First Measurement of the Spin Rotation Parameter Q for p-^{40}Ca Elastic Scattering at 500 MeV

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The spin rotation parameter Q has been measured for p-^{40}Ca elastic scattering at 497 MeV for $0.01 \leq -t \leq 0.15$ (GeV/c)^2 with use of the new high-resolution spectrometer focal-plane polarimeter. In conjunction with analyzing power and differential cross section, this measurement uniquely determines the elastic scattering amplitudes up to an arbitrary phase. A parameter-free analysis within the Glauber diffractive approximation with use of N-N phase-shift solutions is presented.

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Up to the present time, p-nucleus elastic scattering experiments at intermediate energies[1-3] have been limited to the measurement of three observables; σ_T, the total cross section (which constrains the scattering amplitude at $t=0$ only), $(d\sigma/dt)_{\text{unpol}}$, the unpolarized differential cross section, and A_y, the analyzing power, with very few measurements of the spin rotation parameters.[4] $(d\sigma/dt)_{\text{unpol}}$ and A_y do not uniquely determine the scattering amplitude $F(t)$ at finite t. For spin-0 targets, another observable is required. The scattering operator for this system can be written as

$$F(t) = f(t) + g(t)\vec{\sigma}\cdot\hat{n}, \tag{1}$$

where $\vec{\sigma}$ is the proton spin operator and $\hat{n} = (\vec{k}_i \times \vec{k}_f)/|\vec{k}_i \times \vec{k}_f|$ defines the scattering plane, $\vec{k}_i$ and $\vec{k}_f$ being the incident and scattered projectile momenta, respectively. (Note that some authors include an additional factor of i in the spin-orbit amplitude thereby changing the decomposition of real and imaginary parts.) To obtain a third independent observable, and thus to make it possible to extract the scattering amplitude uniquely, requires a double scattering experiment. We have chosen to measure a particular spin rotation Q as suggested by Glauber and Osland.[5] This observable measures the total spin rotation angle β in the plane of scattering through the relation[5]

$$Q = \sin\beta/[1 - P^2]^{1/2}, \tag{2}$$

where P is the induced polarization. It can also be shown that Q is defined in terms of the Wolfenstein spin rotation observables[6] by

$$Q = A\cos\theta_{\text{lab}} + R\sin\theta_{\text{lab}}, \tag{3}$$

where θ_{lab} is the laboratory scattering angle. Similarly, because of the symmetries of the system

$$Q = A'\sin\theta_{\text{lab}} - R'\cos\theta_{\text{lab}}. \tag{4}$$

Noticeable in Eq. (3) is that this expression for Q depends only on analyzing the $\hat{s}$ (transverse horizontal) component of the scattered-proton polarization. Similarly, Q in Eq. (4) depends only on the $\hat{l}$ (longitudinal) component of the polarization.

The measurements of the scattered-proton polarizations (A, R, A', and R') were done with use of the newley developed focal-plane polarimeter at the high resolution spectrometer (HRS) at the Clinton P. Anderson Meson Physics Facility (LAMPF).[7] At the HRS, the $\hat{s}$ component of the polarization lies along the magnetic field and is unprecessed to first order. The $\hat{l}$ and $\hat{n}$ (normal to scattering plane) components are precessed by an angle χ relative to the particle's momentum given by

$$\chi = \gamma(g/2 - 1)\alpha \simeq 269\gamma , \tag{5}$$

where γ is the Lorentz factor, $g/2$ is the magnetic moment of the proton, and α is the bend angle (about 150° at the HRS). At 497 MeV, χ is approximately $360° + 52°$. Thus the components initially along $\hat{n}$ and $\hat{l}$ are precessed and mixed at the focal-plane polarimeter. Consequently, we are able to determine R', A', and P (as well as R and A).

We use the method of Besset *et al.*[8] to determine the polarization of the proton after the first scattering. The angular distribution $I(\theta, \varphi)$ of protons after rescattering in the carbon analyzer target is given by

$$I(\theta, \varphi) = k(d\sigma/d\Omega)_c [1 + P_{n'} A_c(\theta)\cos\varphi - P_{s'} A_c(\theta)\sin\varphi] A(\theta, \varphi) , \tag{6}$$

where θ and φ are the polar and azimuthal angles in the carbon analyzer target. The quantities $(d\sigma/d\Omega)_c$ and $A_c(\theta)$ are the corresponding unpolarized differential cross section and analyzing power of p-^{12}C scattering; $A(\theta, \varphi)$ is the instrumental acceptance; and k is an overall normalization. Since Eq. (6) describes a distribution after the particles are processed in the dipole fields of the HRS, a transformation of coordinate systems is required to relate these polarizations to those in the coordinate system of the target. $P_{s'}$ and $P_{n'}$ refer to the components of the polarization in the $\hat{s}'$ (horizontal transverse) and $\hat{n}'$ ($\hat{n}' = \hat{l}' \times \hat{s}'$ where $\hat{l}'$ is longitudinal) directions *after* precession. To first order $P_s = P_{s'}$ and $P_l = P_{l'}/\cos\chi$, where P_s and P_l now refer to polarizations after the first scattering *before* precession.

The data acquisition system[7] employed a fast microprocessor to reject events not scattered outside of the small-angle multiple-Coulomb-scattering region. A sample of untested events was passed to allow for systematic checks of the system. The final cuts on the data restricted the polar angle in the carbon analyzer to $3.5° < \theta_c < 21°$. The average analyzing power over this interval was found to be 0.32. This was calibrated by running a low-intensity polarized beam through the HRS at 0° and measuring the asymmetries in the polarimeter. The beam polarization was monitored during this calibration by use of a quenching technique at the source, accurate to better than 1%.[7] The measurement of $A_c(\theta)$ at 497 MeV agreed well with that of Besset *et al.*[9] at 483 MeV. Previous measurements of the focal-plane polarimeter have shown instrumental asymmetries to be less than 1%.

Many experimental consistency checks of the data are possible and were made. First, for elastic scattering of protons from a spin-0 nucleus the induced polarization is $P(t) = A_y(t)$. $P(t)$ obtained from our data can be compared with very high-precision A_y data taken recently for ^{40}Ca at 497 MeV at the HRS.[10]

The data were sorted into 0.5° bins. Because of the relatively thick ^{40}Ca target used (0.5 g/cm^2), our angular resolution due to multiple Coulomb scattering was 0.4° (full width at half maximum). Very sharp diffractive structures are seen in the A_y data. Folding of these A_y data with our poorer angular resolution gives good agreement with our $P(t)$ measurement.

Secondly, $A(t) = -R'(t)$ and $R(t) = A'(t)$. As mentioned before, $A(t)$ and $R(t)$ require measurements of the *unprecessed components* (those along the magnetic field direction $\hat{s}$) of the scattered proton's spin; $R'(t)$ and $A'(t)$ require measurements of the spin components *initially oriented along the scattered particle's direction of motion* $\hat{l}$, which are subsequently made to precess in HRS by about $360° + 52°$. The latter are extracted in conjunction with a measurement of $P(t)$. In Fig. 1 we plot $A(t)$ and $-R'(t)$, and $R(t)$ and $A'(t)$. The agreement between these pairs of observables provides another powerful consistency check of the method we use to extract $Q(t)$.

In Fig. 2, we have also plotted curves for P and Q derived from the Glauber diffraction approximation[13] with corrections for finite-energy effects.[11] In the calculation, the latest available *NN* amplitudes from phase-shift analyses[12] at 500 MeV were used. Parameters describing the ^{40}Ca neutron and proton mass distributions were taken from recent papers on proton[14] and electron[15] scattering analyses and measurements.

The contribution from the magnetic moment was included according to Fäldt and Ingemars-

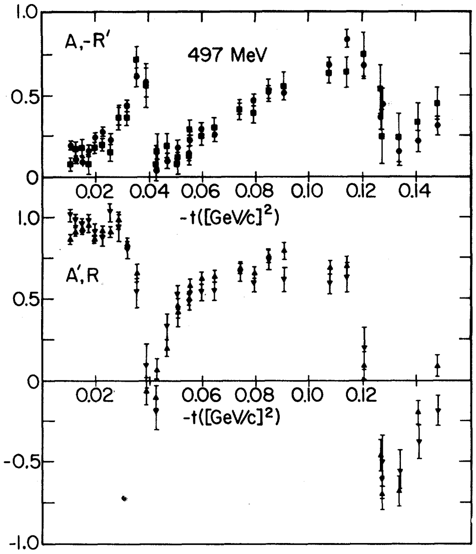

FIG. 1. Spin rotation observables in p-^{40}Ca elastic scattering. (a) A and $-R'$. The circles are A and the squares are $-R'$. (b) R and A'. The upright triangles are R and the inverted triangles are A'.

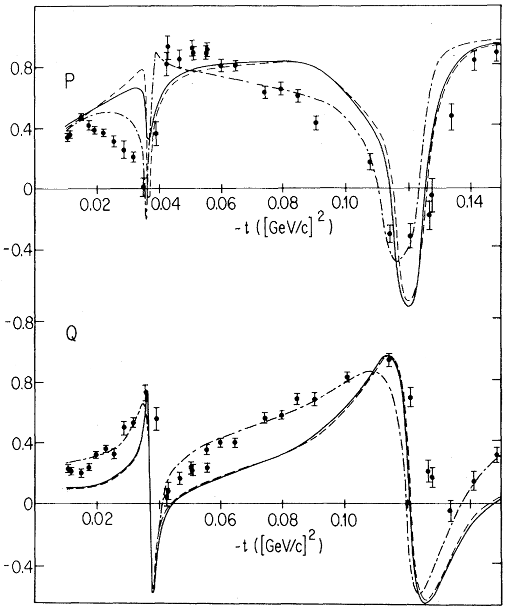

FIG. 2. P and Q for elastic scattering of 497 MeV protons from ^{40}Ca. The solid line corresponds to the full calculation done according to the approach of Ref. 11 with the N-N amplitudes from the phase-shift analysis (Ref. 12). The dashed curve was obtained by neglecting the magnetic moment interaction. The dash-dotted line correspond to the calculation with the Gaussian spin-orbit amplitude. See Eq. (7) in the text.

son,[16] where it was pointed out that in Osland and Glauber[17] there is an error in defining the relative sign of the electromagnetic and nuclear spin-orbit amplitudes. This sign change plays a considerable role in the interference terms in polarization observables at small angles. From Fig. 2, we see that the effect of the magnetic-moment contribution is relatively small except for the first maximum of the polarization. (The recently reported[18] large effects due to the magnetic moment in p-^{208}Pb elastic scattering are mostly due to the relatively large atomic number of this nucleus.) We note that the sign of Q given by the present calculation is generally positive in agreement with the measurements. This sign is controlled by the sign of the real part of the spin-orbit N-N amplitude,[6] i.e., the positive real part corresponds to positive Q. Empirical amplitudes used in most previous analyses at higher energies[4,19] of elastic proton-nucleus data ($d\sigma/dt$ and A_y) to extract the surface characteristics of neutron mass distributions predicted that the overall sign of Q should be negative. These amplitudes were, of course, not unique, since they were obtained from an incomplete set of measurements (two instead of three). More recent proton-nucleus amplitudes consistent with the latest available N-N amplitudes from phase-shift analysis,[12] i.e., which give a positive overall sign for $Q(t)$, are currently under investigation.[10] It is too early to draw firm conclusions about the effect of the new amplitudes on the shape of the neutron density and the difference in the rms radius between the protons and neutrons except to say that agreement with Hartree-Fock calculations cannot be maintained at the present time.

Referring to Fig. 2, we see that our theoretical predictions for Q and P reflect a serious lack of agreement *quantitatively* with these 500-MeV data. The discrepancies may still be attributable to ambiguities in the N-N amplitudes at small $-t$ [up to 0.1–0.2 $(\mathrm{GeV}/c)^2$]. Despite much recent progress both experimentally (N-N measurements) and theoretically (N-N analysis at intermediate energies), little information is available about the N-N amplitudes at small angles since most N-N observables have not been measured at laboratory angles less than 15°, particularly for p-n scattering. We note that N-N amplitudes at small t may be efficiently extracted from multiple-scattering analysis of elastic scattering measurements at small angles from the light elements (deuterium, ^{3}He, and ^{4}He) and from quasielastic p-d scattering. Some of these experiments have been done near 500 MeV.[20] Our understanding of the N-N amplitudes at small t will be considerably clarified when these measurements and their analysis are completed. As is well known, because of the rapid falloff of the nuclear form factor with momentum transfer for small values of t, the amplitude for elastic scattering from a heavy nucleus is sensitive mostly to the N-N amplitudes at very small momentum transfers. As we have just remarked, most of the N-N input data used in the phase-shift analysis come from the experiments at $-t \geqslant 0.1$ $(\mathrm{GeV}/c)^2$ (i.e., 15° in the lab system). We believe that there may still be enough ambiguity in the N-N amplitudes to improve the agreement of the theory with the data. To illustrate this point without attempting to make a fit to the data, we have made a representative calculation in which the central parts of the pp and pn amplitude were taken from phase-shift analyses,[12] whereas the spin-orbit pp and pn amplitudes were assumed to be of the Gaussian form:

$$f_s^{pp(pn)}(q) = q\frac{k\lambda(i+\alpha_s)}{4\pi}\exp\left[-\frac{\beta_s^{pp(pn)}}{2}q^2\right], \qquad (7)$$

with $\lambda = 1.2$ fm^3, $\alpha_s = 0.8$, and $\beta_s^{pp} = 0.6$ fm^2, $\beta_s^{pn} = 0.4$ fm^2. The result is shown in Fig. 2 (dashed-dot curve). Better quantitative agreement is due principally to the increase in the magnitude of the slope parameter β_s. The parameters of the Gaussian [Eq. (7)], which quantitatively replace the spin-orbit N-N amplitude extracted from the phase-shift analysis, are $\lambda \approx 1.2$ fm^3, $\alpha_s \approx 0.4$, and $\beta_s^{pp} = \beta_s^{pn} \approx 0.25$ fm^2. It must be noted, however, that this difference is well outside of the error corridor quoted in the phase-shift analysis. The larger slope of $f_s(q)$ makes the polarization curve sharper. The larger real part of $f_s(q)$ is responsible for increasing Q (compare Ref. 17).

In summary, the present parameter-free theoretical analysis of $P(t)$ and $Q(t)$ for the elastic scattering of protons from ^{40}Ca at 497 MeV correctly predicts the overall sign of Q (positive) and qualitatively reproduces the data. The lack of quantitative agreement may be partly ascribed to an insufficient knowledge of the N-N amplitudes in the region of small momentum transfers.

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[1] G. Igo, in *Proceedings of the Sixth International Conference on High Energy Physics and Nuclear Structure, Santa Fe and Los Alamos, 1975*, edited by D. E. Nagle and A. S. Goldhaber, AIP Conference Proceedings No. 26 (American Institute of Physics, New York, 1975).

[2] G. Bruge, in *Proceedings of the Seventh International Conference on High Energy Physics and Nuclear Structure, Zurich, 1977*, edited by M. P. Locher (Birkhauser-Verlag, Basel and Stuttgart, 1978).

[3] See references in the review article by S. J. Wallace, to be published.

[4] O. N. Jarvis *et al.*, Nucl. Phys. 79, 275 (1966).

[5] R. J. Glauber and P. Osland, Phys. Lett. 80B, 401 (1979).

[6] L. Wolfenstein, Phys. Rev. 96, 1654 (1954), and Annu. Rev. Nucl. Sci. 6, 43 (1956).

[7] J. B. McClelland, J. F. Amann, W. D. Cornelius, and H. A. Thiessen, "A Polarimeter for Analyzing Nuclear States in p-Nucleus Reactions between 300 and 800 MeV," to be published.

[8] D. Besset *et al.*, Nucl. Instrum. Methods 166, 515 (1979).

[9] D. Besset *et al.*, Nucl. Instrum. Methods 166, 379 (1979).

[10] G. W. Hoffmann, private communication.

[11] M. Bleszynski and P. Osland, Phys. Lett. 84B, 157 (1979).

[12] D. Arndt, private communication.

[13] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin, Lecture Notes in Physics Vol. 1 (Interscience, New York, 1959), p. 315.

[14] E. Bleszynski *et al.*, in preparation.

[15] K. Itoh *et al.*, Phys. Rev. C 2, 2181 (1970).

[16] G. Fäldt and I. Ingemarsson, Gustaf Werner Institute Report No. GWI-PH81, 1981 (unpublished), and private communication.

[17] P. Osland and R. Glauber, Nucl. Phys. A326, 255 (1979).

[18] G. W. Hoffmann *et al.*, "$A_y(\theta)$ for $\vec{\mathrm{p}} + {}^{208}$Pb elastic scattering at 0.8 GeV and a test of multiple scattering theory," to be published.

[19] L. Ray, Phys. Rev. C 19, 1855 (1979), and in *Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics—1980*, edited by G. G. Ohlsen, AIP Conference Proceedings No. 69 (American Institute of Physics, New York, 1980).

[20] M. Bleszynski *et al.*, Phys. Lett. 87B, 198 (1979).