will be required to unpin them.

Note that if nonthermal mobile vacancies (ground-state vacancies) exist in ^{3}He, they should have the same effect as thermal ones in unpinning the dislocations. As no evidence of this was observed, the concentration of mobile nonthermal vacancies must be much less than the concentration of thermal vacancies at the lowest temperature at which thermally activated unpinning was observed. For the bcc crystal of Fig. 1, thermal unpinning was observed at 0.55 K, implying that the concentration of mobile nonthermal vacancies was less than about 10^{-10}.

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Tricritical Behavior in $KMnF_3$

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The cubic-to-tetragonal structural phase transition of $KMnF_3$ has been investigated by measurement of the phase diagrams with uniaxial pressure along [110] and [100]. Results indicate the existence of a tricritical point near $p = 0.45$ kbar with p along [110], and two new consecutive tricritical points near $p = 0.25$ kbar with p along [100].

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Among perovskites displaying a structural phase transition from the cubic O_h^1 to the tetragonal D_{4h}^{18} space group, $SrTiO_3$ shows second-order behavior, while the phase transitions in $KMnF_3$ and $RbCaF_3$ are slightly first order.[1] According to classical Landau theory, all of these transitions should be second order, but the $KMnF_3$ and $RbCaF_3$ renormalization-group (RG) analysis explains[2] the first-order character of the transition by lack of an accessible stable fixed point. The phase transition is said to be driven first order by critical fluctuations. The transition of $RbCaF_3$ has been found to go second order[3] through a tricritical point (TCP) when uniaxial pressure is applied in a direction close to [100]. The TCP is believed to be of Lifshitz type[4] as a result of the strongly cubic character of the order-parameter fluctuations. The same TCP is expected to be reached[3] for p along [110].

We have determined the phase diagrams of $KMnF_3$ with uniaxial pressure along [110] and [100] by means of specific-heat measurements. The measured hysteresis, ΔT_c, of the phase transition temperature and the shape of the phase diagram indicate the existence of a TCP near $p = 0.45$ kbar with p along [110]. With p along [100] we found two consecutive TCP's joined by a second-order line near $p = 0.25$ kbar. Outside a temperature region of 0.5 K to both sides of T_c, the measured specific-heat curves were all analyzed by classical Landau theory. We find that the variation of the Landau parameters with p is correlated with the variation of ΔT_c. To our knowledge these measurements constitute the first observation of two consecutive TCP's in a structural phase diagram.

In analyzing the measured specific-heat curves we apply Landau theory where the free energy is expanded to the sixth power of the order parameter Q:

$$F_L = \tfrac{1}{2}a(T - T_0)Q^2 + \tfrac{1}{4}bQ^4 + \tfrac{1}{6}cQ^6. \quad (1)$$

a, b, and c are assumed to depend only weakly on temperature and pressure. The Landau excess specific heat is given by[5]

$$\Delta C_L = \begin{cases} AT(T_d - T)^{-1/2} & \text{for } T < T_c{}^L, \\ 0 \text{ for } T > T_c{}^L, \end{cases} \quad (2)$$

where

$$A = \frac{a^{3/2}}{4c^{1/2}}, \quad (3)$$

$$T_d = T_0 + b^2/4ac, \quad (4)$$

$$T_c{}^L = \begin{cases} T_0 \text{ if } b > 0, \\ T_0 + \dfrac{3b^2}{16ac} \text{ if } b < 0. \end{cases} \quad (5)$$

For $b > 0$ ($b < 0$) we have a second- (first-) order transition. For $b = 0$ we have a Landau tricritical point, where $T_d = T_0$. In our case $b > 0$ and $T_c{}^L = T_0$. The value of the observed phase transition temperature T_c, however, must be expected to be influenced by critical fluctuations which are important close to T_c. As mentioned, the fluctuations cause the transition to go first order at $p = 0$, and thermal hysteresis is observed. But we believe T_0 to be close to the mean value of T_c on cooling and heating. This is the meaning we will assign to T_0 in the following.

Our apparatus for ac specific-heat measurements under uniaxial pressure is fully automated. It has been described in detail elsewhere.[5,6] We present measurements on three samples. Two of these were used for p along [110] and [100], and have the shape of rectangular plates with the dimensions $5.7 \times 1.6 \times 0.75$ mm^3 and $4.7 \times 2.1 \times 0.70$ mm^3, respectively. The third sample, a 0.3-mm thick slab, was used for measurements at $p = 0$.

All $C_p(T)$ curves were measured with an average sweep rate of 0.15 K/h, changing the temperature stepwise by 12 mK each fifth minute. The new pressure was always put on at $T \approx T_c + 1.7$ K, cooling down to $T \approx T_c - 1.7$ K during 24 hours, and heating back to the starting point during the next 24 hours.

The measured phase diagrams, starting at $p = 0$, are shown in Figs. 1 and 2 for p along [110] and [100], respectively, showing $T_c(p)$ for both heating and cooling runs. T_c is usually taken as

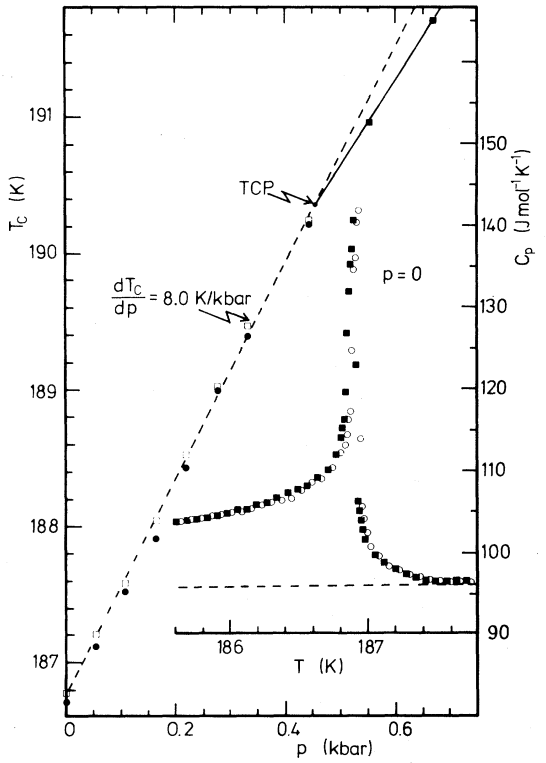

FIG. 1. Phase diagram of $KMnF_3$ for p along [110] showing $T_c(p)$ both for cooling (solid circles) and heating (open squares) runs. The inset shows specific-heat curves at $p = 0$, for both cooling (solid squares) and heating (open circles) runs.

the temperature for the maximum of the $C_p(T)$ curve. We see that for p along [110], the hysteresis, ΔT_c, of T_c is zero within the uncertainty of 12 mK for $p = 0.56$ kbar and $p = 0.67$ kbar. The coefficient dT_c/dp is also changed to a lower value above $p = 0.45$ kbar. This is the same behavior as was observed[3] in $RbCaF_3$ for p in a direction close to [100] but with a component along [010]. We therefore believe that we have a Lifshitz TCP for $p \approx 0.46$ kbar applied along [110].

For p along [100] the behavior is quite different. As we follow curve 1 of Fig. 2, the hysteresis first increases as a function of pressure. At $p = 0$ we have $\Delta T_c = 51 \pm 12$ mK and at $p = 83$ bar, $\Delta T_c = 390 \pm 12$ mK. At higher pressures ΔT_c falls off to 23 ± 12 mK at $p = 219$ bar and 12 ± 12 mK at $p = 262$ bar. At still higher pressure ΔT_c increases once more. This behavior indicates two consecutive TCP's joined by a second-order line close to $p = 0.25$ kbar.

Independent of our measurements Blankschtein and Aharony[7] have made an RG analysis which is relevant for our measurements. They mainly

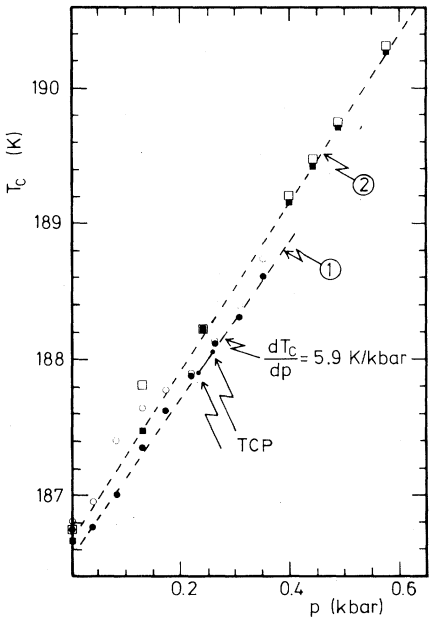

FIG. 2. Phase diagram of $KMnF_3$ for p along [100] showing $T_c(p)$ for both cooling (solid circles, solid squares) and heating (open circles, open squares) runs. The difference between curve 1 (solid circles, open circles) and curve 2 (solid squres, open squares) is explained in text.

studied the influence of quadratic anisotropies on phase transitions which are first order from Landau theory but are driven second order by fluctuations. They also found that in systems which exhibit fluctuation-driven first-order transitions at $p=0$, increasing anisotropic stress may yield two consecutive TCP's. Our results are consistent with their analysis.

In the inset of Fig. 1 is shown specific-heat curves on cooling and heating for $p=0$. Above T_c all C_p curves, also for $p>0$, fall quickly to C_l while ΔC is quite high over an extended range below T_c. This excludes the possibility that ΔC is dominated by critical fluctuations, since these would give rise to nearly symmetric C_p curves. For a Heisenberg system the amplitude of ΔC is expected to be about a factor of 1.5 higher above T_c than below.[8] Fitting our data above T_c in the inset of Fig. 1 to a power law gives an exponent $\alpha = 1.25 \pm 0.15$, which is far from the Heisenberg value $\alpha = -0.1$. The main excess specific heat above T_c must therefore be expected to be caused by defects. We have observed a similar behavior in SbSI.[6]

All C_p curves below T_c have been analyzed according to Eq. (2). From the measured C_p values

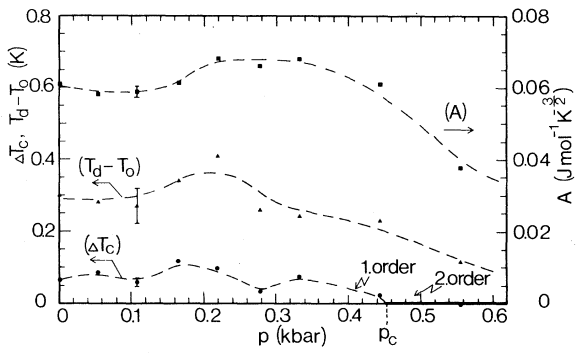

FIG. 3. ΔT_c (solid circles), A (solid squares), and $T_d - T_0$ (solid triangles) for $KMnF_3$ as a function of pressure along [110]. ΔT_c is the hysteresis of T_c. A and $T_d - T_0$ are found by fitting the Landau formula of Eq. (2) to the measurements. p_c is the pressure of the TCP marked in Fig. 1.

we subtracted the lattice specific heat C_l as extrapolated from above T_c (see inset of Fig. 1). C_l was measured for $p=0$ from room temperature to T_c. It falls nicely on a straight line. Figures 3 and 4 show the resulting parameters A and $T_d - T_0$ as a function of p along [110] and [100], respectively, together with ΔT_c. The solid symbols in Fig. 4 correspond to curve 1 of Fig. 2 and open symbols to curve 2. Both curves were measured on the same sample but with p better aligned along [100] for curve 1 as a re-

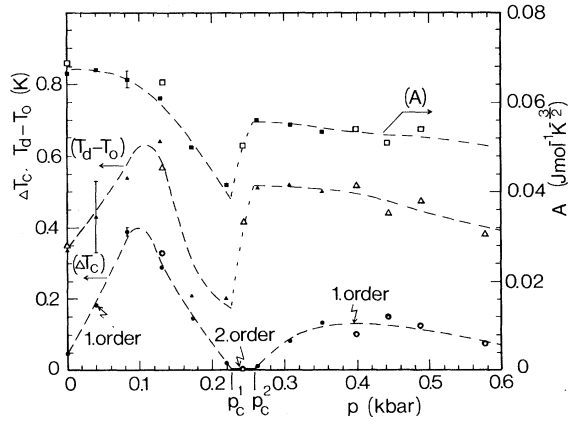

FIG. 4. ΔT_c (solid circles, open circles), A (solid squares, open squares), and $T_d - T_0$ (solid triangles, open triangles) for $KMnF_3$ as a function of pressure along [100]. ΔT_c is the hysteresis of T_c. A and $T_d - T_0$ are found by fitting the Landau formula of Eq. (2) to the measurements. p_c^1 and p_c^2 are the pressures of the two TCP's marked in Fig. 2. Filled symbols correspond to curve 1 in Fig. 2 and open symbols to curve 2.

sult of a better distribution of the pressure across the end surfaces of the sample. A higher component of p along [010] for curve 2 explains the increased dT_c/dp coefficient, since dT_c/dp is higher for p along [110]. Curve 1 was measured first, starting at $p=0$. Then the points above $p=0.35$ kbar on curve 2 were taken. Then, having kept the sample at room temperature at $p=0$ for several days, we measured the points below $p=0.3$ kbar on curve 2. For the sake of clarity, the quantities A, $T_d - T_0$, and ΔT_c, corresponding to points on curve 2 above $p=0.20$ kbar, were adjusted to correct for a shift caused by the imperfect pressure distribution.[9]

We see that both A and $T_d - T_0$ fall off in the second-order region of Fig. 3 and just below the TCP's in Fig. 4, which supports our conclusions that the system has different properties here. This change of Landau parameters, we believe, is connected to an increased influence from critical fluctuations, which must be expected to be larger in a second-order region and close to a TCP. Also, the fact that dT_c/dp is higher for p along [110] than for p along [100] indicates influence of fluctuations. According to Landau theory, it should be the same in these two cases.[10] Note that $T_d - T_0 = 0$ at a Landau tricritical point. Our measurements, which give $T_d - T_0 > 0$ even close to the TCP's, confirm that the change of order of the transition is caused by critical fluctuations and is not a result of pressure-dependent Landau parameters. Landau theory explains the specific-heat curve shape outside the temperature range of 0.5 K around T_c. Close to T_c, critical fluctuations are important for the C_p curves, and they determine the order of the phase transition and influence the shape of the phase diagrams and the values of the Landau parameters.

In conclusion, our measurements of $T_c(p)$ have led us to believe that $KMnF_3$ for p along [110] has a TCP near $p=0.45$ kbar, $T=109.4$ K, and two new consecutive TCP's joined by a second-order line near $p=0.25$ kbar and $T=188.0$ K for p along [100]. The specific-heat curves fit Landau theory with parameters showing a change with pressure which is correlated with the change of the order of the transitions. More details will be published elsewhere.[9]

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Droplet Theory of Low-Dimensional Ising Models

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A droplet theory of Ising systems is constructed, valid near space dimension $d = 1$, and based on a model incorporating only the configurational energy of surface tension. The droplet distribution function is calculated explicitly, with use of renormalization-group methods to control the droplet-shape fluctuation effects essential to its scaling form. Universal quantities are calculated; those (e.g., β) which reflect the droplet concentration have an essential singularity in $\epsilon = d - 1$.

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Droplet models have been studied for many years with a view to obtaining a theoretical description of a two-phase (Ising-like) thermodynamic system.[1] Two problems have attracted particular attention. On the one hand, the existence of large droplets of one phase embedded

