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Nuclear Fluid Dynamics versus Intranuclear Cascade—Possible Evidence for Collective Flow in Central High-Energy Nuclear Collisions

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The predictions of a variety of current theoretical models of high-energy nuclear collisions are compared with recent experimental data for central collisions of ^{20}Ne on ^{238}U at $E_{\rm lab} = 393$ MeV/u. The experimental observation of broad sideward maxima in the angular distributions of low- and medium-energy protons is reproduced by a nuclear fluid-dynamical calculation with final freezeout of the protons. In contrast, the current intranuclear-cascade and simplified collision models predict forward-peaked angular distributions.

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High-energy nuclear collisions offer a unique opportunity to probe nuclear matter at high density and temperature. However, a precise knowledge of the reaction dynamics is required to extract information on the bulk properties of nuclear matter from the experimental data.

It has been pointed out that the large pressure in the high-density, high-temperature matter should cause a collective hydrodynamical sideways flow.[1,2] Quite early experiments[2] reported sideward maxima in the angular distributions of α particles emitted from high-multiplicity selected, i.e., central, collisions. On the other hand, inclusive, i.e., impact-parameter averaged, data[3,4] on light-fragment emission do not show sideward-peaked angular distributions. However, the measured azimuthal correlations between light and heavy fragments[5] exhibit signatures of the hydrodynamical bounce-off effect,[6,7] and so do the two-proton correlations[8] in heavy systems.

From the inclusive data[3] it was in general not possible to differentiate between the existing dynamical models. Possible differences are washed out by the impact-parameter averaging.[9] Hence, recent high-multiplicity selected data[10] for Ne (393 MeV/u) + U → light fragments have received great attention. It is the purpose of the present work to compare the predictions of several distinct model calculations for this reaction and to provide a test of these models by a direct confrontation with the experimental data.[10] The models we use are two versions of the macroscopic nuclear fluid-dynamical (NFD) model,[11,12] two versions of the microscopic intranuclear-cascade (INC) calculations,[13,14] and two thermal models with a simplified participant-spectator geometry.[15,16]

Let us briefly survey the various models used for our comparison. Both versions of the fluid-dynamical approach solve numerically the equations of motion of an Eulerian fluid.[6,7] These equations express the conservation of baryon number, momentum, and energy for a classical fluid. The validity of such an approach requires that the nucleon's mean free path λ is much less than the system's dimensions D. For peripheral collisions, the average number of scatterings is $\langle n \rangle \sim D/\lambda \sim 1$, and hydrodynamics is unlikely to apply. However, for central collisions, $\langle n \rangle$ can be much larger than 1, in particular if high compressions are achieved.[1] Thus, the best chance for hydrodynamics to be applicable is clearly in central collisions of the heaviest available nuclei.

In the first set of NFD calculations,[11] long-range interactions are neglected. The initial nuclei have a sharply cut off surface. The energy and angular distributions are calculated from the particle density and velocity vectors at a time sufficiently long so that the residual thermal energy is negligible, i.e., the densities are very low.[11] However, calculations based on transport theory indicate that during the expansion the thermal equilibrium can only be maintained until the fluid reaches the breakup density $\rho_{BU}/\rho_0 \approx 0.3$–$0.7$.[17] Then the system breaks up into free particles, which reach the detectors with the momentum distribution they had in this freezeout moment.

The incorporation of this freezeout concept is the most prominent difference between the two applied versions of the NFD model. In the second NFD approach,[12] which incorporates[6] realistic surface and binding properties, the particle spectra are calculated by transforming the internal thermal momentum distribution of each fluid element at the breakup density with the corresponding collective flow velocity into the laboratory. Proton distributions are calculated by suppressing the emission of bound nucleons with internal energy $\epsilon < m_p c^2$.[12]

The second class of models we consider are the relativistic intranuclear-cascade approaches (Refs. 13, 14, 18, and 19). They are based on the classical impulse approximation, i.e., a nucleus-nucleus collision proceeds as a sequence of independent two-particle collisions. The scattered particles follow straight-line trajectories until they interact again. This approach neglects the N-N potentials, which form the essential ingredient of the much more complex classical equations-of-motion calculations,[20] as well as all many-body interactions, which can be considerable for the high densities considered. Besides some attempts to include the Pauli blocking, the microscopic approaches are also classical. Initially, the nucleons reside in potential wells, with the momentum distributions of a degenerate Fermi gas.

In the first cascade approach,[13] the nuclei are treated initially as continuous Fermi seas of nuclear matter. The collision process starts via the interactions between projectile Fermi-sea and target Fermi-sea nucleons forming cascade particles, which have Gaussian density distributions. In the course of the collision process, interactions between cascade and Fermi-sea particles, as well as between cascade particles, can occur.

In the second version of the cascade model,[14] the nucleons are represented by pointlike particles and are initially assigned random positions and momenta in the nuclei. The nucleons interact at the point of closest approach if their separation d satisfies $\pi d^2 \leq \sigma_{tot}(E_{c.m.})$, where σ_{tot} is the appropriate N-N total cross section. If this condition is satisfied, the scattering angle is randomly chosen from experimental elastic-scattering angular distributions.

The third class of models we consider is based on the simplified participant-spectator geometry.[21] The firestreak model[15] allows for a calculation of the spectra of different light fragments ($p, d, t, \ldots$) emitted, by assuming thermal equilibrium in streaks of nuclear matter. The second approach, the two-component—direct plus thermal—model,[16] takes into account, in addition to the thermal nucleon component, the single-scattering contribution, which is appreciable at intermediate energies and forward angles.

Figure 1 shows the angular distributions of protons emitted from central collisions of Ne (393) MeV/u) + U. The various models discussed above are compared and confronted with the experimental data.[10] The central-collision data have been obtained by triggering on the highest 15% of the multiplicity distribution associated with a 90° proton. This trigger corresponds to roughly 15% of the inclusive cross section.[10] With the partici-

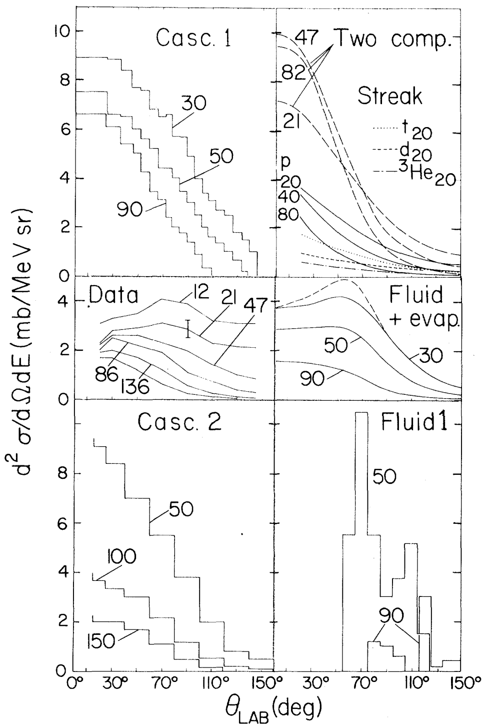

FIG. 1. Angular distributions of protons emitted from central collisions of Ne (393 MeV/u) + U. The numbers indicate the kinetic energies of the emitted fragments in megaelectronvolts. The data (middle left frame) exhibit broad sideward maxima, in contrast to the cascade calculations 1 (Ref. 13) and 2 (Ref. 14) (upper and lower left frames), which yield forward-peaked angular distributions. The two-component model (Ref. 16) (long-dashed curves) and the firestreak calculations (Ref. 15) for protons (p, solid curves), d (short-dashed), t, (dotted), and ^{3}He (dash-dotted) are shown in the upper right frame. The hydrodynamic calculations 1 (Ref. 11) without thermal breakup (lower right frame) yield too-narrow angular peaks. The NFD model with thermal breakup (Ref. 12) (middle right frame) gives a reasonable description of the broad sideward maxima and their forward shift with energy. The dashed curve shows the enhanced sideward peaking for the sum-of-charges distribution at $E_{kin} = 30$ MeV/u. The curve is multiplied by 0.2.

pant-spectator geometry, this leads to a cutoff impact parameter $b^{max} = 1.5$ fm.[16] This small value (used in the macroscopic calculations shown at the right-hand side of Fig. 1) arises from the large contribution of small impact parameters to high-multiplicity inclusive events.[16] The NFD model with thermal breakup and the firestreak model allow for a calculation of the actual proton spectra. The remaining models yield the sum of charges ($p + d + t + 2\,^3$He + . . .) distributions only. However, the preliminary data[5,10] on central selected d and t exhibit similar spectra to that of the protons shown in Fig. 1. In particular, the sum of the p, d, and t angular distributions exhibit even stronger sideward peaking than the proton distributions.

The data exhibit broad sideward maxima at large angles, which shift forward with increasing proton (or d or t) energy. In comparison, the NFD model without thermal breakup,[11] although yielding a sideward peak structure, fails to explain the data in several respects: It gives a peak about 1 order of magnitude too large at low energies, $E < 50$ MeV, too few high-energy particles, peaks that are clearly too narrow, and peak positions that shift to somewhat larger angles with increasing proton energy, opposite to the trend in the data. On the other hand, the NFD model with breakup included[12] describes the data reasonably well, in shape as well as in magnitude. The energy spectra exhibit a similar flatness to the data. This agreement with data demonstrates the importance of a proper treatment of the breakup stage.[12,22]

Next we discuss the results of the cascade models. The results of cascade 1 (Ref. 13) are based on a high-multiplicity selection, $M \geq 21$, while those of cascade 2 (Ref. 14) are for exactly central collisions. For this reason, the shapes of the angular distributions should be compared rather than their absolute values. In sharp contrast to the NFD calculations, both cascade models exhibit forward peaking and thus fail to reproduce the salient features of the data. It is important to emphasize the insensitivity of the cascade results to variations in the multiplicity cutoff and to different impact-parameter cutoffs. In fact, we find that peripheral, inclusive, and central-selected angular distributions in this model[13] all exhibit the same, forward-peaked shape of the angular distributions, in strong contrast to the data.[10] This qualitative failure of the cascade model points towards the necessity of a more realistic treatment of the nuclear interactions, including for instance mean-field effects as in the classical equations-of-motion approach.[20]

Finally, we consider the near-analytic models. Both the firestreak and the two-component model exhibit forward-peaked distributions. It should be pointed out, however, that the relative yields of p, d, and t as calculated in the chemical equilibrium firestreak model[15] agree reasonably well with the preliminary data.[5,10] The direct nucleon component[16] produces a slight sideward peak at very small angles beyond a proton energy of 140 MeV only (not shown here).

In summary, we presented the first detailed comparison between various theoretical calculations and the central-collision data for Ne (393 MeV/u)+U. The main qualitative feature of the data that we focus on is the observed broad sideward maxima in the angular distributions of low- and medium-energy protons. This qualitative feature is not accounted for by existing intranuclear cascade or thermal models. On the other hand, the hydrodynamical models predict sideward emission due to collective matter flow. We find a qualitative agreement of the NFD model with the data once the thermal breakup is included in the calculation. The present proton (and d and t) data support the existence of collective matter flow. However, with the proton data alone no definite conclusion can be reached because the proton spectra are sensitive to the mechanism of composite formation. In particular, enhanced composite production in the forward direction could lead to forward suppression of free protons. Therefore, it is essential to measure the spectra of heavier composites in central collisions to establish conclusively the existence of a collective sideward flow in high-energy nuclear collisions.

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