[2]An incomplete list of references to grand-unified theory predictions on the top mass is as follows: H. Georgi and D. V. Nanopoulos, Phys. Lett. 82B, 392 (1979); K. T. Mahanthappa and M. A. Sher, Phys. Lett. 86B, 294 (1979); R. Barbieri and D. V. Nanopoulos, Phys. Lett. 91B, 369 (1980); S. L. Glashow, Phys. Rev. Lett. 45, 1914 (1980), and Harvard University Report No. HUTP-80/A089 (to be published); G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B181, 287 (1981).

[3]Glashow, Ref. 2.

[4]In such theories $\underline{M}(Q)$ decomposes into two terms: $\underline{M}(Q) = F_1(Q)\underline{G}_1 + F_2(Q)\underline{G}_2$, $F_{1,2}(Q)$ being numbers which are Higgs vacuum expectation values and $\underline{G}_{1,2}$ being Q-independent complex Yukawa coupling matrices. Two examples are: (1) an 0(10) model where a single complex $\underline{10}$ and a $\underline{126}$ of Higgs bosons generate fermion masses, and (2) an E_6 model where fermions become massive by coupling to a $\underline{27}$ and a symmetric $\underline{251}$ of Higgs fields.

[5]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

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[8]V. Barger, W. F. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979); R. E. Shrock, S. B. Treiman, and L-L. Chau Wang, Phys. Rev. Lett. 42, 1589 (1979). Our V (and that of Ref. 7) is the transpose of the matrix defined by these authors.

[9]The adjugate of a matrix A is the transpose of the matrix obtained from A by replacement of each element by its cofactor.

[10]S. Pakvasa, S. F. Tuan, and J. J. Sakurai, University of Hawaii Report No. UH-511-427-80 (unpublished).

[11]The case $\xi_1 = 0 = \xi_2$, corresponding to both $|\xi_1|$ and $|\xi_2|$ being $\ll 1$ in the real world, can be physically disallowed since it is incompatible with the observation of identical orders of magnitude for charged fermion masses within each generation.

Gauge Model of Generation Nonuniversality

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An electroweak gauge model is discussed, where generations are associated with separate gauge groups with different couplings. The observed μ-e universality is the result of a mass-scale inequality, $v_{03} \ll v_{12}$, in much the same way as strong isospin is the result of $m_u, m_d \ll 1$ GeV. However, in contrast to the standard model, it is now possible to have (1) a longer τ lifetime, (2) an observable B^0-$\bar{B}^0$ mixing, and (3) many gauge bosons W_i, Z_i in place of W, Z with $M_{W_i} > M_W$ and $M_{Z_i} > M_Z$.

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The replication of quarks and leptons is now a well-established phenomenon. However, there is still very little understanding as to why each generation is so like another. Certainly, as far as the standard electroweak gauge model[1] is concerned, generation universality is simply put in by hand. As an alternative, we consider in this paper the following approach. We postulate generation nonuniversality as a matter of principle, and then try to identify the observed μ-e universality as the result of an approximate symmetry which is determined by the existence of a mass hierarchy in the model. This is closely analogous to the situation in quantum chromodynamics, where strong isospin invariance is explained by the fact that m_u and m_d are much less than the typical hadronic mass scale of 1 GeV, and has nothing to do with whether or not m_u is equal to m_d. Described below is a successful realization of this approach in terms of an electroweak gauge model for three fermion generations.

We adopt the group $U(1) \otimes [SU(2)]^n$, and assign the ith generation of fermions to $U(1) \otimes SU(2)_i$, where $i = 1, \ldots, n$, as in the standard model for that subgroup. The gauge couplings are g_0 for U(1) and g_i for $SU(2)_i$ with associated vector bosons B and $W_i^{\pm}$, W_i^0. The Higgs bosons are doublets under $U(1) \otimes SU(2)_i$ and self-dual quartets under $SU(2)_j \otimes SU(2)_k$ with vacuum expectation values v_{0i} and v_{jk}. Hence our model has the same structure as that of Barger, Ma, and Whisnant,[2] and differs only in the fermion assignments.[3]

To be specific, we consider here the case of three fermion generations, i.e.,

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L, \tag{1}$$

and

$$\begin{pmatrix}u'\\ d'\end{pmatrix}_L, \begin{pmatrix}c'\\ s'\end{pmatrix}_L, \begin{pmatrix}t'\\ b'\end{pmatrix}_L, \tag{2}$$

where L means left handed, and each generation is coupled to a separate SU(2) with a different gauge coupling. The right-handed states are all singlets coupled only to U(1). The quark states u', d', etc., are not mass eigenstates, but are related to them (u, d, etc.) by unitary transformations:

$$\begin{pmatrix}u'\\ c'\\ t'\end{pmatrix} = U\begin{pmatrix}u\\ c\\ t\end{pmatrix}, \tag{3}$$

and

$$\begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} = D\begin{pmatrix}d\\ s\\ b\end{pmatrix}. \tag{4}$$

The lepton states are assumed to be mass eigenstates for simplicity.[4]

Each generation has its own mass scale according to which Higgs doublet it is coupled to. Assuming that these mass scales are the determining factor[5] in forming the observed mass hierarchy within each fermion charge sector, we then have

$$\nu_{01}^2 \ll \nu_{02}^2 \ll \nu_{03}^2. \tag{5}$$

The Higgs quartet vacuum expectation values ν_{12}, ν_{13}, and ν_{23} are not involved in the quark and lepton mass matrices, and are presumably at least as large as ν_{03} since they must correspond to heavier objects such as the W's and Z's in this model. Consider now the phenomenology of electroweak interactions at low energies. The electromagnetic coupling is given by

$$\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2} + \frac{1}{g_3^2}, \tag{6}$$

as expected, and the Fermi weak coupling G_F as defined by μ decay is given by[6]

$$\frac{4G_F}{\sqrt{2}} = \frac{1}{\nu_{03}^2} + \frac{\nu_{12}^2}{\nu_{12}^2(\nu_{13}^2+\nu_{23}^2)+\nu_{13}^2\nu_{23}^2}. \tag{7}$$

More generally, let

$$\left(\frac{4G_F}{\sqrt{2}}\right)_{ij} \equiv \tfrac{1}{2}g_i g_j \langle W_i^+ W_j^-\rangle = \tfrac{1}{2}g_i g_j (M_w^{-2})_{ij}, \tag{8}$$

then

$$\left(\frac{4G_F}{\sqrt{2}}\right)_{11} = \left(\frac{4G_F}{\sqrt{2}}\right)_{12}\left\{1+\frac{\nu_{03}^2\nu_{23}^2}{(\nu_{12}^2+\nu_{03}^2)\nu_{23}^2+\nu_{13}^2(\nu_{12}^2+\nu_{23}^2)}\right\}, \tag{9}$$

$$\left(\frac{4G_F}{\sqrt{2}}\right)_{22} = \left(\frac{4G_F}{\sqrt{2}}\right)_{12}\left\{1+\frac{\nu_{03}^2\nu_{13}^2}{(\nu_{12}^2+\nu_{03}^2)\nu_{13}^2+\nu_{23}^2(\nu_{12}^2+\nu_{13}^2)}\right\}, \tag{10}$$

and

$$\left(\frac{4G_F}{\sqrt{2}}\right)_{13} = \left(\frac{4G_F}{\sqrt{2}}\right)_{23} = \left(\frac{4G_F}{\sqrt{2}}\right)_{33} = \left(\frac{4G_F}{\sqrt{2}}\right)_{12}\left\{1-\frac{\nu_{03}^2\nu_{12}^2}{(\nu_{03}^2+\nu_{13}^2+\nu_{23}^2)\nu_{12}^2+\nu_{13}^2\nu_{23}^2}\right\}, \tag{11}$$

where $(4G_F/\sqrt{2})_{12}$ is of course $4G_F/\sqrt{2}$ as given by Eq. (7). To accommodate the observed μ-e universality, we must have $(G_F)_{11} = (G_F)_{22} = (G_F)_{12}$ as well as $(G_F)_{13} = (G_F)_{23}$. The latter condition is automatic in this model, whereas the former is equivalent to the statement

$$\nu_{03}^2 \ll \nu_{12}^2. \tag{12}$$

Note that Eq. (12) is independent of the gauge couplings which can be all different in this model. The analogy with $m_u \neq m_d$ for strong isospin in quantum chromodynamics is striking indeed.

Since Eq. (12) by itself does not imply that $(G_F)_{13}$ is equal to $(G_F)_{11}$, the weak-interaction strength of the third generation can be different from that of the first two. From Eq. (11), it is clear that $(G_F)_{13} < G_F$. An immediate consequence is that the lifetime of the τ lepton must be longer than is predicted by the standard model. The enhancement factor is simply given by ξ^2, where

$$\xi \equiv \frac{G_F}{(G_F)_{13}} = 1 + \frac{\nu_{03}^2\nu_{12}^2}{(\nu_{13}^2+\nu_{23}^2)\nu_{12}^2+\nu_{13}^2\nu_{23}^2}. \tag{13}$$

If $\nu_{03}^2 \ll \nu_{13}^2+\nu_{23}^2$, then $\xi = 1$ and we get back the standard model. In fact, ξ differs significantly from unity only if ν_{03}^2 and $\nu_{13}^2+\nu_{23}^2$ are comparable in magnitude; hence we will assume from now on that

$$\nu_{13}^2 + \nu_{23}^2 \ll \nu_{12}^2 \tag{14}$$

for simplicity. As a result,

$$\xi = 1 + \frac{\nu_{03}^2}{\nu_{13}^2 + \nu_{23}^2} . \qquad (15)$$

Recently, the first nonzero measurement of the τ lifetime has been made with the Mark II detector at PEP at the Stanford Linear Accelerator Center, and it has been reported[7] that

$$t_\tau = (4.9 \pm 1.8) \times 10^{-13} \text{ sec}, \qquad (16)$$

to be compared with the standard theoretical prediction of $(2.8 \pm 0.2) \times 10^{-13}$ sec. This implies for our model

$$\xi < 1.6. \qquad (17)$$

Consider now weak neutral-current interactions. First, we have in this model the identity[6]

$$\langle g_i W_i^0 - g_0 B, g_j W_j^0 - g_0 B \rangle = g_i g_j \langle W_i^+ W_j^- \rangle, \qquad (18)$$

which ensures the equality of neutral-current to charged-current effective strengths in a natural way. However, because of mixing in the quark sector, the effective neutral-current parameters for $\nu_\mu N$ and $\bar{\nu}_\mu N$ interactions become[8]

$$\begin{aligned} \epsilon_L^u &= \tfrac{1}{2}[1 - (1 - \xi^{-1}) |U_{31}|^2] - \tfrac{2}{3} \sin^2\varphi_W, \\ \epsilon_R^u &= -\tfrac{2}{3} \sin^2\varphi_W, \\ \epsilon_L^d &= -\tfrac{1}{2}[1 - (1 - \xi^{-1}) |D_{31}|^2] + \tfrac{1}{3} \sin^2\varphi_W, \\ \epsilon_R^d &= \tfrac{1}{3} \sin^2\varphi_W, \end{aligned} \qquad (19)$$

where

$$\sin^2\varphi_W = \left(1 - \frac{e^2}{g_0^2}\right) - \frac{e^2}{g_3^2}\left(1 - \frac{1}{\xi}\right). \qquad (20)$$

Nevertheless, the quantities $|U_{31}|^2$ and $|D_{31}|^2$ are expected to be much less than 1, so that Eq. (19) is hardly any different from what we get in the standard model.

For $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, the effective interaction is[2]

$$(4G_F/\sqrt{2})\{(j^{(3)} - \sin^2\varphi_W j^{\text{em}})^2 + C(j^{\text{em}})^2\}, \qquad (21)$$

where

$$C = \frac{e^4}{g_3^4}\left(1 - \frac{1}{\xi}\right)\frac{1}{\xi}. \qquad (22)$$

Using Eqs. (20) and (22), we then obtain the constraint

$$\xi > 1 + C/\sin^4\varphi_W. \qquad (23)$$

Given that $\xi < 1.6$ from Eq. (17), and $\sin^2\varphi_W = 0.23$ from present data, we get $C < 0.03$ from Eq. (23), which result is remarkably the same as that given by recent PETRA data.[9] For $e^+e^- \to \tau^+\tau^-$, our model gives

$$(8G_F/\sqrt{2})\{\xi^{-1} j_e^{(3)} j_\tau^{(3)} - \xi^{-1}[\sin^2\varphi_W + (\xi - 1)^{1/2} C] j_e^{\text{em}} j_\tau^{(3)} - \sin^2\varphi_W j_e^{(3)} j_\tau^{\text{em}} + (\sin^4\varphi_W + C) j_e^{\text{em}} j_\tau^{\text{em}}\}, \qquad (24)$$

but it may be difficult to use for extracting ξ and C from the scant data presently available.

In our model, since each generation is coupled to different gauge bosons, the Glashow-Iliopoulos-Maiani mechanism[1] is not in operation, and we expect first-order flavor-changing neutral currents. The effective $K_L \to \mu^+\mu^-$ amplitude is given by

$$\frac{G_F}{\sqrt{2}} D_{31}^* D_{32}\left(1 - \frac{1}{\xi}\right)\frac{f_K p_K^\alpha}{\sqrt{2}} \bar{\mu}\gamma_\alpha\gamma_5\mu, \qquad (25)$$

which, when combined with data,[10] yields the inequality

$$|D_{31}^* D_{32}|(1 - \xi^{-1}) \lesssim 10^{-5}. \qquad (26)$$

Similarly, from the K_L-K_S mass difference, we get

$$|D_{31}^* D_{32}|^2(1 - \xi^{-1}) \lesssim 10^{-8}. \qquad (27)$$

From the above two equations, it is clear that a natural hierarchy exists for flavor-changing neutral currents, and it is determined by mixing angles. Since we expect the hierarchy $|D_{31}| \ll |D_{32}| \ll |D_{33}| \simeq 1$, the strong suppression of the rare kaon processes indicated by Eqs. (26) and (27) are not difficult to understand, even if ξ is as large as 1.6, as allowed by Eq. (17). For the same reason, D^0-$\bar{D}^0$ mixing is expected to be negligible because $|U_{31}^* U_{32}|^2$ is involved. However, for the neutral B mesons, the relevant factors are $|D_{31} D_{33}^*|^2$ and $|D_{32} D_{33}^*|^2$, which are expected to be significant. Hence B_s^0-$\bar{B}_s^0$ mixing is likely to be maximal, and B_d^0-$\bar{B}_d^0$ mixing should be observable if ξ differs significantly from 1. Direct flavor-changing neutral-current decays such as $b \to s\mu^+\mu^-$ are also possible. Using $\xi < 1.6$, we find

$$\frac{\Gamma(b \to s\mu^+\mu^-)}{\Gamma(b \to c\mu^-\bar{\nu}_\mu)} \lesssim 0.035 \frac{|D_{32}|^2}{|D_{23} + \xi^{-1} U_{32}^*|^2}, \qquad (28)$$

which is consistent with the recently obtained[11] experimental bound of 0.08. As more data become available from the Cornell University electron-positron storage ring, these ideas can be readily tested.

We now mention briefly the consequences of our model with regard to the vector gauge bosons. Since each generation is coupled to a different set of $W^{\pm}$ and W^0, there will be three sets of W and Z bosons. At least one set will be very heavy, corresponding to the requirement of μ-e universality, i.e., Eq. (12). Of the remaining two sets, if we take $\xi < 1.6$, $C < 0.03$, and $\sin^2\varphi_W = 0.23$, then

$$1 < M_{W_1}/M_W < 1.005, \tag{29}$$

$$1 < M_{Z_1}/M_Z < 1.106, \tag{30}$$

and

$$M_{W_2} \simeq M_{Z_2} \gtrsim 10 M_W. \tag{31}$$

In the above, M_W and M_Z are the standard-model mass values, i.e., 80 and 93 GeV, respectively. The condition that $M_{W_i} > M_W$ and $M_{Z_i} > M_Z$ is a general, but not unique,[12] feature of our model. Furthermore, it is possible to have $C = 0$, in which case $M_{W_1} = M_W$ and $M_{Z_1} = M_Z$, without requiring ξ to be 1.

In conclusion, we have put forward in this paper a radical, if not heretical, point of view that both the observed μ-e universality and the known suppression of flavor-changing neutral-current kaon processes are in fact accidents, in much the same way that strong isospin is an accident. We thus predict a hierarchy of generations, in analogy with strong SU(2), SU(3), SU(4), etc., in which each succeeding generation breaks the universality of weak interactions more and more.[13]

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[3]The immediate price to pay for this unconventional approach is the existence of first-order flavor-changing neutral currents. However, the suppression of $K_L \to \mu^+\mu^-$ and the K_L-K_S mass difference is still obtained in our model for very reasonable values of the mixing angles, as shown in the text.

[4]It is obviously straightforward to incorporate mixing also in the lepton sector, but present data do not require it, as they do for quarks.

[5]This is consistent with the observation that mass ratios within each fermion charge sector are very approximately universal for all sectors. In fact, an often proposed prediction for m_t is $m_t/m_c \approx m_b/m_s$. See, for example, S. Pakvasa and H. Sugawara, Phys. Lett. 82B, 105 (1979).

[6]For details of the calculational technique, see, for example, V. Barger, W. Y. Keung, and E. Ma, Phys. Rev. D 22, 727 (1980), and Phys. Lett. 94B, 377 (1980).

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[13]This idea must have occurred to many people in the past, and may even have been put down in writing. However, our model is the first concrete realization of such a possibility.