of four parameters A_1, A_2, B, and C. However, for the observed states, only B and C enter. As a measure of the deviation of the observed energies from Eq. (4), we define the quantity $\varphi=\sum_i|E_i^{\mathrm{theor}}-E_i^{\mathrm{expt}}|/\sum_i E_i^{\mathrm{expt}}$, where the sum goes over all observed states. From Table I, we find $\varphi\approx 19\%$. A similar test for the pair ${}^{192}_{76}\mathrm{Os}_{116}$-${}^{192}_{77}\mathrm{Ir}_{116}$ gives $\varphi\simeq 18\%$.

In addition, we have performed other tests of the U(6/4) supersymmetry predictions. These tests, which will be reported in our longer article, include (i) a comparison of two-neutron separation energies in Os and Ir; (ii) a study of electromagnetic ($E2$) transition rates in Os, Ir, Pt, and Au; and (iii) a study of one- and two-nucleon transfer reactions in Os, Ir, Pt, and Au. Related tests have been performed by Wood,[14] Harekeh *et al.*,[15] Iwasaki *et al.*,[16] Vergnes *et al.*,[17] and Cizewski.[18]

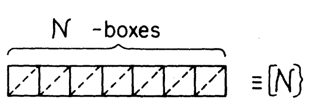

FIG. 1. The supertableau characterizing the completely supersymmetric representations of U(n/m).

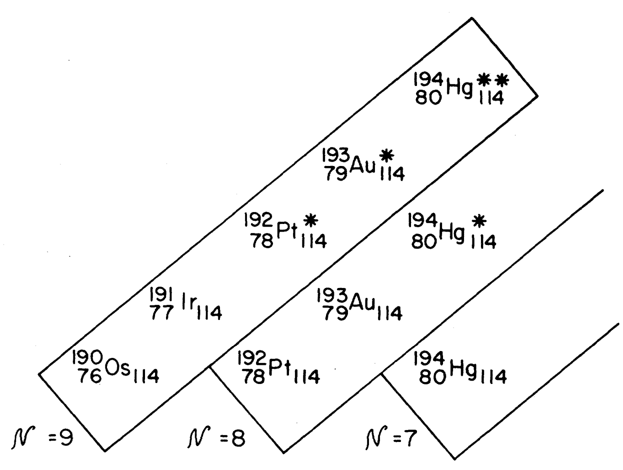

FIG. 2. Possible supersymmetric multiplets in the Os–Pt region. $\mathfrak{N}$ denotes the total number of bosons plus fermions. The states with two, three, and four unpaired nucleons are excited configurations in the corresponding nuclei. They are labeled by one or two stars.

TABLE I. Excitation energies in $^{190}_{76}Os_{114}$ and $^{191}_{77}Ir_{114}$. The theoretical energies are calculated with use of Eq. (5) with $\frac{1}{6}B = 40$ keV and $C = 10$ keV for both nuclei. The experimental energies are taken from Refs. 12 and 13, and $\Delta = E^{theor} - E^{expt}$.

Nucleus	σ_1	τ_1	J^π	E^{theor} (keV)	E^{expt} (keV)	Δ (keV)
$^{190}_{76}Os_{114}$	9	0	0^+	0	0	0
		1	2^+	220	186	+34
		2	4^+	600	548	+52
			2^+	460	557	−97
		3	6^+	1140	1050	+90
			4^+	920	955	−35
			3^+	840	756	+84
			0^+	720	912	−192
$^{191}_{77}Ir_{114}$	17/2	1/2	$3/2^+$	0	0	0
		3/2	$7/2^+$	320	343	−23
			$5/2^+$	250	129	+121
			$1/2^+$	170	82	+88
		5/2	$11/2^+$	800	832	−32
			$9/2^+$	690	503	+187
			$7/2^+$	600	686	−86
			$5/2^+$	530	351	+179
			$3/2^+$	480	179	+301
		7/2	$15/2^+$	1420	1418	+2
			$13/2^+$	1270	1004	+266
			$11/2^+$	1140	1207	−67
			$9/2^+$	1030	945	+85
			$9/2^+$	1030	812	+218
			$7/2^+$	940	504	+436
			$5/2^+$	870	748	−122

All tests performed so far appear to indicate that the U(6/4) scheme reproduces the observed data in the pairs of nuclei $^{190}_{76}Os_{114}$-$^{191}_{77}Ir_{114}$, $^{192}_{78}Os_{116}$-$^{193}_{77}Ir_{116}$, $^{192}_{78}Pt_{114}$-$^{193}_{79}Au_{114}$, and $^{194}_{78}Pt_{116}$-$^{195}_{79}Au_{116}$ within 30%. The only large discrepancy observed appears to be that reported by Vergnes *et al.*[17] in the (^{3}He,^{2}H) reaction leading from a member of a $\mathfrak{N}=8$ supermultiplet ($^{193}_{77}Ir_{16}$) to a member of a $\mathfrak{N}=7$ supermultiplet ($^{194}_{78}Pt_{116}$). Since all other reactions involving transfer of particles appear to follow, within 30%, the supersymmetry predictions it is not at all clear whether or not the discrepancy observed by Vergnes *et al.*[17] is related to the structure of the transfer operator or to a breakdown of the supersymmetry itself. Little or no information at all is available so far on the other members of the supermultiplets, Pt*, Au*, and Hg**.

The 30% deviations from the supersymmetry scheme observed in the excitation energies, Table I, and electromagnetic transition rates, can be attributed mainly to admixtures of the $3s_{1/2}$ orbital to the dominant $2d_{3/2}$. This orbital, together with the other positive-parity orbitals in the proton shell 50–82, $2d_{5/2}$ and $1g_{7/2}$ can be included within the framework of the supersymmetry scheme presented here, by increasing the dimension of the fermionic space. Since at this stage our purpose is to provide a semiquantitative, but simple, description of the experimental situation, we have preferred not to include the other orbitals. Numerical calculations which include these orbitals have already been done, both within the framework of the interacting-boson-fermion model[19] and of other models.[20]

In conclusion, even with deviations of order 30%, we find it remarkable that such a complex and highly accidental type of symmetry appears to exist in nature.

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Intermediate Structure Resonances in ^{56}Ni

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Detailed excitation functions of angle-integrated cross sections for ^{28}Si + ^{28}Si elastic scattering and reactions show narrow, highly correlated structures. These resonances have enhanced partial widths for decay into two ^{28}Si fragments and are interpreted as having quasimolecular or fission-isomeric nature.

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The intense interest in the intermediate width structure ($\Gamma_{c.m.} \sim 100$ keV) observed in excitation functions for some heavy-ion reactions stems from the belief that this signifies the existence of highly clustered or moleculelike doorway states in the composite system.[1] For the ^{12}C + ^{12}C and ^{12}C + ^{16}O systems, at energies not too far above the Coulomb barrier, extensive experimental studies have demonstrated the nonstatistical origins of the observed structure and strongly support an interpretation as a true resonance phenomenon.[2-6] For these systems at higher energies[7,8] and for heavier systems,[9-12] the situation is less clear. Structure of width comparable to that seen in much lighter systems has been observed. In most cases, however, the data are insufficient to distinguish between a resonance or statistical origin of the observed structures.[13] The connection between these heavier systems and the ^{12}C + ^{12}C and ^{12}C + ^{16}O systems therefore remains to be made. The possibility that this connection exists is an exciting one, as it would signify the observation of nuclear structure effects at higher excitation energies and larger angular momentum than previously directly accessible, the understanding of which will

