Measurement of Pair Emission from the 2.8-MeV Parity-Mixed Doublet of ^{21}Ne

A. B. McDonald and E. D. Earle
Atomic Energy of Canada Limited, Chalk River Nuclear Laboratories, Chalk River, Ontario K0J 1J0, Canada

and

J. J. Simpson
University of Guelph, Guelph, Ontario N1G 2W1, Canada

and

R. G. H. Robertson
Michigan State University, East Lansing, Michigan 48824

and

H. B. Mak
Queen's University, Kingston, Ontario K7L 3N6, Canada
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The branching ratio for pair emission for the ground-state transition from the $J^\pi = \frac{1}{2}^-$, 2789-keV level of ^{21}Ne has been measured relative to the branching ratio for the transition from the $J^\pi = \frac{1}{2}^+$, 2796-keV level. The result indicates that the former transition is predominantly $E1$ in multipolarity [$\delta(M2/E1) < 0.6$] and therefore that measurements of γ-ray circular polarization for this transition are very sensitive to parity mixing in the spin-$\frac{1}{2}$ doublet.

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A parity-nonconserving measurement[1] of the circular polarization of 2789-keV γ rays from the closely spaced opposite-parity doublet in ^{21}Ne provided a low upper limit [$P_\gamma = (2.4 \pm 2.9)\times 10^{-3}$] compared with early predictions[2] of this polarization which were as large as 5×10^{-2}. Recent theoretical calculations[3] with large basis shell models indicate that a smaller circular polarization may arise from a cancellation between isoscalar parity-mixing matrix elements and isovector matrix elements which are enhanced by the presence of weak neutral currents in the Glashow-Weinberg-Salam model.[4] All calculations to date have assumed that the observed transition from the 2789-keV, $J^\pi = \frac{1}{2}^-$ level to the ground state is a nearly pure $E1$ transition so that the circular polarization is a very sensitive measure of parity mixing. However, this $E1$ transition is extremely weak (8×10^{-8} Weisskopf units) and difficult to calculate,[3,5] as is the competing $M2$ transition. Therefore the measurement of the multipolarity of this transition is essential for the unambiguous interpretation of the circular-polarization results.

The present paper describes a measurement of the internal-pair-emission branching ratio of the 2789-keV transition. This branching ratio can be used to define the $M2/E1$ mixing ratio which can then be combined with previous measurements of lifetimes and branching ratios to define accurately the sensitivity of circular-polarization measurements to the parity-mixing matrix elements between the 2789-keV, $J^\pi = \frac{1}{2}^-$ and 2796-keV, $J^\pi = \frac{1}{2}^+$ levels of ^{21}Ne.

The low-lying energy levels of ^{21}Ne, together with information on lifetimes, branching ratios, and mixing ratios are illustrated in Fig. 1. For the ground-state transitions from the spin-$\frac{1}{2}$ levels, the determination of the multipolarity is dif-

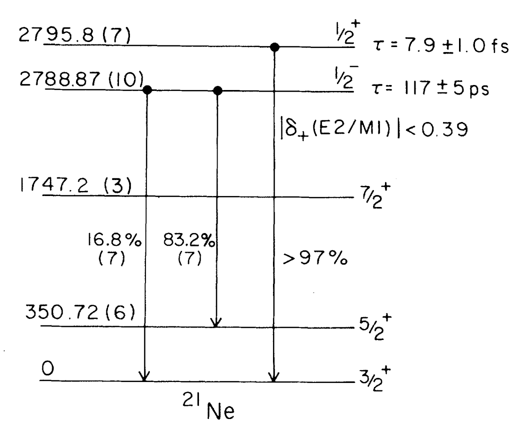

FIG. 1. Low-lying energy levels of ^{21}Ne. Information on lifetimes and energies are as summarized in Refs. 5 and 6. The upper limit for δ_+ is obtained from the measured lifetime and the extreme assumption $B(E2) = 30$ Weisskopf units (see Ref. 7).

ficult. The technique suggested by Millener *et al.*[5] of measuring the circular polarization of γ rays from a polarized initial state was attempted during previous parity-nonconserving measurements[1] but was unsuccessful because of small polarization transfer in the reaction $^{21}\mathrm{Ne}(p_{\mathrm{pol}},p')$.

The internal pair emission process for these transitions, although small in intensity ($\Gamma_{\mathrm{pair}}/\Gamma_\gamma \lesssim 10^{-3}$), is directly dependent upon the mixing ratios and the relative emission angle (θ) of the electron-positron pairs.[8] The electron-positron angular correlation and the absolute pair-emission probabilities are similar for $E2$ or $M1$ transitions so that the present upper limit on δ_+, the $E2/M1$ mixing ratio of the $\frac{1}{2}^+$ level (see Fig. 1), is sufficient to define the pair-emission probability for $\theta = 90°$ to better than 7%. On the other hand, for the $\frac{1}{2}^-$ level the pair-emission probability ($\theta = 90°$) is 6 times larger for $\delta_-(M2/E1) = 0$ than for $\delta_- = \infty$. Therefore the present experiment sought to determine the relative pair-emission probability ($\theta = 90°$) for the two spin-$\frac{1}{2}$ levels as a sensitive measure of δ_-.

The measurement of relative pair-emission rates avoided significant experimental difficulties associated with uncertainties in electron and positron scattering and detection efficiencies for solid-state detectors. The necessity of resolving the pair transitions from the closely spaced levels was avoided by populating the levels via the reaction $^{18}\mathrm{O}(\alpha,n)^{21}\mathrm{Ne}$ and observing the sum of the 2789- and 2796-keV transition rates at three incident energies where the relative populations of the two levels were significantly different. These populations were monitored by simultaneously observing the γ rays from these transitions in a Ge(Li) detector. The variation of pair-emission rate with level population was then used to extract the relative pair-emission rates for the two levels.

A 250-μg/cm^2-thick target of $\mathrm{W}^{18}\mathrm{O}_3$ (>90% enrichment) on a 125-μm-thick carbon foil backing (GRAFOIL) was bombarded by a 1-μA α-particle beam from the Chalk River MP tandem accelerator. Electrons and positrons were detected in a pair of silicon detector telescopes, each subtending about 2% of 4π at angles of $\pm 135°$ to the beam direction. Each telescope consisted of a 300-μm-thick surface-barrier detector followed by a 3000-μm-thick lithium-drifted silicon detector at room temperature. The use of telescope systems reduced the counting rate from γ-ray background by a factor of 10 in each system, so that random coincidence counting rates were less than 10% of the rate for real electron-positron coincidences. The energy resolution for each telescope system was about 50 keV full width at half maximum for 662-keV electrons from ^{137}Cs and the detection efficiency varied slowly with energy above a cutoff imposed at 480 keV.

Related-address data for energies and times of coincidences between the two telescope systems were accumulated at incident energies of 4.48, 4.85, and 4.95 MeV. At all energies a clear peak was observed at 1.77 MeV in the sum of the energies deposited in the two telescopes (see Fig. 2). In addition, a continuum was observed at lower energies as expected for events where the electron or positron lost energy by scattering in the detector or surroundings. The area of the 1.77-MeV peak was extracted from each spectrum after subtraction of a background interpolated linearly between the continuum levels on each side of the peak. For coincidence events falling in the peak, the distribution of counts versus electron or positron energy was relatively flat as expected for the energy range $0.48 < E < 1.29$ MeV.

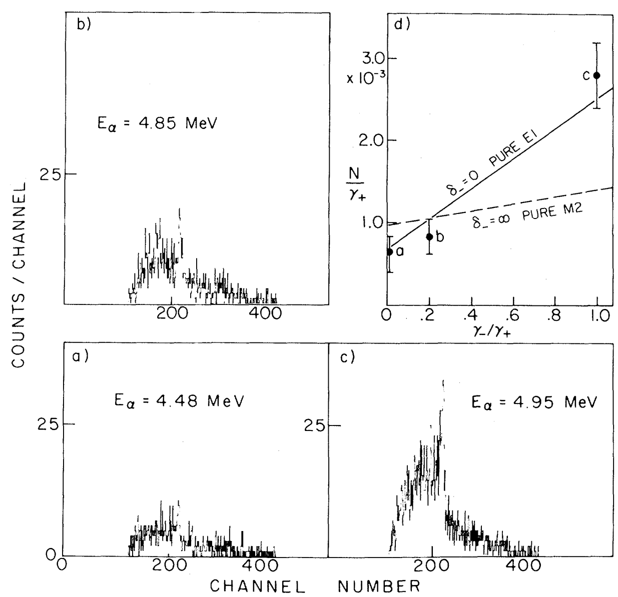

FIG. 2. (a)–(c) Spectra showing number of coincidences vs the sum of the energy deposited in the two telescopes. The peak at 1.77 MeV (channel 216) arises from pair emission from the $J^\pi = \frac{1}{2}^+$ and $\frac{1}{2}^-$ levels. (d) Number of counts in the 1.77-MeV peak in spectra (a) to (c) plotted to correspond to Eq. (2). The straight lines are the expected results for pure $E1$ and pure $M2$ transitions from the $\frac{1}{2}^-$ level.

The number of pairs (N) detected at each incident energy can be expressed as follows:

$$N = K[\alpha_+\gamma_+ + \alpha_-\gamma_-], \quad (1)$$

where $\alpha_{+(-)}$ is the pair-emission probability ($\theta = 90°$) for the $J^\pi = \frac{1}{2}^{+(-)}$ level, $\gamma_{+(-)}$ is the number of γ rays detected in the Ge(Li) detector from the $\frac{1}{2}^{+(-)}$ level, and K is a constant involving the γ-ray and charged-particle detection efficiencies, which are assumed to be similar for each of the ground-state transitions from the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ levels. Therefore,

$$N/\gamma_+ = K[\alpha_+ + \alpha_-(\gamma_-/\gamma_+)], \quad (2)$$

and if one plots N/γ_+ vs γ_-/γ_+ the ratio of slope to intercept is α_-/α_+. Such a plot is shown in Fig. 2 for the data obtained at the three incident energies. A weighted least-squares fit to the data implies $\alpha_-/\alpha_+ = 4.6 \pm 2.3$ with a χ^2 of 0.49.

The theoretical expressions of Rose[8] for electron and positron energies within the accepted region 0.48 MeV $< E <$ 1.29 MeV were averaged over detector solid angles to predict $\alpha_- = 8.44 \times 10^{-7}$ ($E1$) or 1.32×10^{-7} ($M2$) and $\alpha_+ = 2.80 \times 10^{-7}$ ($M1$) or 4.20×10^{-7} ($E2$). For a light nucleus like neon and a transition energy as high as 2.8 MeV, the first Born approximation is sufficiently accurate for these calculations.[9] For the same reason, we should not expect penetration matrix elements to influence the conclusions regarding multipolarity.[10]

For a mixed transition

$$\alpha_- = [\alpha(E1) + \alpha(M2)\delta_-^2]/(1 + \delta_-^2),$$

with a similar expression for α_+. With $|\delta_+(E2/M1)| < 0.39$ (see Fig. 1), the allowed range of α_+ is $2.80 \times 10^{-7} < \alpha_+ < 2.96 \times 10^{-7}$. The best fit to the data for $\delta_- = 0$ (pure $E1$) has $\chi^2 = 1.1$, and for $\delta_- = \infty$ (pure $M2$) has $\chi^2 = 16$ (see Fig. 2). For one standard deviation, the data imply $|\delta_-| < 0.6$, in agreement with the "best" theoretical result of Millener *et al.*,[5] $\delta_- = 0.13$.

The circular polarization of 2789-keV γ rays emitted by the unpolarized $\frac{1}{2}^-$ level may be expressed[5] as

$$|P(2789)| = \left| \frac{2\langle \frac{1}{2}^- | H^w | \frac{1}{2}^+ \rangle}{\Delta E} \frac{\langle || M1 || \rangle}{\langle || E1 || \rangle} \left(\frac{1 + \delta_+\delta_-}{1 + \delta_-^2} \right) \right|,$$

where ΔE is the energy separation of the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ levels, $\langle \frac{1}{2}^- | H^w | \frac{1}{2}^+ \rangle$ is the matrix element for the parity-nonconserving weak Hamiltonian, and $\langle || M1 || \rangle$ and $\langle || E1 || \rangle$ are the reduced matrix elements for $M1$ and $E1$ transitions to the ground state from the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ levels, respectively. Expressing $P(2789)$ in terms of partial lifetimes for the ground-state transitions, we have

$$|P(2789)| = \left| \frac{2\langle \frac{1}{2}^- | H^w | \frac{1}{2}^+ \rangle}{\Delta E} \left(\frac{\tau_-}{\tau_+} \right)^{1/2} \times \frac{1 + \delta_+\delta_-}{(1 + \delta_-^2)^{1/2}} \frac{1}{(1 + \delta_+^2)^{1/2}} \right|.$$

With the present measurement of δ_-, and current knowledge of energies, lifetimes, and branching ratios of the levels,[5,6] $P(2789)$ is calculated to be

$$|P(2789)| = |[8.6^{+1.1}_{-2.0} \times 10^{-2}\ \text{eV}^{-1}] \langle \tfrac{1}{2}^+ | H^w | \tfrac{1}{2}^- \rangle|. \quad (3)$$

This result confirms the previous assumptions that $P(2789)$ is very sensitive to parity mixing between the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ levels. The parity-mixing matrix element derived from Eq. (3) and the measurements[1] of $P(2789)$ is

$$\langle \tfrac{1}{2}^- | H^w | \tfrac{1}{2}^+ \rangle = 0.028 \pm 0.034\ \text{eV},$$

almost an order of magnitude smaller than has been measured[11] for ^{19}F ($\langle \frac{1}{2}^- | H^w | \frac{1}{2}^+ \rangle = 0.47 \pm 0.14$ eV).

The relationship between these matrix elements and the basic weak interaction has recently been calculated using a large basis shell model[3] and a meson exchange model[4] for the weak nucleon-nucleon interaction. These calculations indicate that the experimental results are consistent with theory, using values of the meson-nucleon-nucleon coupling constants derived[4] from the Glashow-Weinberg-Salam model of the basic weak interaction with reasonable renormalization corrections.

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Higher-Order Binding Corrections to the Lamb Shift

Jonathan Sapirstein
Department of Physics, University of California, Los Angeles, California 90024
(Received 4 September 1981)

A new method of calculating the one-loop self-energy contribution to the Lamb shift is presented. The technique relies on treating as perturbations certain terms in the equation for the Dirac Coulomb Green's function, in the absence of which the equation can be solved in terms of a simple integral representation. A new result for the 1S Lamb shift is obtained and compared with previous calculations.

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The classic high-precision tests of QED, the electron anomaly and the Lamb shift, have calculational difficulties of very different character.[1] The anomaly calculation involves the relevant external field, a constant magnetic field, only to first order, with energy shifts induced by repeated actions of the external field being totally negligible for laboratory magnetic fields. The difficulty of this calculation arises in treating the numerous complicated graphs associated with higher-order loops. On the other hand, the external field in the case of the Lamb shift, the Coulomb field of the nucleus, cannot be taken to act only once, or even a finite number of times, but must be taken into account to all orders even in the one-loop self-energy term. An attempt to expand in powers of the external field leads to a false expansion, in which the explicit powers of $Z\alpha$ from n Coulomb interactions, $(Z\alpha)^{2n+2}$, are compensated when the loop integration is performed by $(Z\alpha)^{-2(n-1)}$, making all terms $O((Z\alpha)^4)$.[2] This means that the full Green's function for the electron in an external Coulomb field must be used. Thus, even the lowest-order Lamb-shift calculation involves evaluating the Bethe logarithm, which is obtained as an integral over the nonrelativistic Coulomb Green's function. In the exact relativistic calculation, the Dirac Coulomb Green's function must be used, and unlike free-particle propagators, no closed form is known for this expression, although a form involving an infinite sum over partial waves does exist.[3] The lack of a convenient form for this Green's function is the essential difficulty of the Lamb-shift calculation. The main aim of this paper is to circumvent this difficulty by exploiting the fact that a simple integral representation can be given for the Dirac Coulomb Green's function if two terms, both of which are small if $Z\alpha \ll 1$, are treated as perturbations.

The usual approach to the evaluation of the 1S Lamb shift is to expand in powers of $Z\alpha$, the strength of the Coulomb potential, forming the series

$$\Delta E = \frac{m\alpha(Z\alpha)^4}{\pi}\{A_{40}+A_{41}\ln(Z\alpha)^{-2}+(Z\alpha)A_{50}+(Z\alpha)^2[A_{62}\ln^2(Z\alpha)^{-2}+A_{61}\ln(Z\alpha)^{-2}+G(Z\alpha)]\}, \tag{1a}$$

$$G(Z\alpha)=A_{60}+O(Z\alpha). \tag{1b}$$

The constant A_{40} includes the Bethe logarithm, and thus involves the full nonrelativistic Coulomb Green's function in its evaluation. However, of the remaining terms that have been directly evaluated, namely A_{41}, A_{50}, A_{62}, and A_{61}, the full Green's function is not needed, a fact that can be traced to the nonanalyticity of their coefficients in $(Z\alpha)^2$.[4] All such terms come from the action of three or fewer

