[1]H. Hamber, Phys. Rev. D 24, 951 (1981); E. Marinari, G. Parisi, and C. Rebbi, following Letter [Phys. Phys. Rev. 47, 1795 (1981)]; D. Weingarten, unpublished.
[2]E. Marinari, G. Parisi, and C. Rebbi, to be published.
[3]K. Wilson, Phys. Rev. D 10, 2445 (1974), and in *New Phenomena in Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1977).
[4]T. Banks, S. Raby, L. Susskind, J. Kogut, D. Jones, P. Scharbach, and D. Sinclair, Phys. Rev. D 15, 1111 (1977).
[5]M. Weinstein, S. Drell, H. Quinn, and B. Svetitsky, Phys. Rev. D 22, 1190, 490 (1980).
[6]N. Kawamoto and J. Smit, Institute of Theoretical Physics, University of Amsterdam Report No. ITFA-81-2, 1981 (to be published), and references therein.
[7]M. Creutz, Phys. Rev. Lett. 45, 313 (1980); E. Pietarinen, Nucl. Phys. B190, 349 (1981).
[8]F. Fucito, E. Marinari, G. Parisi, and C. Rebbi, Nucl. Phys. B180, 369 (1981).
[9]I. Yoffe, to be published, and references therein.
[10]M. Nauenberg, T. Schalk, and R. Brower, Phys. Rev. D 24, 548 (1981).
[11]D. Weingarten and D. Petcher, Phys. Lett. 99B, 333 (1981).
[12]D. J. Scalapino and R. Sugar, Phys. Rev. Lett. 46, 519 (1981).
[13]A. Duncan and M. Furman, to be published.

Computer Estimates of Meson Masses in SU(2) Lattice Gauge Theory

E. Marinari
Istituto di Fisica "G. Marconi," Università degli Studi, Roma, Italy, and Istituto Nazionale di Fisica Nucleare, Roma, Italy

and

G. Parisi
Istituto di Fisica, Facoltà di Ingegneria, Università degli Studi, Roma, Italy, and Istituto Nazionale di Fisica Nucleare, Frascati, Italy

and

C. Rebbi
Brookhaven National Laboratory, Upton, New York 11973
(Received 9 October 1981)

It is shown that in an SU(2) lattice gauge theory, in the approximation where internal quark closed loops are neglected, chiral symmetry is broken. With use of partially conserved axial-vector current f_π, the bare masses of the u and d quarks, and the ρ and δ masses are estimated.

PACS numbers: 11.10.Np, 05.50.+q, 12.70.+q, 14.40.-n

Recently some progress has been made in numerical simulations of theories with fermions.[1-3] Although in a complete computation the effects of fermionic closed loops must be taken into account, a reasonable estimate of the hadron spectrum can be obtained by eliminating all internal quark loops (quenched case, see Ref. 2). In this way the Zweig rule is enforced for all flavors. In this note we present a study of chiral-symmetry breaking and of the π, ρ, and δ masses for the SU(2) gauge theory in the quenched approximation. A similar study for the SU(3) gauge theory, including also baryons, can be found in Ref. 4. The results obtained are rather satisfactory.

Let us begin discussing our strategy in the continuum case; later we will adapt it to the lattice version of the model. We consider the fermionic Euclidean action

$$S_f = \int d^D x\, \bar{\psi}(\not{D} + m)\psi\,, \tag{1}$$

where D_μ is the covariant derivative in presence of a gauge field A_μ. If $G(x,0|A)$ is the fermionic Green function with A_μ as background, and $d\mu[A]$ is the probability distribution of the field A (normalized to 1), the following relations hold:

$$\langle \bar{\psi}(0)\psi(0)\rangle = \int d\mu[A]\,\mathrm{Tr}[G(0,0|A)],$$
$$\langle \bar{\psi}(x)\gamma_5\psi(x)\bar{\psi}(0)\gamma_5\psi(0)\rangle = \int d\mu[A]\,\mathrm{Tr}[G(x,0|A)G^*(x,0|A)]. \tag{2}$$

© 1981 The American Physical Society

The fact that $\not{D}$ is an anti-Hermitean operator with spectral density $\rho(i\lambda)$ implies that

$$\lim_{m\to 0} \langle \bar{\psi}(0)\psi(0)\rangle = \frac{\pi}{V}\rho(0),$$

$$\mathrm{Tr}[G(0,0|A)] = m\int d^D x \,\mathrm{Tr}[G(x,0|A)G^*(x,0|A)]. \quad (3)$$

If $\rho(0)\neq 0$, when $m\to 0$ chiral symmetry is broken, and if the integrand in Eq. (2) is finite in this limit then the Goldstone theorem holds.

A natural procedure to evaluate the expectation values of composite field operators is the following: A_μ field configurations are generated with probability distribution $d\mu[A]$ by a Monte Carlo simulation (suitably generalized if one wants to include the effects of inner fermionic loops[1-3]); the propagator $G(x,0|A)$ is then calculated by Monte Carlo like techniques or by relaxation methods. If the effect of closed loops is included, Eq. (2) (and its obvious generalizations) holds for those operators which do not have the internal quantum numbers of the vacuum.

In the relaxation method one obtains the Green functions as the $t\to\infty$ limit of $G_t(x,0|A)$ satisfying

$$dG_t(x,0|A)/dt = (\not{D}+m)G_t(x,0|A) + \delta(x). \quad (4)$$

On the contrary, direct Monte Carlo simulations cannot be performed with a first-order formalism.[1,2] However, one can adapt the standard Langevin formulation to this case by writing

$$d\varphi_i(x,t)/dt = [(-1)^i\not{D}+m]\varphi_i(x,t) + \eta(x,t) \quad i=1,2, \quad (5)$$

where η is a Gaussian stochastic white noise: $\langle\langle \eta(x,t)\eta(x',t')\rangle\rangle = 2\delta(t-t')\delta(x-x')$ (the double angular bracket denotes an average over the noise). It is straightforward to check that

$$\lim_{t\to\infty}\langle\langle \varphi_1(x,t)\varphi_2^\dagger(y,t)\rangle\rangle = \left\langle x \left| \frac{1}{\not{D}+m} \right| y\right\rangle \equiv G(x,y|A). \quad (6)$$

Let us briefly underline the main differences between the Langevin and the relaxation techniques: Using the Langevin equation we can complete $G(x,y|A)$ for all x and y at the same time, while in a comparable computer time the relaxation procedure gives only $G(x,0|A)$. On the other hand the relaxation procedure gives exact results for $G(x,y|A)$, while statistical errors are present with the Langevin method. So we can conclude that to measure $G(x,y|A)$ at $x\sim y$, where G is large, the Langevin approach is the most suitable, whereas for computing G in the large-$|x-y|$ region, where G itself is small, the relaxation method should be used. The second is the situation one encounters in the computation of the mass spectrum of the theory.

On the lattice we used the fermionic action[5-8]

$$S(\psi) = \sum_i \bar{\psi}_i[(D_x\psi)_i + (-1)^x(D_y\psi)_i + (-1)^{x+y}(D_z\psi)_i + (-1)^{x+y+z}(D_t\psi)_i + m\psi_i], \quad (7)$$

where D_i ($i=x,y,z,t$) is the covariant version of the central first derivative ∂_i $\{(\partial_i\psi)_j = \frac{1}{2}[\psi(\vec{j}+\vec{n}_i) - \psi(\vec{j}-\vec{n}_i)]\}$. It is known that this action describes four fermion flavors and is invariant under an SU(4) internal flavor group. As discussed in Ref. 2, the quenched correlation functions for the two-flavor theory can be obtained simply by dividing by a factor of 2 the correlation functions computed with the full action (7).

No multiplicative factor is needed in the computation of the masses. However, if

$$P(\vec{i}-\vec{j}) = \int d\mu[A]\,\mathrm{Tr}[G(\vec{i},\vec{j}|A)G^*(\vec{i},\vec{j}|A)], \quad (8)$$

particles with different spin parity will appear as singularities at different corners of the Brillouin zone; this effect is typical of the approach of Refs. 5 and 8.

If π and ρ are the lowest-mass particles it is easy to check that asymptotically

$$\sum_{n_x,n_y,n_z} P(\vec{n}) \equiv \Delta_\pi(n_t) \simeq \exp\{-n_t m_\pi\}, \quad (9)$$

$$\sum_{n_x,n_y,n_z} P(\vec{n})\{(-1)^{n_x}+(-1)^{n_y}+(-1)^{n_z}\} \equiv \Delta_\rho(n_t) \simeq \exp\{-n_t m_\rho\}. \quad (10)$$

Similar expressions are also valid for the other particles of the theory.

This completes the description of all the basic machinery we used to perform the computation. As a first step we generated a few equilibrium configurations for the pure gauge theory, defined by the 120-element subgroup of SU(2), $\tilde{Y}$ (the covering group of the symmetry group of the iscosahedron): We worked on an $8\times 8\times 8\times 8$ lattice, with periodic boundary condition and the standard Wilson action.[9]

We concentrated our attention to the range $\beta = 2.1-2.4$, where $\beta = 4/g^2$ is the coupling parameter of the gauge theory. This is the region where the asymptotically free behavior of the string tension appears to set in. To relate lattice spacing a to $\tilde{\Lambda}_{\rm mom}$ we use[10]

$$\tilde{\Lambda}_{\rm mom} = (\pi/a)[(6\pi^2/11)(\beta - 1.08]^{51/121} \times \exp\{-(3\pi^2/11)(\beta - 1.08)\}. \quad (11)$$

With $\tilde{\Lambda}_{\rm mom} \simeq 250$ MeV, we find that the size of the box goes from 2.7 to 1.3 fm. The momentum cutoff $C_M \equiv \pi/a$ (i.e., the boundary of the Brillouin zone) ranges from 1.8 to 3.9 GeV. In Ref. 9 a parameter

$$\Lambda_{\rm mom} = \tilde{\Lambda}_{\rm mom}(1 - 1.08/\beta)^{-0.42} \quad (12)$$

was used. $\tilde{\Lambda}_{\rm mom}$ and $\Lambda_{\rm mom}$ are asymptotically equal, but in the β range we are considering they differ by about 30%. With the value of the string tension K determined in Refs. 9 and 11 we obtain $\tilde{\Lambda}_{\rm mom} \simeq 250$ MeV if $\sqrt{K} = 500$ MeV.

We treated both Eq. (4) and (6) implementing the time derivative by a second-order Runge-Kutta algorithm[12]; $\langle\bar{\psi}\psi\rangle$ has been computed with both methods, obtaining compatible results (the values found by use of the Langevin equation have small statistical errors). As a check we have computed $\langle\bar{\psi}\psi\rangle$ as function of m at $\beta = 0$. The results are shown in Fig. 1; the continuous line is the prediction from the limit $N \to \infty$.[7,8] The very good agreement implies that the $1/N^2$ corrections are negligible (as expected) for $N = 2$.

In the whole β range we have explored, we find clear evidence of the fact that $\langle\bar{\psi}\psi\rangle \neq 0$ in the limit $m \to 0$ [the value at $m = 0$ is computed by extrapolating the data obtained with m varying in the range $(0.05-0.5)a^{-1}$]. From renormalization-group arguments we expect

$$\langle\bar{\psi}\psi\rangle_{m=0} \sim \tilde{\Lambda}^3 \alpha_B^{-6/11} \quad [\alpha_B = 1/\pi(\beta - 1.08)] \quad (13)$$

for $\beta \to \infty$. In Fig. 2 we plot $(\frac{3}{2}\langle\bar{\psi}\psi\rangle)^{1/3}$ versus β. The continuous line represents the quantity $R\tilde{\Lambda}(2\alpha_B)^{-2/11}$ with $R = 1.75$. The fit is satisfactory, and we can provisionally assume $R = 1.75 \pm 0.1$.

The pion and ρ masses have been estimated by looking at the large-distance decay of the correlation functions; for the computation of these quantities we used the relaxation method. The number of iterations needed for a good convergence ranges from 50 to 500. The accuracy reached with use of the relaxation procedure can be estimated by checking the validity of the sum rule (3); this is also a consistency check for the algorithm. With a lattice of size 8^4, the largest distance at which correlation functions can be computed is 4; in order to remove finite-size effects from the t direction we have constructed lattices of size $8^3 \times 16$ and $8^3 \times 32$, respectively, by duplicating and quadruplicating the same gauge-field configuration. (This procedure is justified by the short range of the gauge field correlations. Indeed an 8^4 lattice would be adequate if one could obtain the exact spectrum for the propagation of fermions. Iterating the gauge-field configuration in time allows a good determination of the lowest masses in this spectrum, through the rate of decay of the Green functions.) Most of our estimates for the values of the masses (in the β range

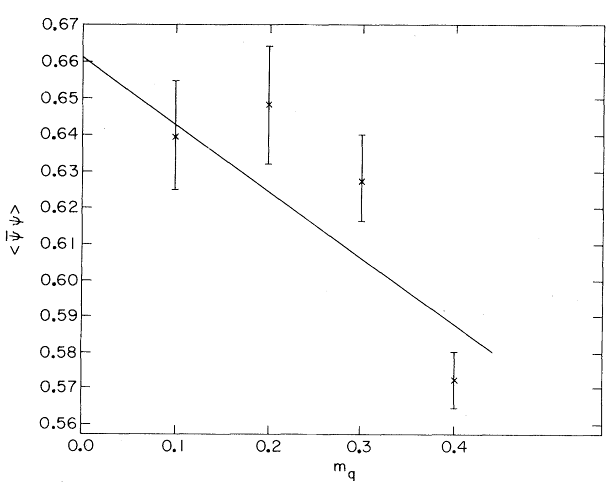

FIG. 1. $\langle\bar{\psi}\psi\rangle$ vs m at $\beta = 0$. The continuous line is the prediction from the limit $N \to \infty$ (Refs. 7 and 8).

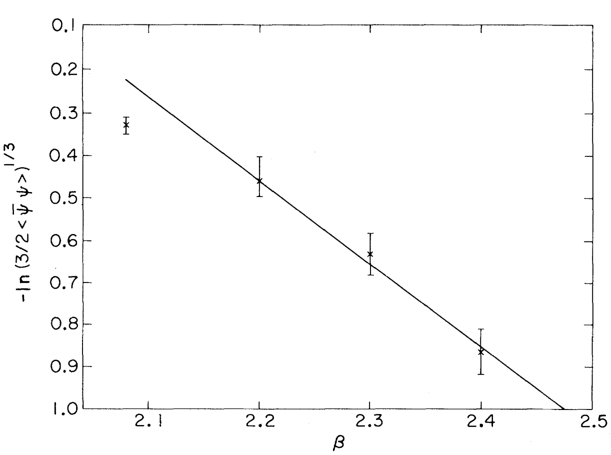

FIG. 2. $(\frac{3}{2}\langle\bar{\psi}\psi\rangle)^{1/3}$ vs β. The continuous line fits $R\Lambda\alpha_B^{-2/11}$ with $R = 1.75$.

we are considering) have been obtained from the $8^3\times 16$ lattice (this could be impossible for higher values of β). So the results from the $8^3\times 32$ lattice were mainly used as a check.

We concentrated our efforts on $\beta = 2.2$, which is already in the scaling region for $\langle\bar\psi\psi\rangle$. We estimated the masses by looking at the rate of exponential decay of the correlation functions. We obtained all our results by averaging over four configurations of the gauge fields. We have a good control of the π correlation functions (i.e., $\bar\psi\gamma_5\psi$) at all distances (see Fig. 3), while our statistical accuracy for the ρ ($\bar\psi\gamma_\mu\psi$) and δ ($\bar\psi\psi$) correlation functions is reasonable up to distances 7 and 5, respectively. In our range of quark masses (0.3 – 0.1 in units of a^{-1}) the data can be fitted by

$$m_\pi^2 = (6.5 \pm 0.1) m_q/a,$$

$$m_\rho^2 = (1.0 \pm 0.1)/a^2 + m_\pi^2,$$

$$m_\delta^2 - m_\rho^2 = (0.4 \pm 0.1)/a^2.$$

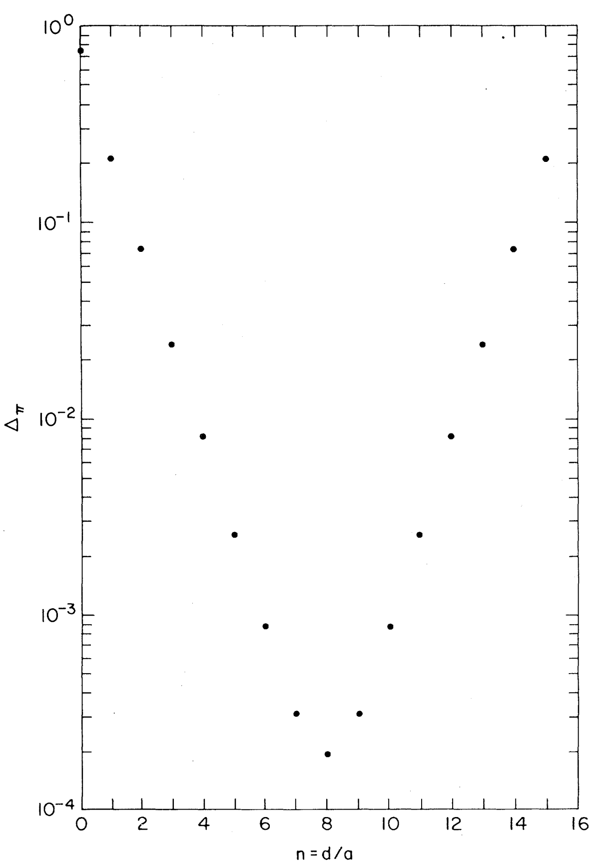

FIG. 3. Correlation function for the π with $m_q = 0.2$ and $\beta = 2.2$

Using the partial conservation of axial-vector current relation $m_\pi^2 f_\pi^2 = \frac{3}{2}\langle\bar\psi\psi\rangle_{SU(2)} m_q$ ($f_\pi^{\rm exp} = 0.95$ MeV), we get $f_\pi = (0.19 \pm 0.01)a^{-1}$. With $K = (500\ \mathrm{MeV})^2$ we finally obtain

$$f_\pi = 150 \pm 10\ \mathrm{MeV},$$

$$m_\rho = 800 \pm 80\ \mathrm{MeV},$$

$$m_\delta = 950 \pm 100\ \mathrm{MeV}.$$

(These errors do not reflect the possible uncertainty in the Monte Carlo determination of the string tension.) $m_I = m_q \alpha^{-6/11}$ is renormalization-group invariant. Its value turns out to be 7 MeV, in agreement with phenomenological estimates.

Data with lower statistics at $\beta = 2.4$ seem to indicate

$$m_\rho \sim 710\ \mathrm{MeV}, \quad f_\pi \sim 120\ \mathrm{MeV}.$$

It is clear that one should extend this computation to smaller lattice spacing to check the reliability of our results. Doing this would not present any problem of principle, and the only difficulty would be the larger amount of computer time required. The central processing unit time needed for the computations we described here can be estimated to be about 100 h of VAX 780 (the equivalent of about 10 h of CDC), which is actually not a lot.

We are at present extending our analysis to include the effects of fermionic loops, following the method of Ref. 1.

We are grateful to all the people around the VAX in Frascati and in Rome, and in particular to M. Ferrer, F. Marzano, and E. Valente for their generous help. One of us (G.P.) is happy to thank H. Quinn for the warm hospitality at the Stanford Linear Accelerator Center, the Mark 3 group for the permission of using its computer, and H. Hamber for clarifying discussions at the Santa Barbara 1981 Summer Meeting.

[1]F. Fucito, E. Marinari, G. Parisi, and C. Rebbi, Nucl. Phys. B180, 369 (1981); F. Fucito and E. Marinari, Nucl. Phys. B190, 266 (1981).

[2]E. Marinari, G. Parisi, and C. Rebbi, Nucl. Phys. B190, 734 (1981).

[3]D. J. Scalapino and R. L. Sugar, Phys. Rev. Lett. 46, 519 (1981); D. Weingarten and D. Petcher, Phys. Lett. 99B, 333 (1981); H. Hamber, Phys. Rev. D 24, 951 (1981); A. Duncan and M. Furman, Columbia University Report No. CU-TP-194 (to be published).

[4]H. Hamber and G. Parisi, preceding Letter [Phys.

Rev. Lett. 47, 1792 (1981)].
[5]T. Banks, S. Raby, L. Susskind, J. Kogut, D. R. T. Jones, P. N. Scharbach, and D. K. Sinclair, Phys. Rev. D 15, 1111 (1977); L. Susskind, Phys. Rev. D 16, 3031 (1977).
[6]B. Svetitsky, S. D. Drell, H. R. Quinn, and M. Weinstein, Phys. Rev. D 22, 490 (1980); M. Weinstein, S. D. Drell, H. R. Quinn, and B. Svetitsky, Phys. Rev. D 22, 1190 (1980).
[7]H. Kluberg-Stern, A. Morel, O. Napoly, and B. Petersson, Centre d'Etudes Nucléaires de Saclay Report No. Dph-T/19, 1981 (to be published).
[8]N. Kawamoto and J. Smit, Institute for Theoretical Physics, University of Amsterdam, Report No. ITFA-81-2, 1981 (to be published), and references therein.
[9]G. Bhanot and C. Rebbi, Nucl. Phys. B180, 469 (1981).
[10]A. Hasenfratz and P. Hasenfratz, Phys. Lett. 93B, 165 (1980).
[11]M. Creutz, Phys. Rev. Lett. 45, 313 (1980).
[12]Periodic boundary conditions have been imposed on the fermionic fields.

Initial-State Interactions and the Drell-Yan Process

Geoffrey T. Bodwin and Stanley J. Brodsky
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

G. Peter Lepage
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 8 September 1981)

It is shown that initial-state interactions violate the usual QCD factorization predictions for massive lepton-pair production in leading twist. The initial-state collisions correct $d\sigma/dQ^2 dx_F$ and also smear the lepton-pair transverse momentum distribution.

PACS numbers: 12.40.Cc, 13.85.Kf

A puzzling aspect of standard factorized-QCD predictions[1] for massive lepton-pair production in hadronic collisions is the absence of any corrections to the cross section due to the effects of initial-state interactions. For example, the elastic and inelastic collisions of a hadron propagating in a nuclear target might be expected to alter profoundly its constituents' transverse and longitudinal momentum distributions as well as their color quantum number correlations. Such initial-state interactions might destroy any simple connection between the Drell-Yan[2] cross sections and the projectile's structure functions as measured in deep-inelastic scattering. It thus seems all the more remarkable that the standard QCD predictions for $d\sigma/dQ^2$ are quite consistent with experiment[3] (up to uncertain normalization factors) including the important feature that lepton-pair production in hadron-nucleus collisions is additive in the nucleon number A at high Q^2.

The standard derivations of QCD factorization for hard inclusive reactions are based on the organization of all collinear divergences into universal factors that can be incorporated into hadronic structure functions, and the demonstration that infrared divergences cancel in the physical cross section. In this paper we are concerned with (s-independent) contributions from initial-state interactions.[4] These come from the region of integration near the fermion poles and are away from the collinear region. Such contributions correspond physically to the usual Glauber singularities, which occur, for example, when a fermion scatters in a target and then propagates nearly on shell over a finite distance before annihilating. Because one is dealing with nearly on-shell scattering-matrix elements one cannot use Ward identities or a choice of gauge to eliminate these contributions.[5]

In order to illustrate the physics of the initial-state interactions, we analyze the process $\pi N \to \mu^+\mu^- X$ to all orders in perturbative QCD. We neglect only terms of higher order in $1/s$. The leading-order gluon exchange contribution to initial-state elastic scattering of the active $\bar{q}$ on a spectator quark in the nucleon is shown in Fig. 1(a). To leading order in s the energy denominator after the gluon exchange has the form $y r_\perp^2 - 2 r_\perp \cdot l_\perp + i\epsilon$, where $r_\perp^2 = s$ and $l_\perp^2/1-y = -t$.

© 1981 The American Physical Society