Relativistic Effects in Deuteron Form Factors at Large q^2

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Deuteron electromagnetic form factors are calculated in the relativistic impulse approximation for $q^2 \leq 6$ (GeV/c)^2. Shift in the proton-neutron relative momentum due to Lorentz transformation to the deuteron rest frame is shown to be the most important relativistic kinematical effect. A new treatment of this effect results in a remarkable agreement with the available data. Predictions are made for the tensor polarization in ed elastic scattering. These differ markedly from the standard results.

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Recent measurements[1] of ed elastic-scattering cross sections for $q^2 \leq 6$ (GeV/c)^2 have shown a large disagreement with calculations based on nonrelativistic deuteron wave functions. A typical result of such calculations based on Reid soft-core wave functions[2] appears as a dashed line in Fig. 1. Calculation of the form factor $A(q^2)$, in the impulse approximation (IA), is based on the triangle diagram shown in Fig. 2. Here the standard prescription has been to take the spectator nucleon (N) on its mass shell.[3] In Ref. 3 it was shown that relativistic calculations based on this prescription only led to worsening of the disagreement with the data (dotted line in Fig. 1). However, these results may be strongly dependent upon the prescription used. For instance, taking the struck N (Fig. 2) on its mass shell (see below) one obtains a large enhancement of the calculated form factors (dot-dashed line in Fig. 1). Our treatment of the Bethe-Salpeter equation (BSE) allows us to go beyond the above prescription, with results in very good agreement with the available data (solid lines in Fig. 1). In the following we describe the salient features of this new approach and present our results. Detailed derivation will be given elsewhere.[4]

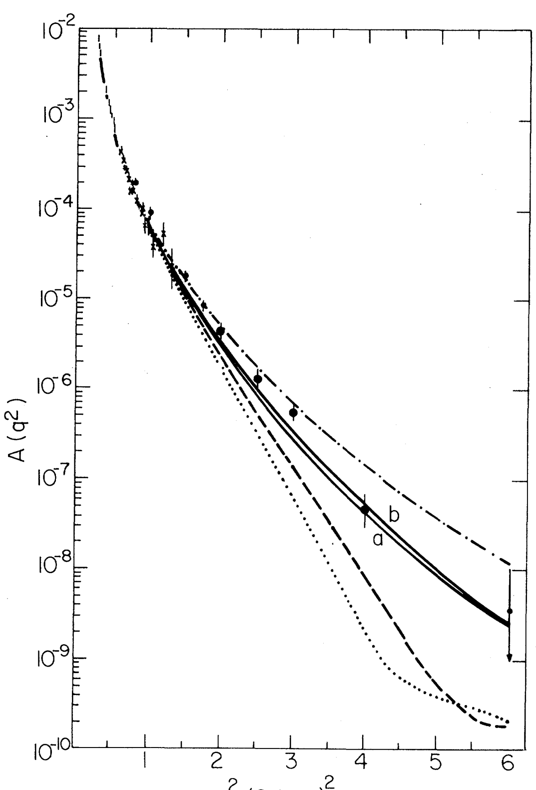

FIG. 1. Results of the present calculations compared with the data of Refs. 1 (circles), 5 (asterisks), and 6 (vertical bars), where the dashed curve corresponds to the nonrelativistic calculations and dotted and dot-dashed curves to relativistic calculations with spectator and struck nucleons on-the mass shell, respectively, all using the Reid soft-core wave functions. Solid curves are our predictions using (a) Reid soft-core and (b) Hamada-Johnston hard-core wave functions.

The quantity $A(q^2)$ is obtained from ed elastic cross-section data[1,5,6] by use of

$$d\sigma_{ed}/d\Omega = (d\sigma^{\mathrm{Mott}}/d\Omega)[A(q^2) + B(q^2)\tan^2\tfrac{1}{2}\theta]. \quad (1)$$

According to IA $A(q^2)$ can be written as[7]

$$A(q^2) = (G_E{}^p + G_E{}^n)^2(S_S{}^2 + S_Q{}^2) + (G_M{}^d)^2, \quad (2)$$

where $G_E{}^p$ and $G_E{}^n$ are electric form factors of proton and neutron, S_S and S_Q are scalar and

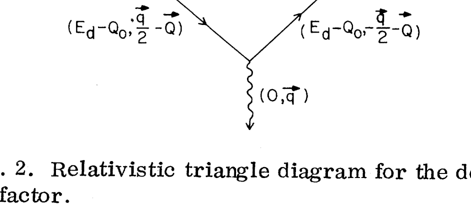

FIG. 2. Relativistic triangle diagram for the deuteron form factor.

quadrupole form factors of deuteron, and $G_m{}^d$ is the small contribution of the deuteron magnetic form factor.[3] In the present calculations $G_E{}^n$ was taken to be zero and $G_E{}^p = (1+q^2/0.71)^{-2}$, where q^2 is in $(\mathrm{GeV}/c)^2$. The form factors S_S and S_Q can be written in terms of deuteron nonrelativistic wave functions as

$$S_S(q) = \int [U(|\vec{Q}-\tfrac{1}{4}\vec{q}\,|)U(|\vec{Q}+\tfrac{1}{4}\vec{q}\,|) + W(|\vec{Q}-\tfrac{1}{4}\vec{q}\,|)W(|\vec{Q}+\tfrac{1}{4}\vec{q}\,|)P_2(Z_{+-})]\, d^3Q/4\pi, \tag{3a}$$

$$S_Q(q) = \int [2U(|\vec{Q}-\tfrac{1}{4}\vec{q}\,|)W(|\vec{Q}+\tfrac{1}{4}\vec{q}\,|)P_2(Z_+) + (1/2\sqrt{2})(3Z_+Z_-Z_{+-}-1)W(|\vec{Q}-\tfrac{1}{4}\vec{q}\,|)W(|\vec{Q}+\tfrac{1}{4}\vec{q}\,|)]\, d^3Q/4\pi, \tag{3b}$$

where U and W are deuteron S- and D-state wave functions in the momentum representation, $Z_\pm = \cos(\vec{Q} \pm \frac{1}{4}\vec{q}, \vec{q}\,)$, and $Z_{+-} = \cos(\vec{Q}+\frac{1}{4}\vec{q}, \vec{Q}-\frac{1}{4}\vec{q}\,)$ [we use $(\vec{A}, \vec{B}\,)$ to denote the angle between $\vec{A}$ and $\vec{B}$].

Figure 1 shows that the disagreement between the data and calculations (dashed line) begins near $q^2 \sim 1.5$ $(\mathrm{GeV}/c)^2$. A different choice of deuteron wave functions does not reduce the disagreement substantially.[3] Although the disagreement appears for rather large q^2, the values of pn relative momentum in the deuteron wave functions which mainly contribute to the form factors Eqs. (3a) and (3b) are not so large. So it is hoped that the use of nonrelativistic wave functions could be a starting point in the calculations for $q^2 \gtrsim 1.5$ $(\mathrm{GeV}/c)^2$.

The question arises whether the explanation of the data requires inclusion of processes such as exchange currents, etc., which are beyond the scope of IA, Eq. (2).[8,9] Before one can answer this question one should analyze the effects of relativistic kinematics carefully. The most important of such effects is the following one: In the (symmetric) Breit frame the deuteron is not at rest (Fig. 2), while the wave functions U and W appearing in Eqs. (3a) and (3b) should be calculated at the values of the pn relative momentum obtained in the *deuteron rest frame*. Since the momenta $\vec{Q} \pm \frac{1}{4}\vec{q}$ are not Lorentz invariant they undergo shifts as a result of the Lorentz transformation (LT) to the deuteron rest frame. The wave functions U and W are extremely sensitive to small variations in the relative momentum and therefore this effect can be very important (cf. Noble and Weber[10]).

In order to carry out the LT we need to know the energy as well as momentum of each of the target nucleons. For this purpose one usually uses a prescription of taking the spectator N on the mass shell.[4] The energy of the spectator N is thus $Q_0 = (\vec{Q}^{\,2} + m^2)^{1/2}$, and the LT can be performed. Replacing the pn relative momenta $\vec{Q} \pm \frac{1}{4}\vec{q}$ in Eqs. (3a) and (3b) by their values in the deuteron rest frame [and taking into account an overall reduction factor $(M_d/E_d)^{1/2}$ due to relativistic normalization of the wave function], we get the dotted curve in Fig. 1. This curve is practically indistinguishable from the corresponding one obtained using the same prescription by Arnold, Carlson, and Gross,[3] who have used, however, more complicated expressions including additional kinematical effects. This means that the shift in the pn relative momentum due to LT to the deuteron rest frame is indeed the most important relativistic kinematical effect.

Even without the numerical integration in Eqs. (3a) and (3b) one can see that if the spectator is on the mass shell the relativistic calculations lead to a reduction of the form factor. Indeed the major contribution to the integrals (3a) and (3b) comes from the regions $\vec{Q} = \pm\vec{q}/4$ (though the region around $\vec{Q} = 0$ may also be important). Let $\vec{Q}_{pn}{}^{(1)}(L)$ and $\vec{Q}_{pn}{}^{(1)}(R)$ denote the pn relative momenta in the deuteron rest frame obtained at the left-hand and the right-hand vertex (Fig. 2), respectively, by keeping the spectator N on the mass shell, and let $\vec{Q}_{pn}{}^{NR}$ denote the corresponding nonrelativistic quantities. We can see that if $\vec{Q} = \vec{q}/4$, then $Q_{pn}{}^{NR}(L) = Q_{pn}{}^{(1)}(L) = 0$. However, $Q_{pn}{}^{NR}(R) = q/2$, and $Q_{pn}{}^{(1)}(R) = q(1+q^2/16m^2)^{1/2}/2$. Thus $\Delta^{(1)} \equiv Q_{pn}{}^{NR}(R) - Q_{pn}{}^{(1)}(R) < 0$ [for $q^2 = 2$ $(\mathrm{GeV}/c)^2$, $\Delta^{(1)} \simeq -48$ MeV/c] which tends to reduce the relativistic form factor compared to the nonrelativistic one. Consider next the struck N in the right-hand vertex to be on the mass shell. We find that the pn relative momentum in the deuteron rest frame is

$$Q_{pn}{}^{(2)}(R) = (q/4)[3(1+q^2/16m^2)^{1/2} - (1+9q^2/16m^2)^{1/2}].$$

Thus $\Delta^{(2)} \equiv Q_{pn}{}^{NR}(R) - Q_{pn}{}^{(2)}(R) > 0$. For $q^2 = 2$ $(\mathrm{GeV}/c)^2$, we have $\Delta^{(2)} \simeq 107$ MeV/c, which is a very large value considering the sensitivity of the momentum-space deuteron wave functions. If we simply perform the integration in (3a) and (3b) keeping the struck N on the mass shell in each of the two wave functions we obtain the dot-dashed curve in Fig. 1. Now it is kinematically impossible to have the struck N on the mass shell at both the vertices. In spite of this inconsistency, such a large difference between the dotted and dot-dashed curves shows the extreme sensitivity of the result to the relativistic kinematical shift of the pn relative momenta. It is thus very important to derive a new procedure to carry out LT of the wave function to the deuteron rest frame.

Here we only describe the main features of our approach; the detailed derivation will be presented in a separate publication.[4] We consider the case where the deuteron wave function obeys the Schrödinger equation in the deuteron *rest frame*. Nucleons will be assumed to be scalar, and photons spinless and pointlike in our derivation. First we should find the connection between the deuteron wave function and the covariant dpn vertex, Γ_{dpn}, which satisfies the BSE. We thus need to do a three-dimensional reduction of this equation. The usual procedure consists in closing the contour of energy (Q_0) integration over one of the nucleon poles (thus taking that N on the mass shell). Strictly speaking this procedure cannot be carried out because there are threshold meson-production cuts in the Q_0 plane, which lie on both sides of the real axis. (We neglect there the far-away singularities corresponding to the negative energy N's, "pair currents".[9]) However, we can show that in the ladder approximation the BSE for Γ_{dpn} can be rewritten as a system of two coupled equations for $\Gamma_{dpn}{}^{(1)}$ and $\Gamma_{dpn}{}^{(2)}$, where $\Gamma_{dpn} = \Gamma_{dpn}{}^{(1)} + \Gamma_{dpn}{}^{(2)}$. In each of these equations the cuts lie on only one side of the real Q_0 axis (cf. Ref. 11). The Q_0 integration can thus be carried out in each of these equations by closing of the contour of integration over that N pole which is not accompanied by cuts. As a result of this integration we obtain similar equations, for $\tilde{\Gamma}_{dpn}{}^{(1)}$ and $\tilde{\Gamma}_{dpn}{}^{(2)}$, say, which now involve only three-dimensional integrals. Here $\tilde{\Gamma}_{dpn}{}^{(1)}$ corresponds to one N on the mass shell and $\tilde{\Gamma}_{dpn}{}^{(2)}$ to the other N on the mass shell. However, in the deuteron rest frame we find that $\tilde{\Gamma}_{dpn}{}^{(1)} = \tilde{\Gamma}_{dpn}{}^{(2)} = \frac{1}{2}\tilde{\Gamma}_{dpn}$ and the equation for $\tilde{\Gamma}_{dpn}$ has form of Schrödinger equation where $\tilde{\Gamma}_{dpn}(\vec{Q})/(\epsilon_d m + \vec{Q}^2) = \Phi_d(Q)$ is the deuteron wave function. We thus show that the three-dimensional reduction of the BSE leads to the deuteron wave function which is a sum of two components: $\Phi_d(Q) = \Phi_d{}^{(1)}(Q) + \Phi_d{}^{(2)}(Q)$, where $\Phi_d{}^{(1)}$ corresponds to one N on the mass shell and $\Phi_d{}^{(2)}$ to the other N on the mass shell.

Now we consider the relativistic triangle diagram, Fig. 2. We cannot just close the contour of Q_0 integration over the pole of the spectator (or the struck) N because of the presence of the meson-production cuts. In the ladder approximation, however, we can represent the deutron form factor as an infinite series of time-ordered diagrams of the old-fashioned perturbation theory (where the Q_0 integration is already done). We assume that the contribution of that class of diagrams which leads to strong energy dependence of effective pn potential is small. Then we obtain Eqs. (3a) and (3b) where U and W, as discussed above, are deuteron wave functions obtained by the three-dimensional reduction of the Bethe-Salpeter equation. It means that $U(\vec{Q} \pm \frac{1}{4}\vec{q})$ in Eq. (3) should be replaced by $\frac{1}{2}[U(Q_\pm{}^{\rm rel}(1)) + U(Q_\pm{}^{\rm rel}(2))]$, where

$$[Q_\pm{}^{\rm rel}(1)]_z = Q_z(1+\vec{q}^{\,2}/16m^2)^{1/2} \pm (q/4)(1+\vec{Q}^2/m^2)^{1/2},$$

$$[Q_\pm{}^{\rm rel}(2)]_z = (Q_z \pm q/2)(1+\vec{q}^{\,2}/16m^2)^{1/2} \mp (q/4)[1+(\vec{q}/2 \pm \vec{Q})^2/m^2]^{1/2},$$

and $(Q_\pm{}^{\rm rel})_{x,y} = Q_{x,y}$, where the z axis is taken along $\vec{q}$. $Z_\pm$ and Z_{+-} are also calculated accordingly. Similar replacement should be made for $W(\vec{Q} \pm \vec{q}/2)$. This result does not mean that the spectator or the struck N are really on the mass shell during the scattering. It only tells how to treat the effects of the LT to the deuteron rest frame.

The result of these calculations is shown in Fig. 1 [solid line (a)]. For comparison we have also shown the result of a calculation based on the Hamada-Johnston hard-core wave functions[12] [solid line (b)]. A sensitive test of our results would be a measurement of the tensor polarization $T(q^2)$ of recoil deuterons in ed elastic scattering.[4] Our predictions for this quantity (Fig. 3) are significantly different from those obtained by using the standard prescription.[3]

Recently a covariant calculation that goes beyond the prescription of taking the spectator on the mass shell has been performed by Zuilhof and Tjon.[13] However, they have treated the kinematic effects associated with the LT to the deuteron rest frame rather approximately. Since these effects are shown to be very important in

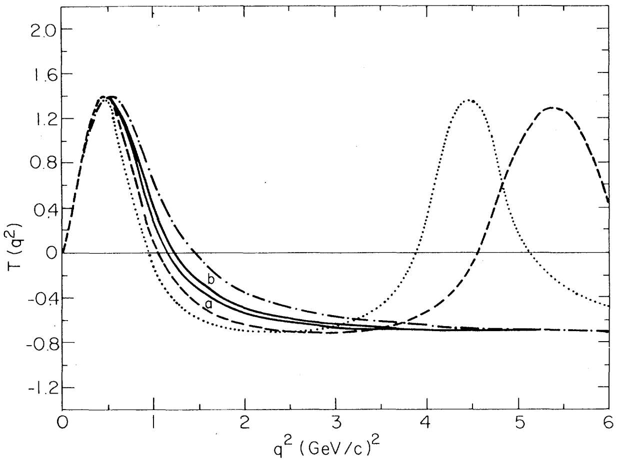

FIG. 3. Our predictions of the tensor polarization of the recoil deuterons. Details of the curves as in Fig. 1.

the present work a direct comparison of our results with theirs is difficult. However, their results for $A(q^2)$ are different from ours and are close to those of Ref. 3.

To conclude, we have been able to describe all the available data for $A(q^2)$ [and $B(q^2)$] in the framework of the relativistic IA. Nevertheless, the following limitations should be kept in mind. (i) Our result has a covariant form even though the covariance is broken in the derivation. First, we neglect negative-energy N poles which are far from the physical region. Second, our derivation is based on the use of time-ordered terms of old-fashioned perturbation theory, which are separately noncovariant. However, we always deal with sums of such terms, albeit with different N's on the mass shell. So it is hoped that the breaking of covariance is of minor importance. (ii) We neglected in the derivation those time-ordered diagrams where the photon interacts with a nucleon at the same time as a meson is present in the intermediate state. These terms depend on the energy derivative of the effective pn potential, which we assume is small. (iii) Although our final result for the shift of pn relative momenta has been derived only for the spinless case, we formally applied it to Eqs. (3a) and (3b), where the spin was taken into account. We assume that it is a correct generalization for the spin case, since if we take the spectator N on the mass shell the same procedure leads to the same result as exact calculations.[3]

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